Numeriske beregninger af betonkonstruktioner; Automatisk kontrol af duktilitet og stabilitetsberegning af revnede skiver

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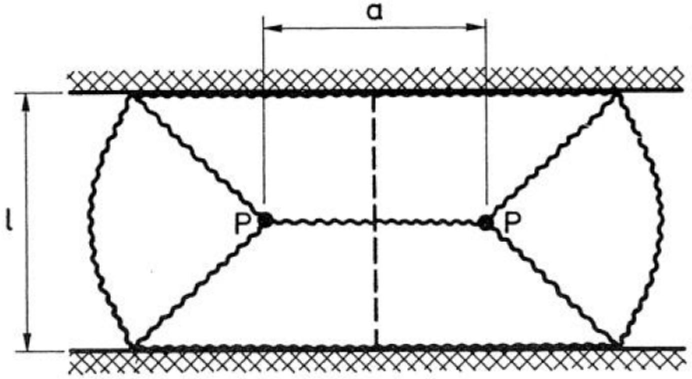
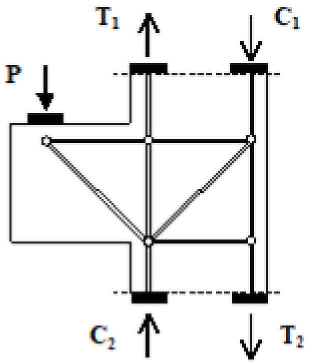
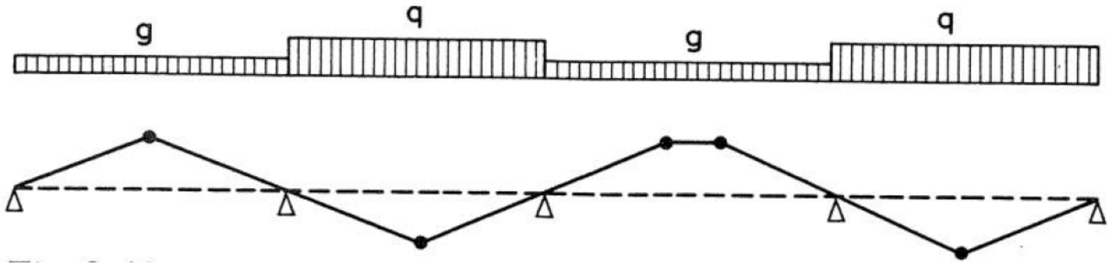
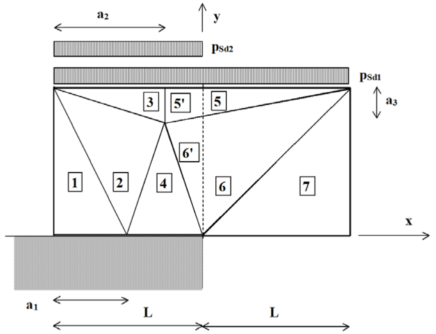
The theory of plasticity has been used in Denmark for more than 100 years for calculating the load bearing capacity of concrete structures.

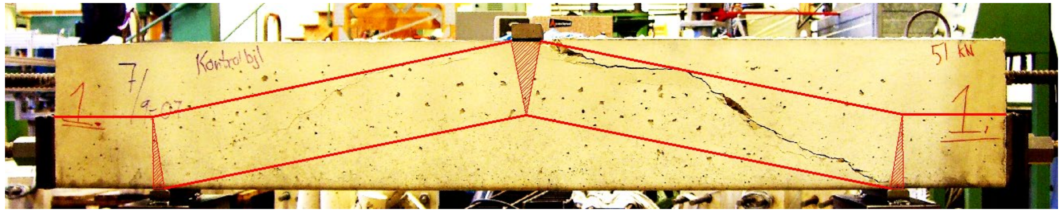
Rules for using the theory of plasticity have been connected to simple structures where hand calculations have been used.

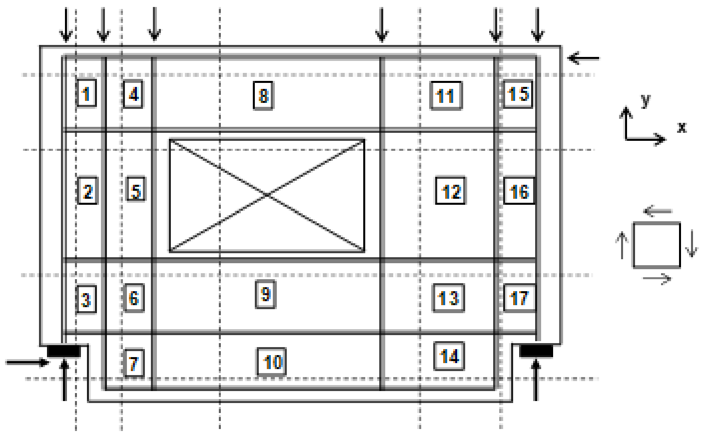




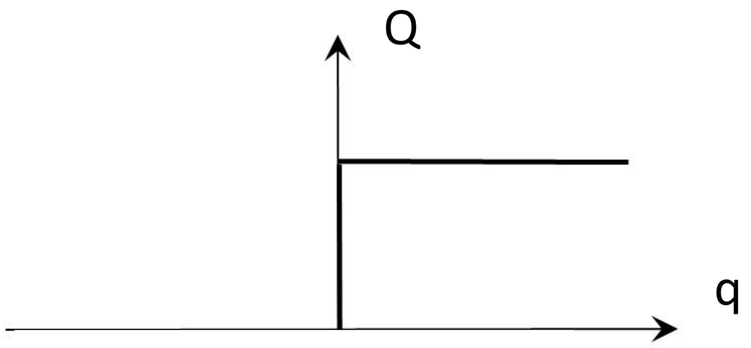




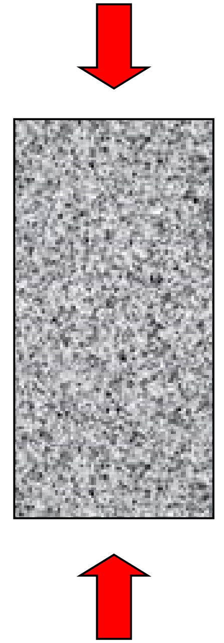


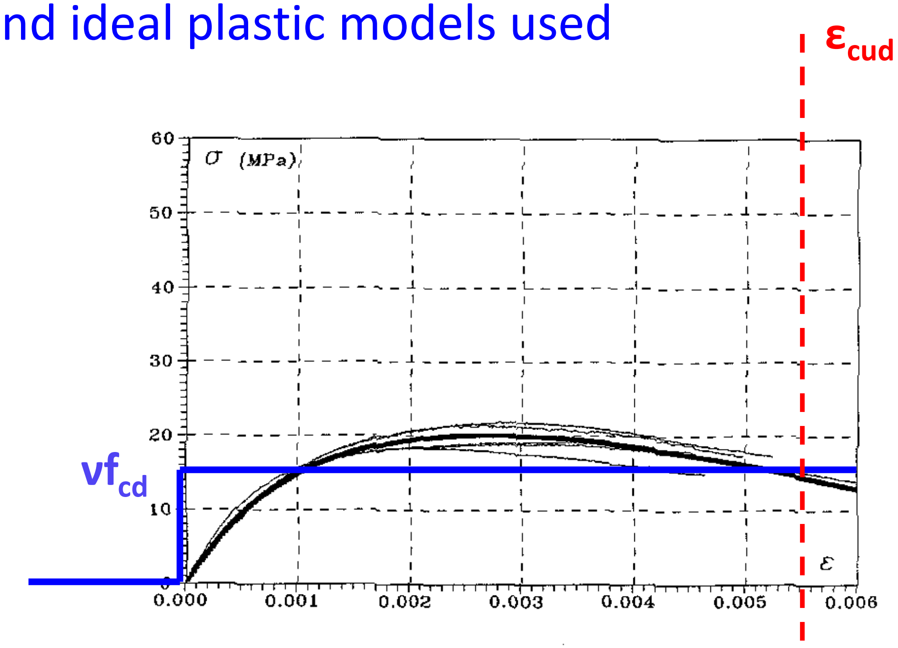


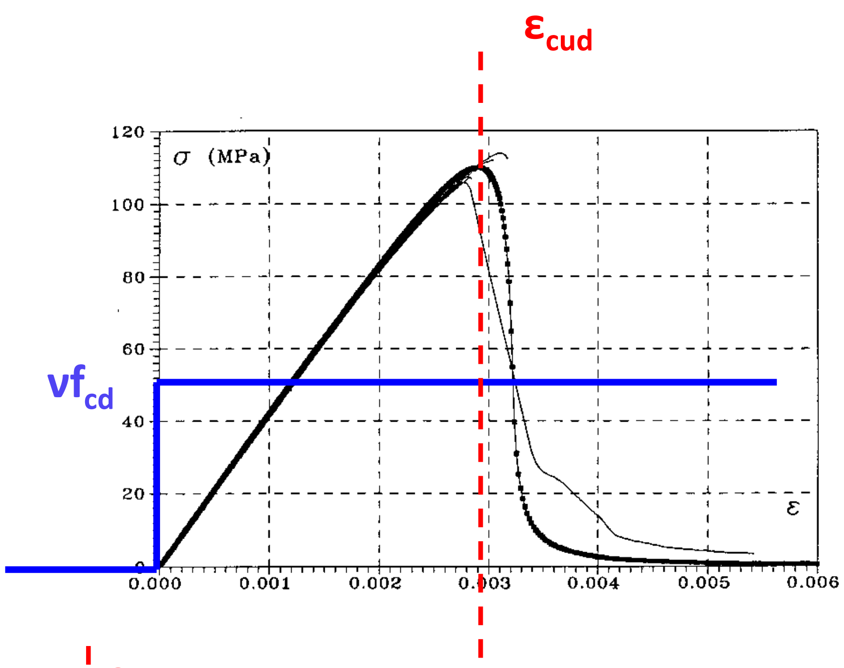
Use of theory of plasticity, and thus use of redistribution of forces and stresses in a concrete structure, require the materials to behave in a ductile manner.

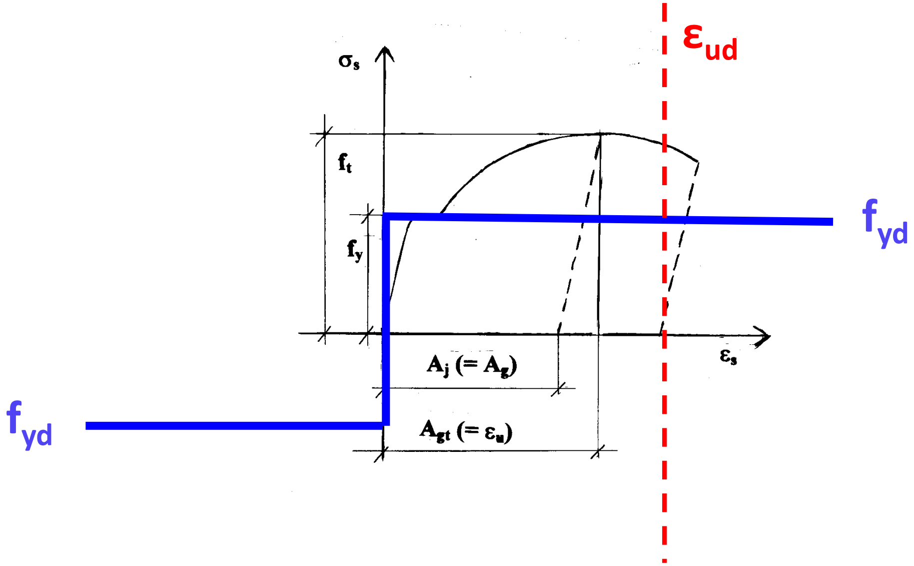


Real stress-strain curves and ideal plastic models used





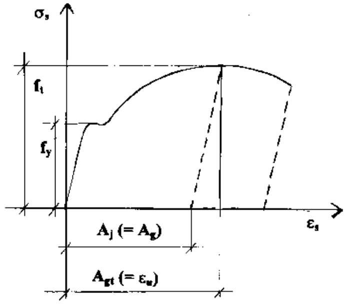


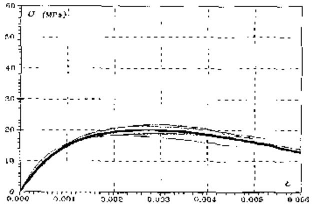


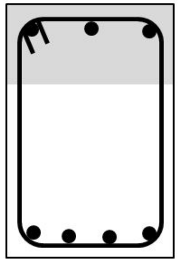
Rules in codes, In general

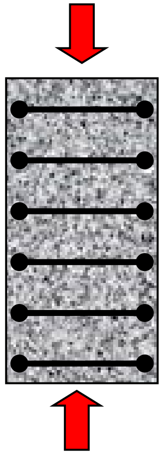
The codes for concrete structures contain basic rules to secure ductility of reinforced concrete structures:

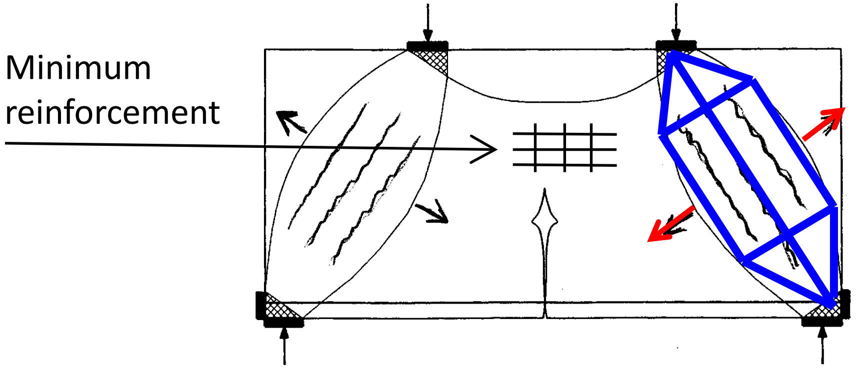
- Ductility of materials (as an example Class B and Class C)
- Detailing rules for structural members, as an example securing confinement (see as an example chapter 9 in DS/EN 1992-1-1)
- Minimum reinforcement preventing brittle tensile failures.











Rules in codes, In general

- The determination of internal forces may be based on the theory of plasticity using the generally acknowledged approximations.

  Adoption of the theory of plasticity presupposes that the structure has adequate ductility, i.e. yielding in the reinforcement will develop to a sufficient extent before other failure modes such as instability intervene in a progressing, ductile failure. When applying the theory of plasticity, verification of sufficient yield capacity can be omitted if the following conditions are fulfilled.

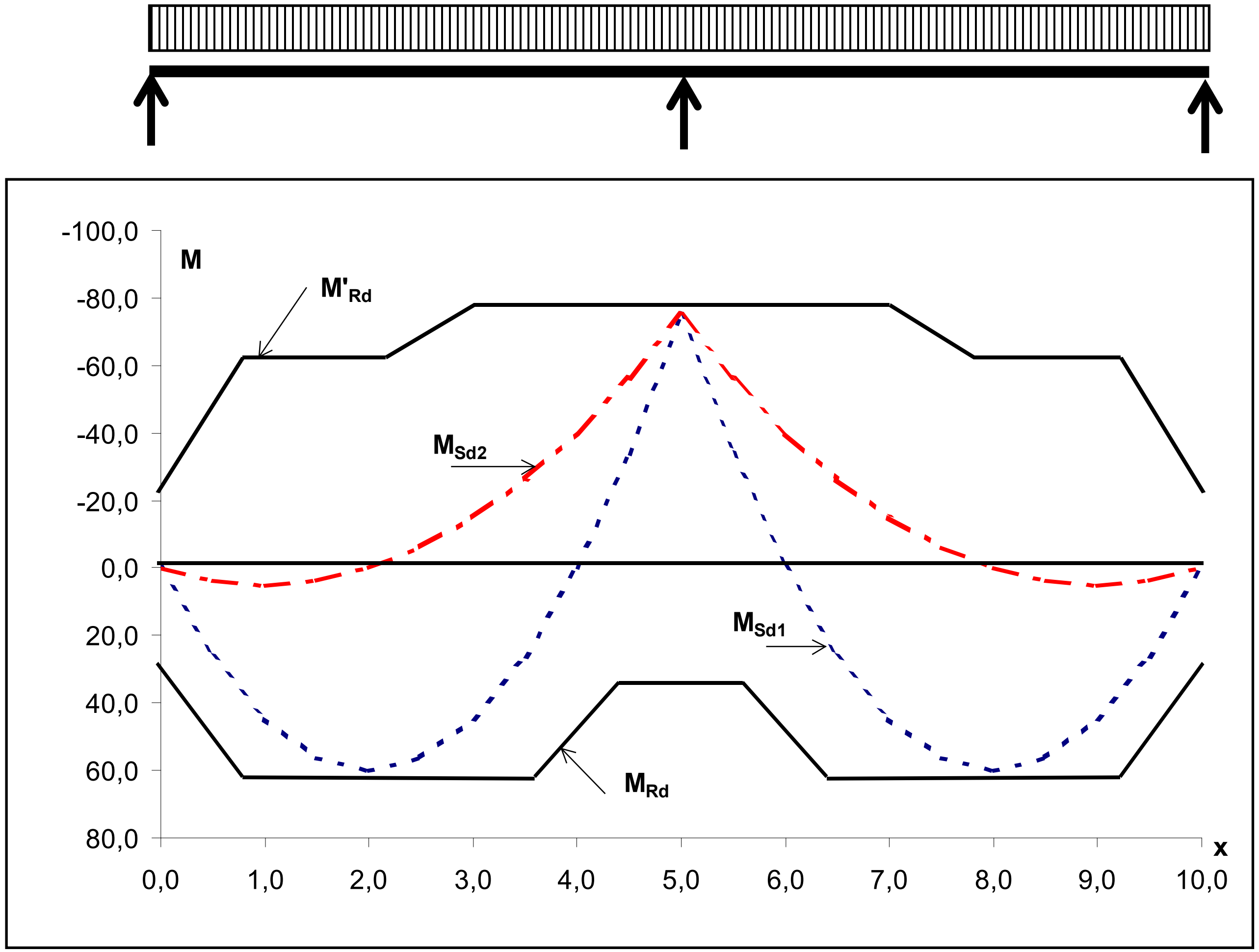
Rules in codes, In general

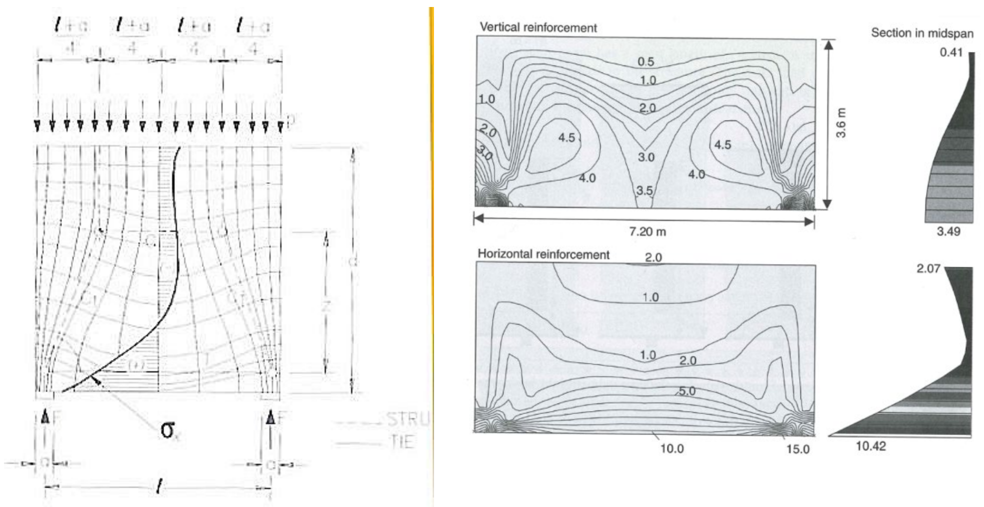
- The distribution of internal forces does not deviate strongly from that corresponding to the theory of elasticity. An accurate calculation of the distribution of internal forces corresponding to the theory of elasticity is not required. It will normally be adequate to apply a qualified estimate or simple approximation methods.

  For lower-bound solutions, the following principle may be used: Where the reinforcement area associated with plastic design at any point of the structure is denoted A_{sP} and the reinforcement area associated with the elastic solution at the same point of the structure is denoted A_{sE}, the above may be assumed to be fulfilled if $1/3\ A_{sE} \leq A_{sP} \leq 3\ A_{sE}$ for all points of the structure. The elastic solution may be assumed to correspond to the plastic solution where the overall design reinforcement for the structure is a minimum.

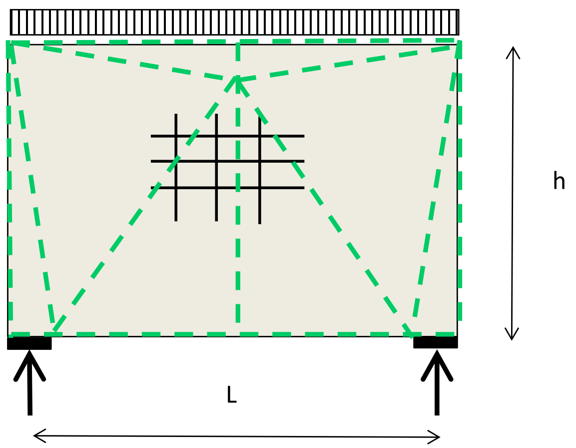
Rules in codes, Continuous beams and slabs

Restraining moments are chosen between the values found by the theory of elasticity and one third thereof. For continuous beams and slabs of approximately equal spans and uniformly distributed loads, verification of the position of the restraining moments in relation to the theory of elasticity may be omitted if at restraints and intermediate supports reinforcement is applied for restraining moments which are taken numerically as not less than 1/3 and not more than twice the maximum design moments in adjacent spans.





Elastic solution. The solution does not take account of reinforcement layout and cracked section. Solution requires reinforcement for highest tension stresses. Reinforcement reduction demands complicated curtailment of the reinforcement.

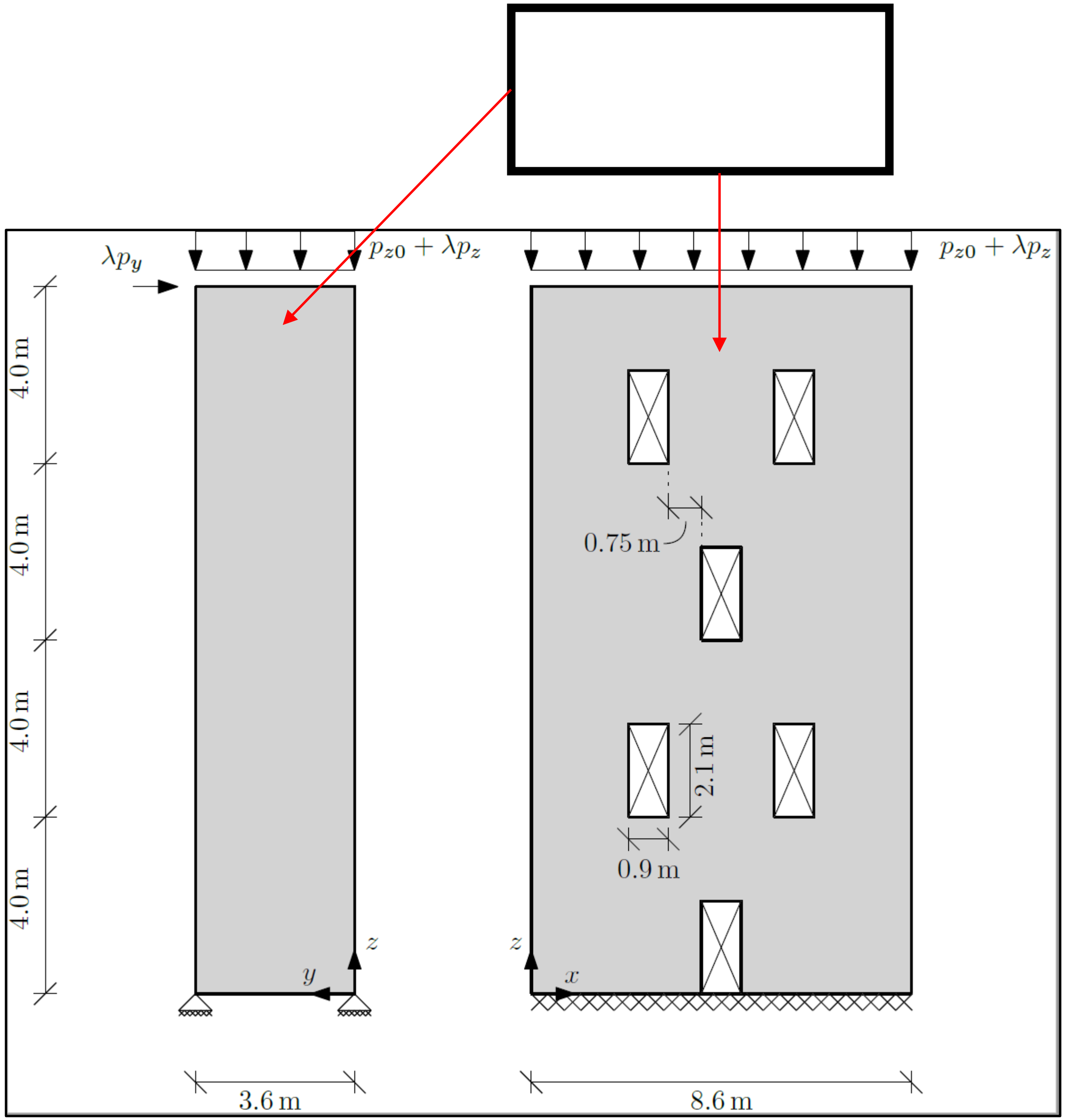


Plastic solution. The solution results in a homogeneous mesh reinforcement giving a simple and suitable layout of the reinforcement. Reinforcement is used optimally by internal redistribution.

Use of numerical calculation methods

The load bearing capacity is calculated according to the theory of plasticity.

How do we check that the calculated load bearing capacity for the structure shown in the figure is correct, i.e. does the structure have the necessary ductility?

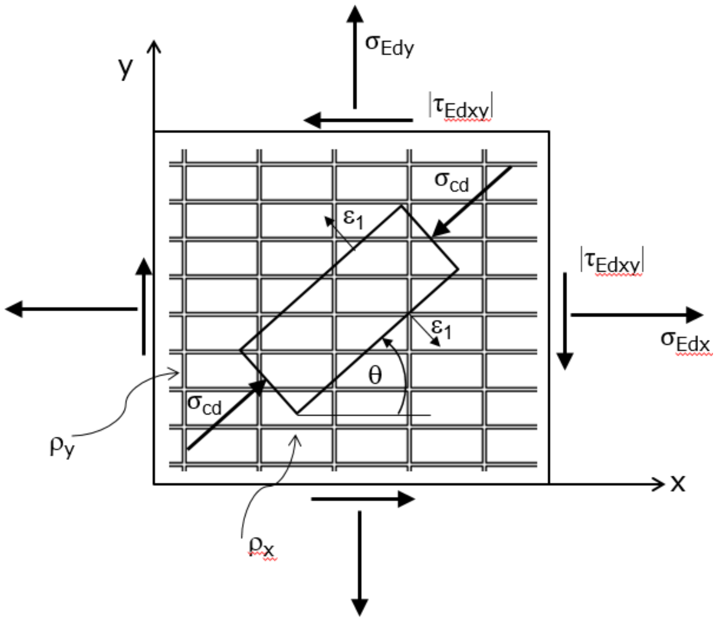


New Eurocode for concrete, Effectiveness factor, orthogonal reinforcement, plain stress field

$$\nu = \eta_{f_c} \eta_{\varepsilon}$$

$$\eta_{f_c} = \left(\frac{30}{f_c} \right)^{1/3} \leq 1 \; ; \; f_c \text{ in MPa}$$ Effect of f_c (brittleness of concrete)

$$\eta_{\varepsilon} = \frac{1}{k_1 + k_2 \varepsilon_1} \leq 1,0$$ Effect of transverse stress/strain

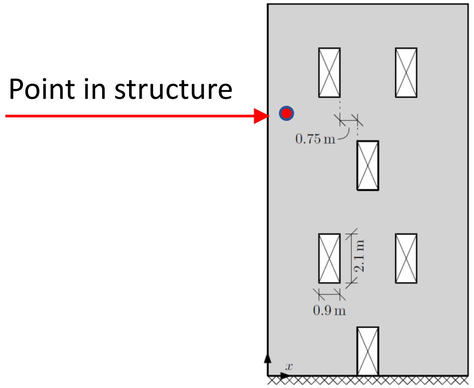


where:

- k_1 and k_2 are constants to be calibrated with tests. If no better information are available, k_1 = 1 and k_2 = 100 may be used.

- ε_1 is the principal strain transverse to the direction of the compression field and determined by accounting for strain compatibility in the member, which is assumed fully cracked.

General check of ductility



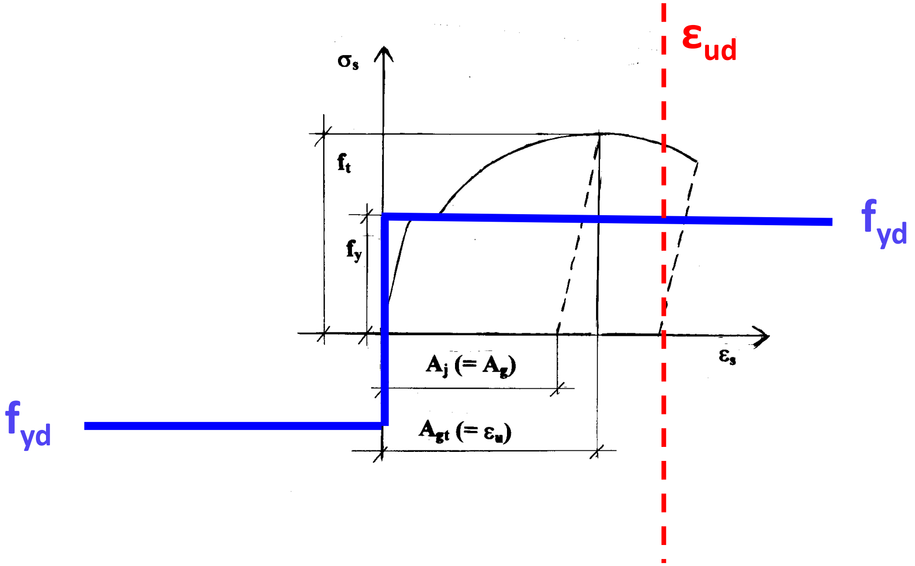
For all points in a concrete construction, the following conditions must be met:

$\sigma_c \leq \nu f_{cd}$

The value of ν depends on the load level - transverse strain -, therefore νf_{cd} reflects the real load bearing capacity of the concrete at the current point for the current load.

$\varepsilon_s \leq \varepsilon_{ud}$

where ε_{ud} is the design limit strain in the reinforcement.



Fulfilment of the specified requirements ensures the ductility of the reinforced concrete structure.

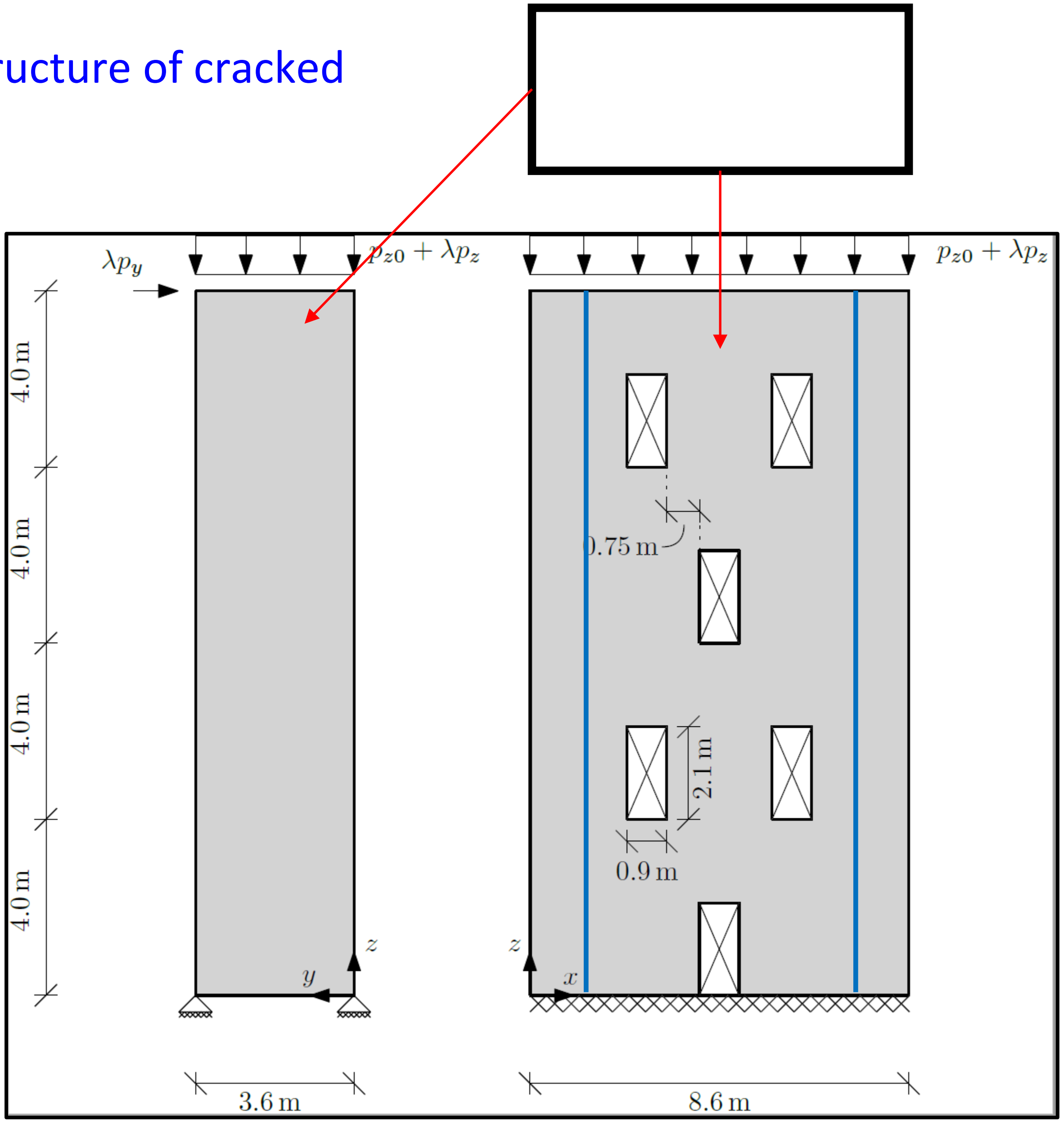
What is the stability capacity of a complex wall structure of cracked reinforced concrete, with holes, cross walls etc.?

How do we calculate the stability capacity of the concrete structure shown in the figure?

Today, simplified principles are used with the insertion of simple columns in the wall structure. This is conservative as it does not take into account:

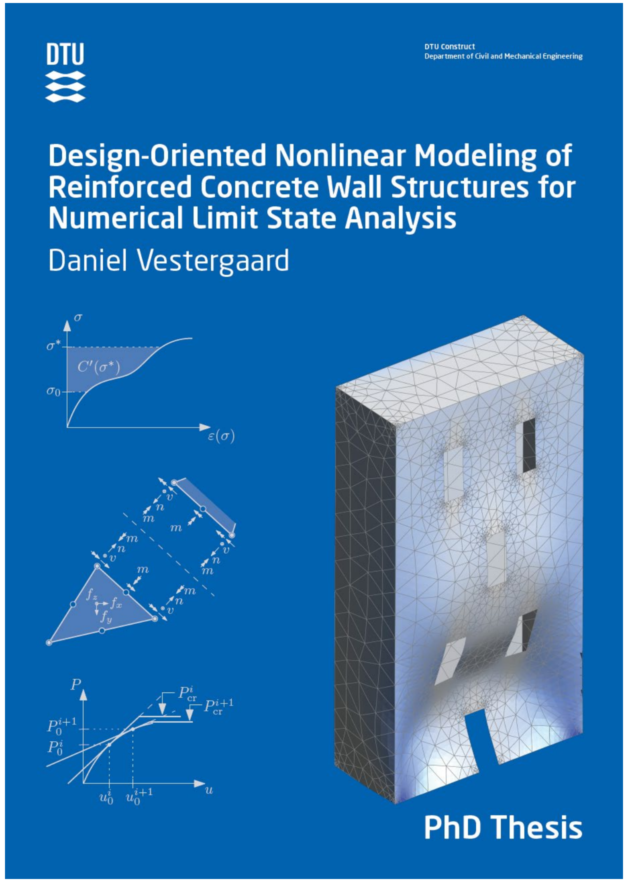
- Support by transverse walls
- stiffness between holes
- importance of the reinforcement lay-out in the wall

and what about fire?



Agenda

- Model principles
  - Plasticity & ductility
  - Stability
  - Fire
- Examples
- Concluding remarks



Direct approach

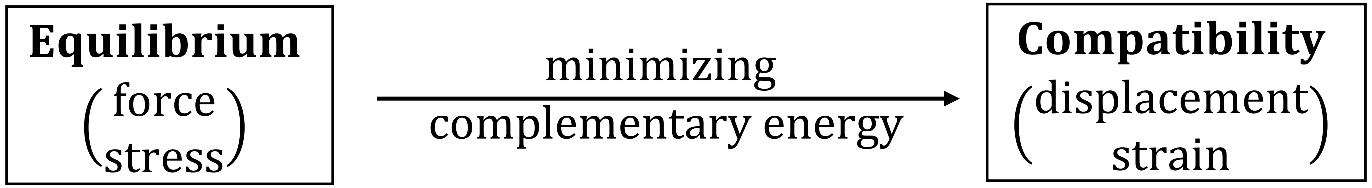
- Traditional approach:
  - Equilibrium (stresses $\leftrightarrow$ loads)
  - Compatibility (strains $\leftrightarrow$ displacements)
  - Constitutive law (stresses $\leftrightarrow$ strains)

$$\mathbf{K}(\mathbf{u})\mathbf{u} = \mathbf{r}$$

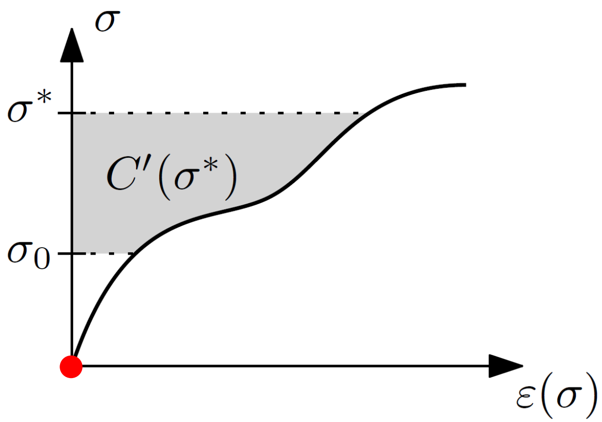
- Complicated except for linear elasticity
  - Reinforced concrete cracks and yields
  - Requires incremental load-stepping

Indirect approach: Principle of minimum complementary energy

- Replace explicit compatibility condition with minimum principle:



- Nonlinear elasticity (positive stiffness)
  - OK for cracked response to static load cases
- No need for explicit stress-strain relation



Minimizing complementary energy using convex optimization

Rigid-plastic analysis

maximize **Load bearing capacity** (**load factor**)
given **Equilibrium**

Stress field and collapse mechanism

ULS
- Material failure

Nonlinear-elastic analysis

minimize **Complementary energy**
given **Equilibrium**

Stress field and deformations

ULS
- Material failure
- Instability
- Fire, incl. instability

SLS
- Displacements
- Crack widths

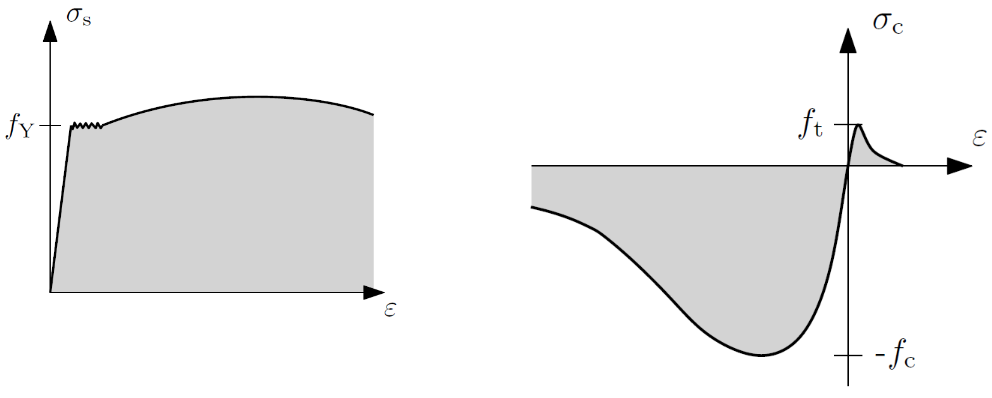
- Efficient and robust algorithms
- Low modelling complexity
- Path-independent solution

Constitutive model

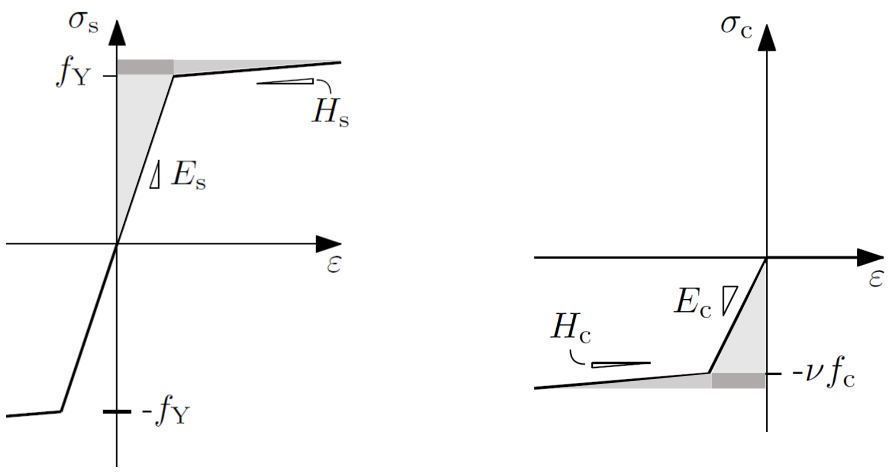
- Reinforced concrete (stress + energy) = Reinforcement + Concrete

- Material stress-strain curves

Actual curves



Model curves

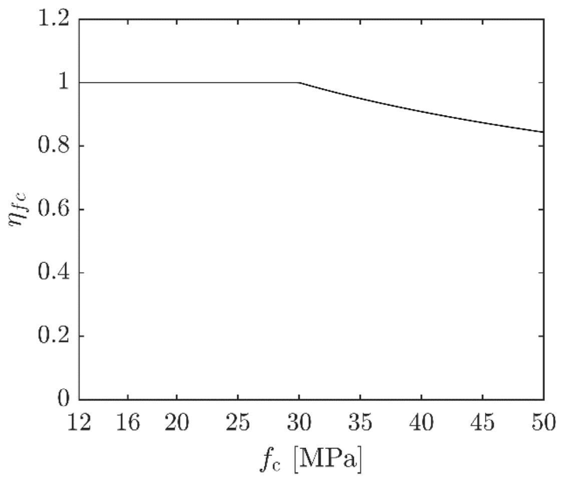


Effectiveness factor

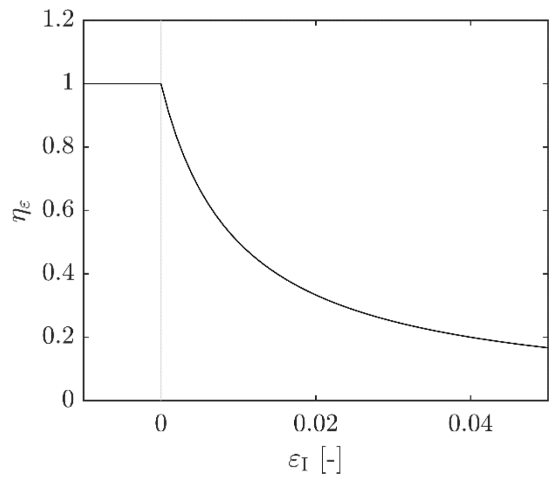
- Plastic concrete strength:

$$f_{\mathrm{cp}} = \nu(\varepsilon_{\mathrm{I}}) f_{\mathrm{c}} \qquad \nu(\varepsilon_{\mathrm{I}}) = \eta_{fc}\eta_{\varepsilon}(\varepsilon_{I})$$

Brittleness factor η_{fc}

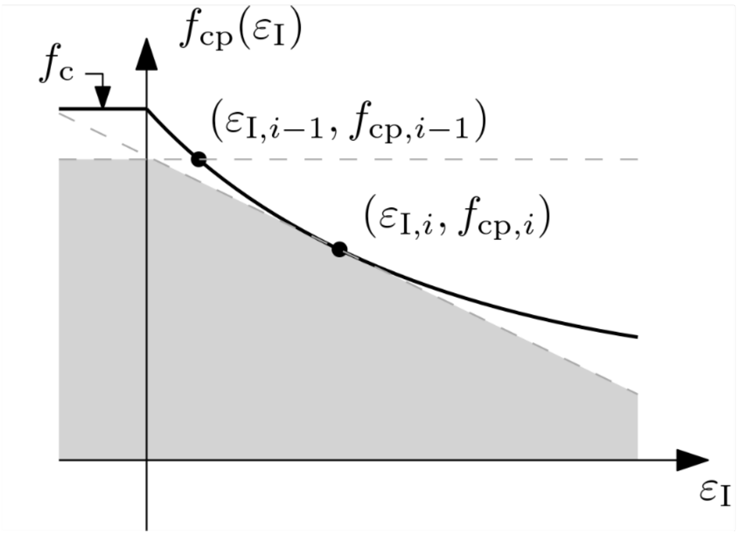


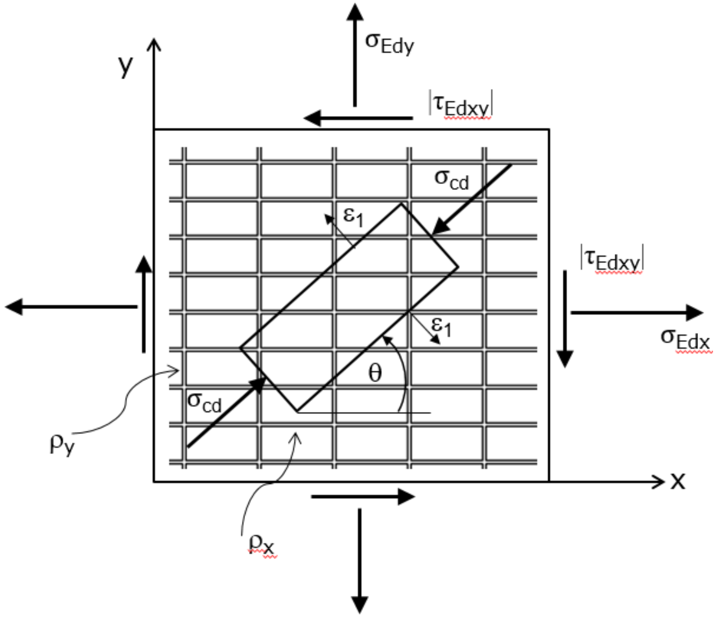
Transverse cracking factor η_{ε}



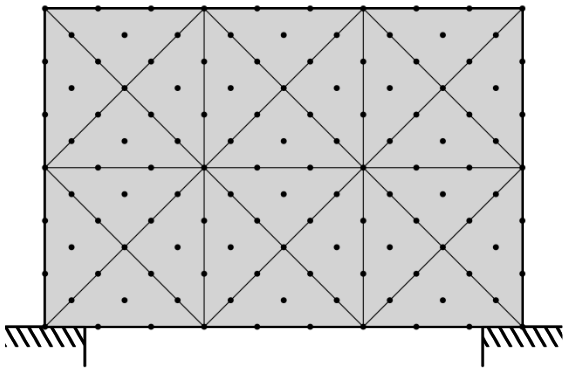
- Model implementation:

$$\varepsilon_{\mathrm{I}} = \varepsilon_{xx}(\sigma_{\mathrm{s}x}) + \varepsilon_{yy}(\sigma_{\mathrm{s}y}) - \varepsilon_{\mathrm{II}}(\sigma_{\mathrm{cII}})$$

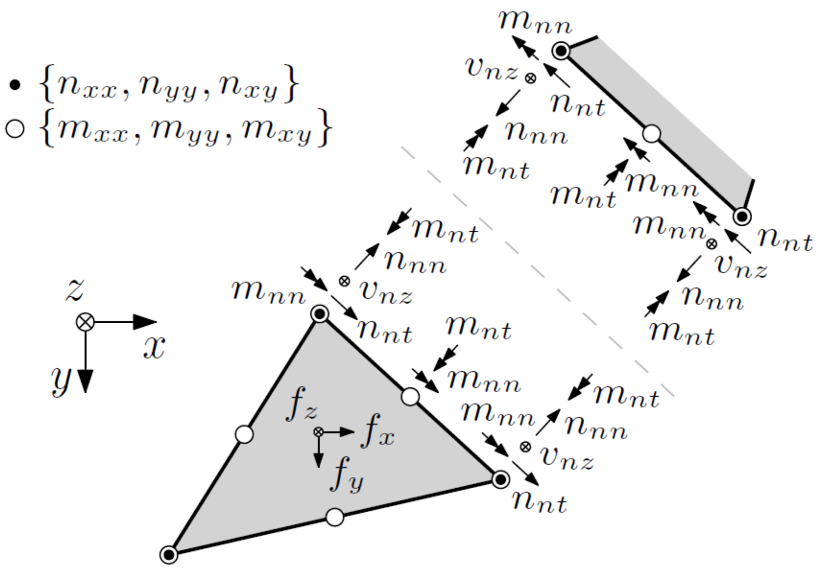


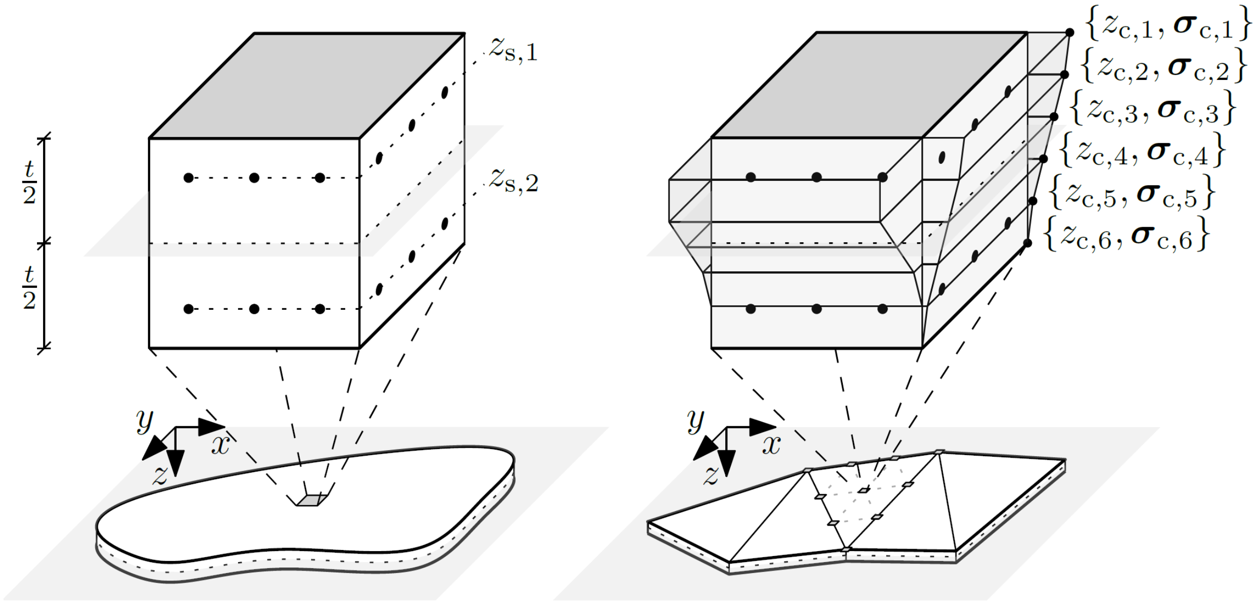


Finite element model



- Elements ensuring section force equilibrium
- Section model
  - Stress, strain, and v-factor in all points

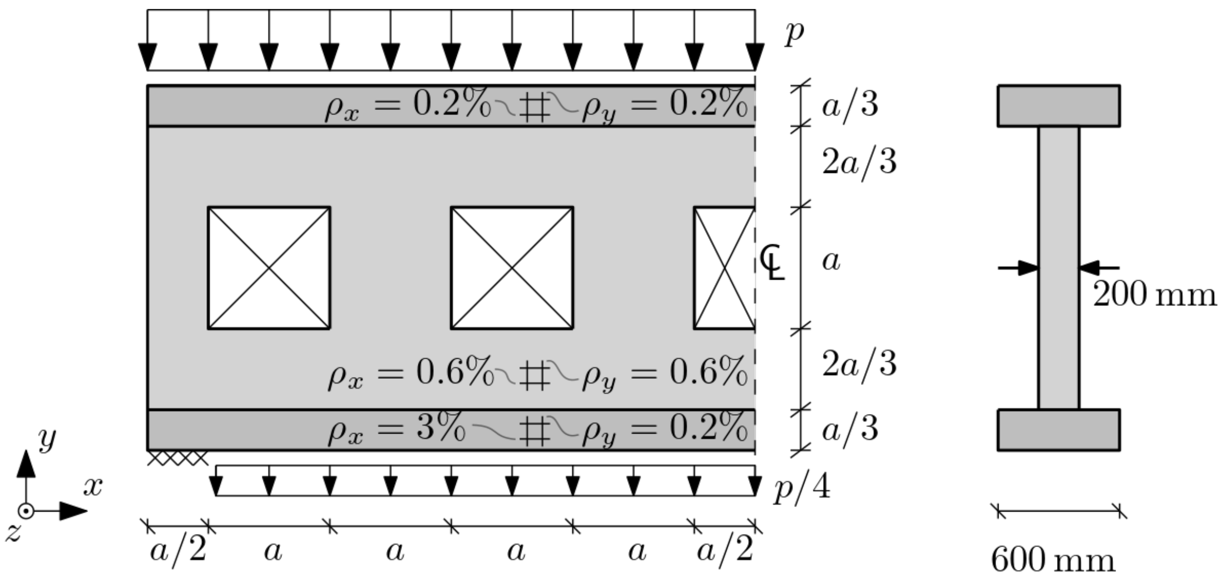


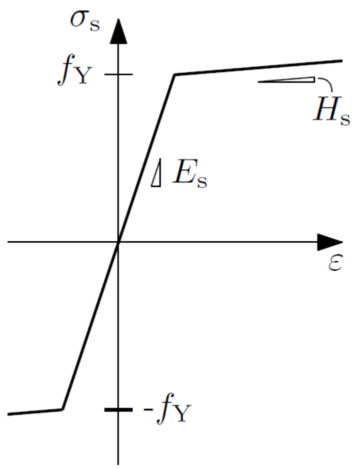


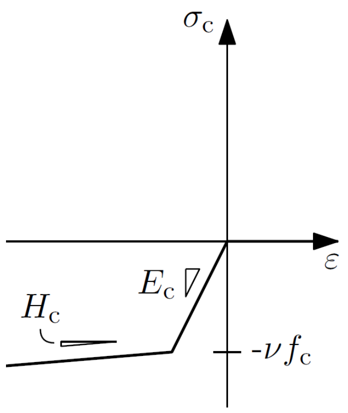
Example: Deep beam with openings

$$E_s = \begin{cases} 200\ \text{GPa} & \text{for} \quad \sigma_s < f_Y = 500\ \text{MPa} \\ 0.84\ \text{GPa} & \text{for} \quad \sigma_s \geq f_Y = 500\ \text{MPa} \end{cases}$$

$$E_c = \begin{cases} 33\ \text{GPa} & \text{for} \quad |\sigma_c| < f_{cp} = \nu \cdot 30\ \text{MPa} \\ 0.013\ \text{GPa} & \text{for} \quad |\sigma_c| \geq f_{cp} = \nu \cdot 30\ \text{MPa} \end{cases}$$







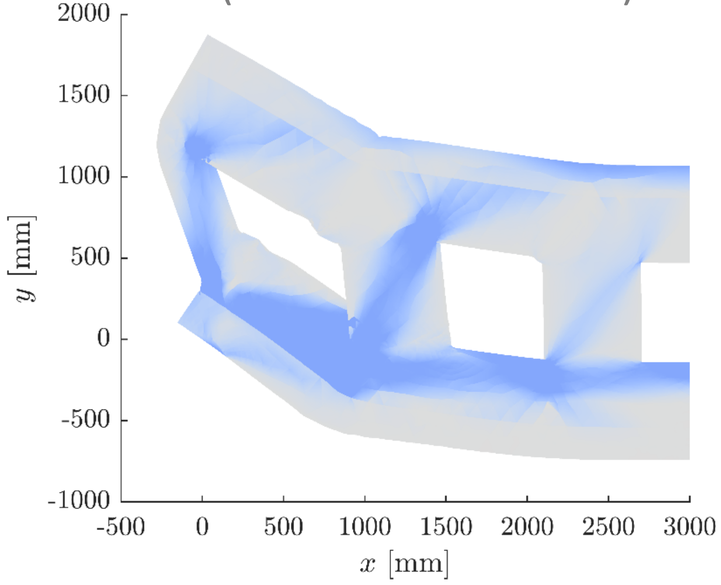
Effect of ν-factor

- Constant vs. Variable ν-factor
  - Actual concrete strength → Ductility is ensured

$\nu = 0.55$

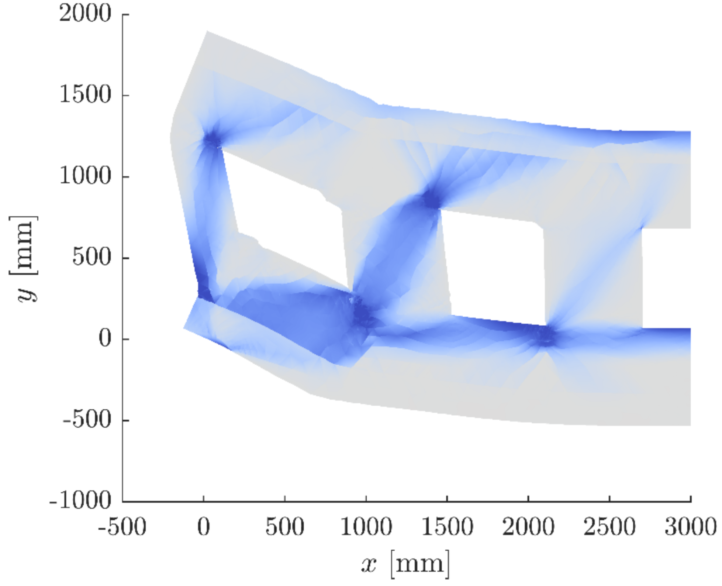
(same value in all points)

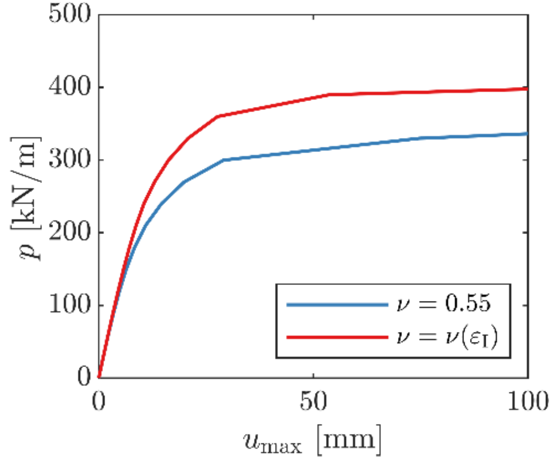
(min. value for shear)



$\nu = \nu(\varepsilon_{\mathrm{I}})$

(varies between points)

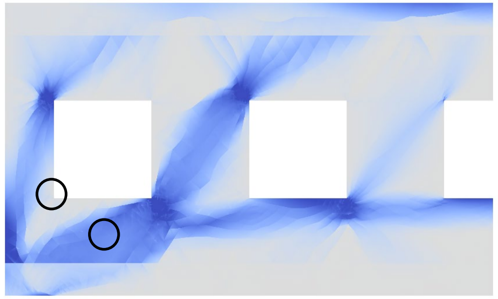




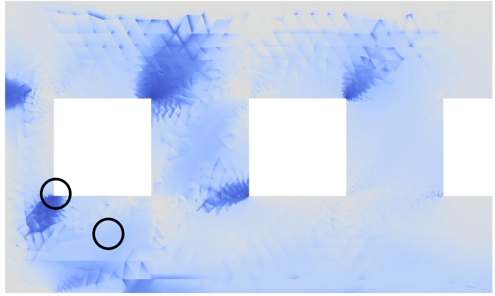
Effect of ν-factor

- Fields for p = 400 kN/m

σ_{cII}
(compressive stress)

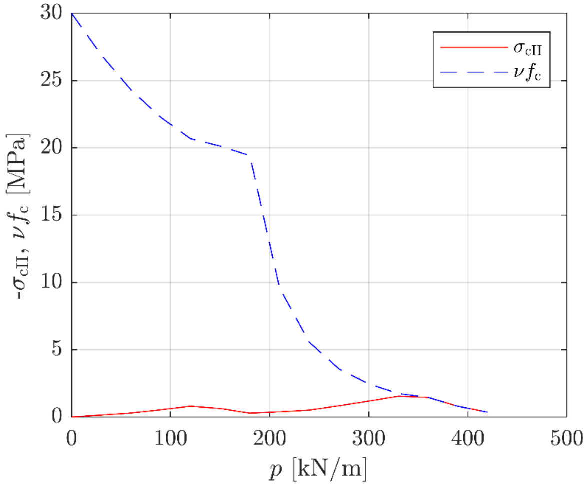


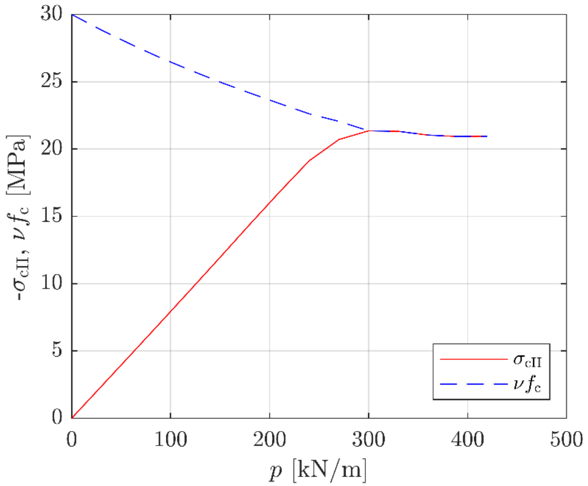
ν
(effectiveness factor)



$\max(\varepsilon_{xx}, \varepsilon_{yy})$
(reinforcement strain)

Evolution of ν and σ_{cII}

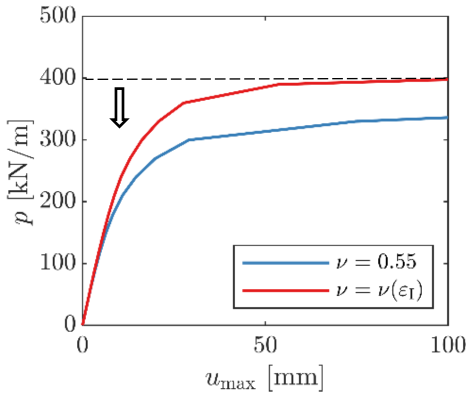




- σ_{cII} is reduced due to ν
- ν is reduced due to strain

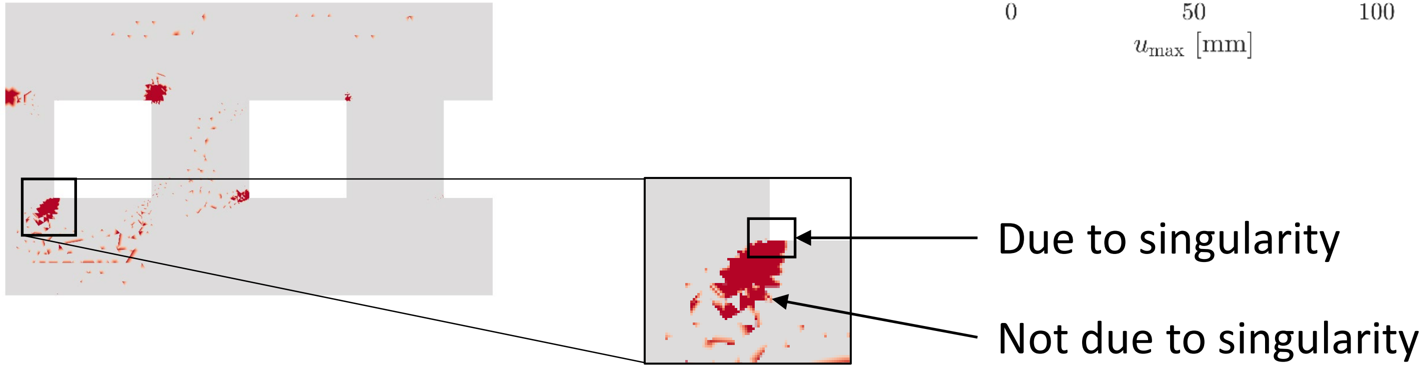
Reinforcement strain

- Reinforcement strain must not exceed ultimate strain
  - $\varepsilon_s \leq \frac{\varepsilon_{uk}}{\gamma}$

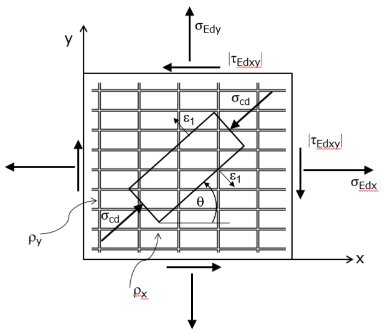


$\max(\varepsilon_{xx}, \varepsilon_{yy})$
(reinforcement strain)

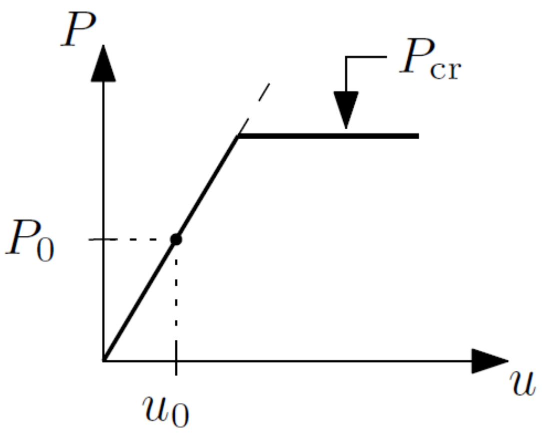
$\max(\varepsilon_{xx}, \varepsilon_{yy}) > \frac{\varepsilon_{uk}}{\gamma} = \frac{0.05}{1.0}$
(exceeding ultimate strain)

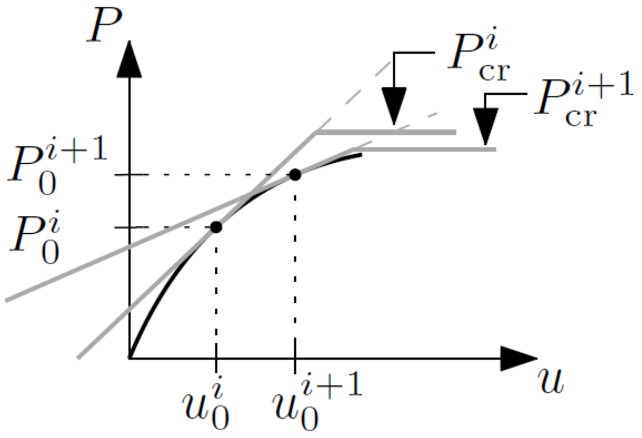


Instability analysis



- Classical buckling problem: Scale P_0 until stiffness vanishes
  - Linear elasticity: Pre-buckling stiffness is constant
  - Nonlinear elasticity: Stiffness depends on deformations
- Approach:
  - Get pre-buckling response to P_0
  - Use cracked stiffness to estimate P_{cr}





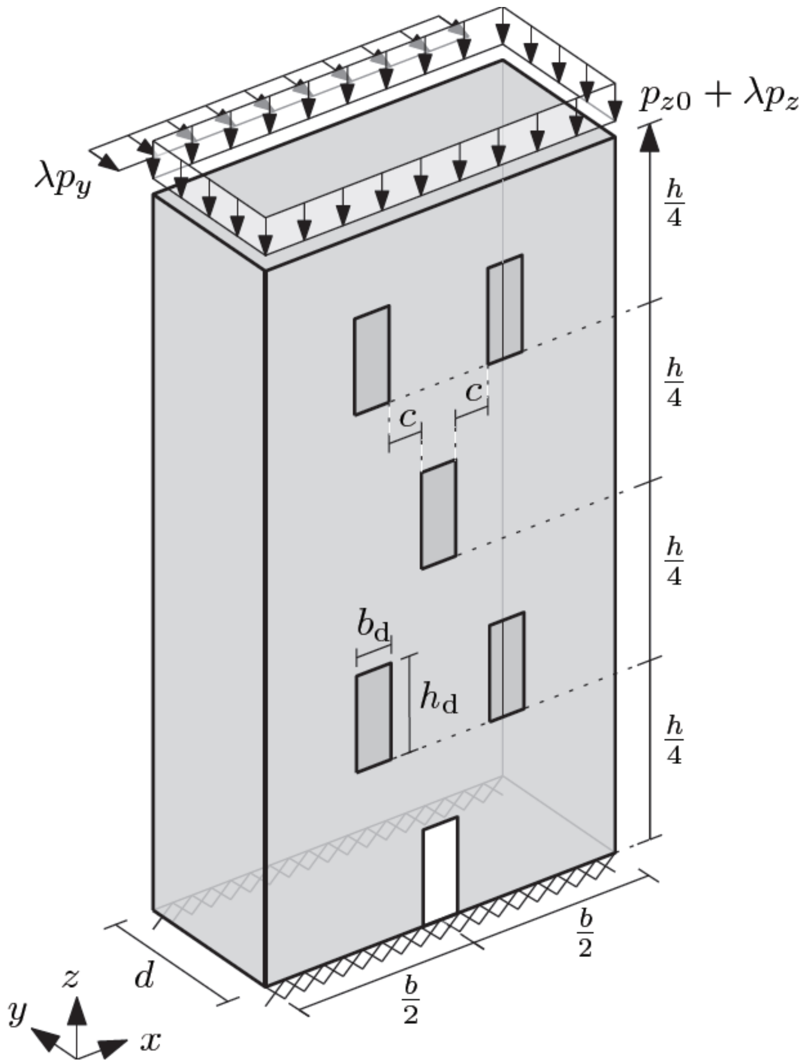
Example: Stairwell with openings

- Material parameters: (prEN-1992-1-2:2021)
  - Concrete: C30
  - Reinforcement: Y500
- Dimensions:

$$h = 16 \text{ m} \qquad b_d = 0.90 \text{ m} \qquad t = 100 \text{ mm}$$
$$b = 8.60 \text{ m} \qquad h_d = 2.10 \text{ m} \qquad \rho_{sx} = 0.0105$$
$$d = 3.60 \text{ m} \qquad c = 0.75 \text{ m} \qquad \rho_{sy} = 0.0105$$

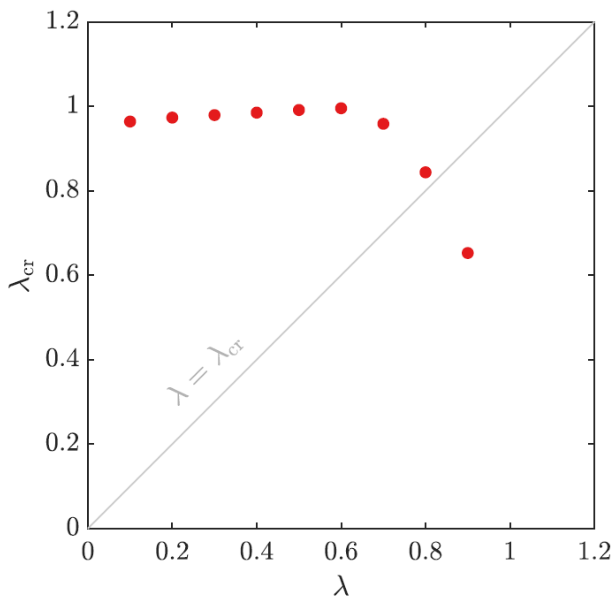
- Load:

$$p_y = 112.5 \text{ kN/m}$$
$$p_{z0} = 300 \text{ kN/m}$$
$$p_z = 100 \text{ kN/m}$$

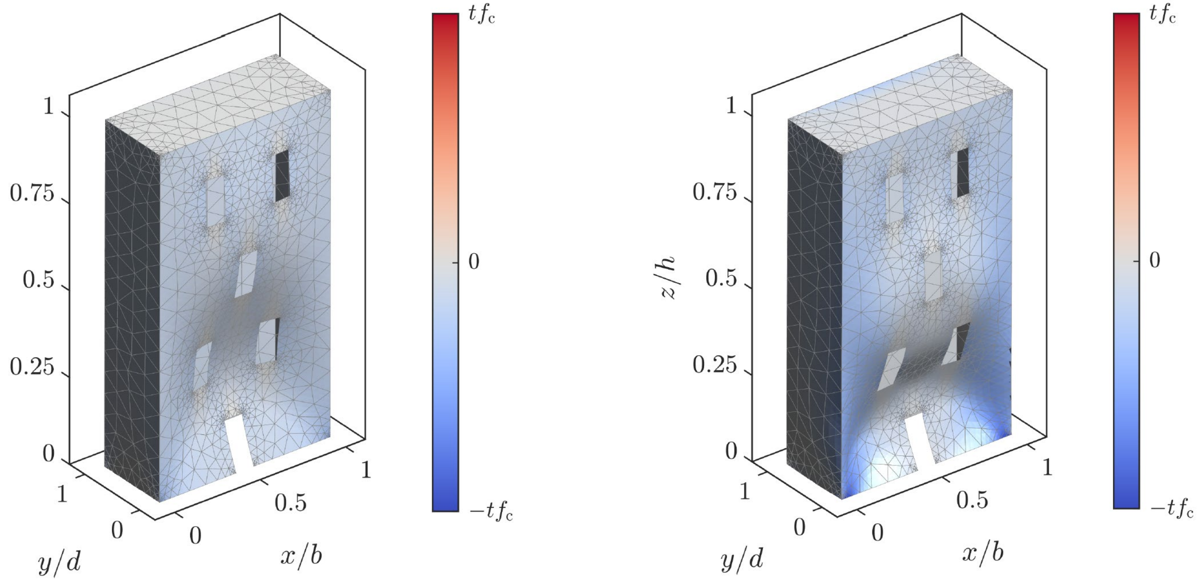


Example: Stairwell with openings

- Evolution of critical load factor
  - When $\lambda = \lambda_{\text{cr}}$, model is exact → $\lambda_{\text{cr}} = 0.84$



- Evolution of buckling mode
  - Lower stiffness in compressive struts

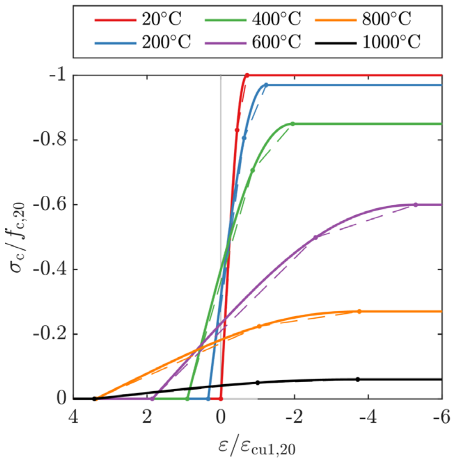


(a) Initial load step **(b)** Final load step

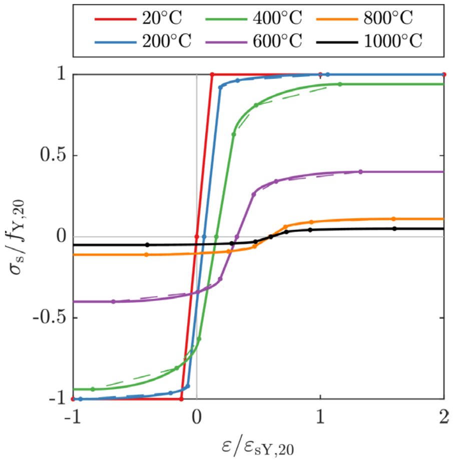
Fire-induced effects

- prEN-1992-1-2:2021:
  - Standard fire resistance
    - →Temperature profile
    - →Reduced material strength & stiffness + Thermal strains
- Stress-strain curves

Concrete



Reinforcement

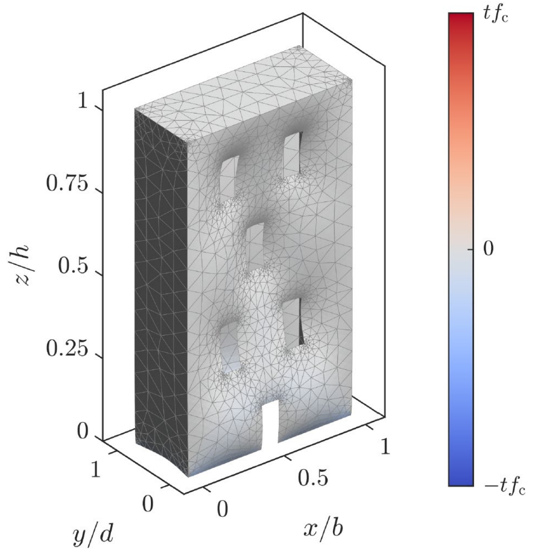


Example: Stairwell with openings

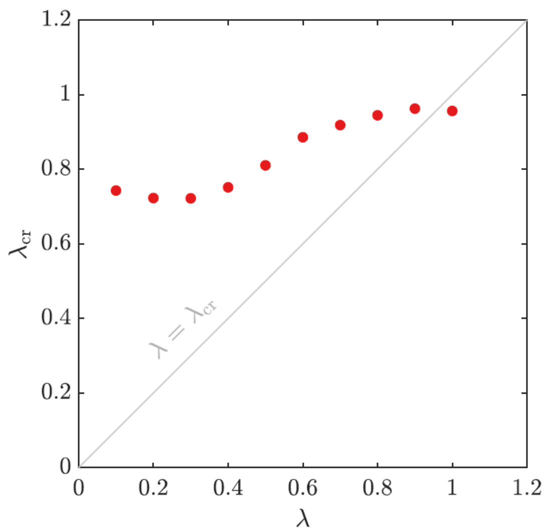
- Load:

$$p_y = 67.5 \text{ kN/m} \quad (112.5 \text{ kN/m})$$
$$p_{z0} = 90 \text{ kN/m} \quad (300 \text{ kN/m})$$
$$p_z = 60 \text{ kN/m} \quad (100 \text{ kN/m})$$

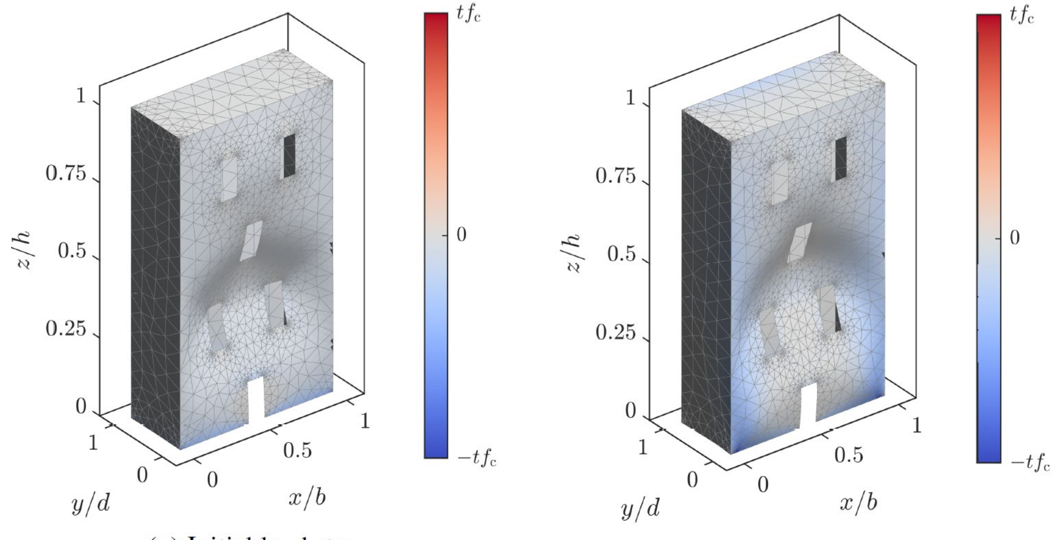
Thermal deformation
(30 min. fire, no load)



Evolution of critical buckling load
($\lambda_{cr} = 0.96$)



Evolution of critical buckling mode



(a) Initial load step

(b) Final load step

- Lower buckling load + different buckling mode

Why can't we just use existing tools?

- High modelling complexity
  - Detailed reinforcement layout
  - Fracture energy (uncertain and difficult to determine)
  - Example: DIANA model → 1 month modelling vs. 1 day
- High computational cost and low robustness
- Loading history vs. path dependence
  - Detailed response to applied loads
  - What about cracks from previous loading history?

Outlook

- When do we need to take $\nu(\varepsilon_l)$ into account?
- When do we face ductility issues?
  - Singularities vs. actual issues

- Tension stiffening
- Post-tensioning