BEHAVIOUR AND DESIGN OF CONCRETE STRUCTURES UNDER THERMAL GRADIENTS

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ABSTRACT

This paper concerns a study of secondary stresses produced in reinforced concrete structures as a result of temperature effects. These stresses often cause premature cracking of a structure and, in exceptional cases, may also have an unfavourable effect on the capacity of the structure. Typical structures which are liable to the stresses in question are first discussed briefly as well as the principles of the heat transfer theory for the sake of repetition. The special features of stresses and their consideration in design are then dealt with on the basis of different design practice. Some design methods are also introduced. In conclusion, the preliminary test results are compared with those obtained from calculations.

Key words: temperature effects on concrete beams, concrete beams under temperature gradients

1. INTRODUCTION

Load-bearing reinforced concrete structures are usually designed to certain design values of external loads which are generally very high compared to actual loads occurring on average. This often results in heavy reinforcement, in which case a crack load is relatively small with regard to the capacity of the structure. In this kind of structure cracks are not discernible at the load acting in practice. If some cracks do occur, the reason lies in that secondary stresses have probably remained without consideration in the design. Such usual stresses are those caused by the prevented deformation of the structure or cross section. In most cases differences in shrinkage and temperature again give rise to deformations. The settlement of supports and prestressing of the structure can produce similar stresses. Some structures, on the other hand, are intentionally designed for external loads only, although it is known that indirect forces will also be produced in these structures. For example, in the design of structures such as external surfaces of facade elements, balcony railings, chimney-stacks and statically indeterminate structures subjected to great temperature differences, temperature effects are not usually taken into account. The reason for this neglect is often undoubtedly the fact that the building regulations and specifications fail to include the procedures for handling of the stresses in question; functional requirements alone are given, as in Finnish regulations.

The temperature stresses due to a non-stationary temperature distribution are of no practical significance within normal temperature ranges, except where a structure is required not to crack. On the other hand, the restraint forces being developed in statically undetermined structures by temperature differences can bring about unexpected cracking. The application of the customary cracking theory leads to overdimensioning and thereby to the growth of restraint forces. This is due to the fact that the cracking phase and mechaninsm are different from those required by the customary cracking theory. In addition to the different serviceability limit state design, the behaviour of restraint forces also differs with regard to external forces in that they relax in the state of failure provided that the structure possesses sufficient deformability. At the same time the external forces are redistributed. If the external loads act simultaneously with restraint forces, failure in some cases can occur before the calculated ultimate state of external forces.

2. TYPICAL REINFORCED CONCRETE STRUCTURES UNDER THERMAL GRADIENTS

Typical concrete structures subjected to large temperature gradients are those of the process industry, concrete chimneys, and the massive cooling water basins of nuclear power plants. In the field of house building the facade elements, which belong to a general group of structural members, are subjected to considerable thermal stresses. Considerable temperature gradients may occur in late winter, especially in sunny locations, in the external surfaces of walls facing south and west. In the walls or roofs with poor thermal insulation there may be a thermal gradient in wintertime. In water towers and bridge structures, among others, similar stresses may exist.

Hydratation heat produced in massive structures also forms a thermal gradient within the structure.

Under special conditions, such as those in arctic regions or in very hot areas, structural members may have been subjected to thermal stresses. For instance the temperature of oil in the North Sea is about 80 - 90 ^oC whereas the temperature of the sea-water outside the tank can be as low as $\sim$5 ^oC.

3. PRINCIPLES OF HEAT TRANSFER IN CONCRETE

The heat conduction theory of mathematical physics is based on the fact that a material is examined as a continuum. The basic equations of one-dimensional heat transfer and the related approximate values of material constants based on the theory in question are discussed briefly in this chapter. The examination is based on the law of energy indestructibility and on the heat quantity which flows through an extremely homogeneous and isotropic slab.

The temperature field in most general form can be given by /2/:

$$\bar{T} = \bar{T}(x, y, z, t) \left[{}^{o}C\right] \quad \text{, where} \qquad (1)$$

$\bar{T}$	is	temperature
x, y, z		coordinates
t		time

If the temperature at certain points in the continuum at the same moment has equal value, niveau surfaces will be formed from these points. The vector perpendicular to the niveau surfaces will determine changes in the temperature of the temperature field. This vector is termed temperature gradient. The heat flux, the direction of which is always the direction of a falling temperature gradient, is described by the vector

$$\bar{\tau} = -\lambda \text{ grad } \bar{T} \left[\frac{\text{kcal}}{m^2 \text{ h}}\right] , \quad \text{where} \qquad (2)$$

constant λ is the heat conductivity factor of concrete material. For concrete λ = 1.5...4.0 kcal/m · h · ^{o}C /2/. The heat quantity which is transferred from the element dV = dx · dy · dz is, by means of heat leakage, expressed in the one dimensional case as:

$$dQ_1 = \lambda \frac{\partial^2 \bar{T}}{\partial x^2} \text{ dx dt [kcal]} \qquad (3)$$

Correspondingly, the heat quantity given up by the element

$$dQ_2 = c \cdot \rho \cdot \frac{\partial \bar{T}}{\partial t} \cdot \text{dx dt [kcal]} , \quad \text{where} \qquad (4)$$

c	is	the specific heat of the material [kcal/kg · ^{o}C] and
ρ	is	the density of the material [kg/m^3]

For concrete $c = 0.21 \ldots 0.24$ kcal/kg · °C and
$\rho = 2300 \ldots 2500$ kg/m³ /2/.

By setting the heat quantities equal, the Fourier differential equation of heat conduction, in the one-dimensional case, is obtained.

$$\frac{\partial^2 \overline{T}}{\partial x^2} - \frac{c \cdot \rho}{\lambda} \cdot \frac{\partial \overline{T}}{\partial t} = 0 \qquad (5)$$

In order to solve the equation the boundary conditions must also be known; one of these can be e.g. an assumption concerning temperature distribution at a certain moment or an assumption concerning the surface temperature of a specimen.

Heat transfer through concrete is thus dependent, not only on the material properties of concrete, examination point in time and thickness of the structure, but also on the surface temperature of the structure. In Fig. 1 the time-dependent temperature distribution in a concrete specimen, the upper surface of which warms up and the lower side of which cools is presented qualitatively.

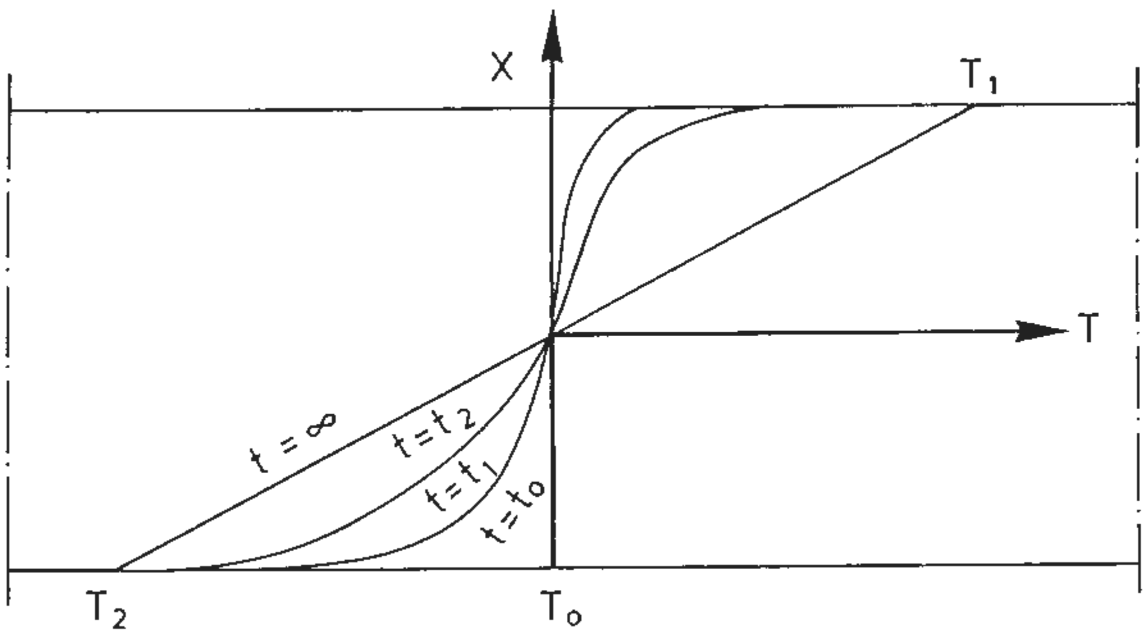


Fig. 1 Temperature distribution in a concrete slab at different points in time.

The final temperature distribution, corresponding to the point in time $t = \infty$, is stationary and all others are non-stationary. In the stationary temperature field the temperature changes almost linearly at the cross-section of the tested specimen.

4. NATURE OF THERMAL STRESSES AND EFFECT ON THE BEHAVIOUR OF REINFORCED CONCRETE STRUCTURES

4.1 Nature of stresses

The stress types can be classified according to their origin as follows:

- the stress caused by external, static or dynamic loads
- the stresses due to restraint forces.

The origin of the stress types can be defined as follows /6/:

1. The stresses due to external loading originate from the self weight of the structure, the acceleration of the structure and from the external static or dynamic loads affecting structures.

2. The forces and stresses due to restraint deformations of the structure are produced in statically indeterminate structures. The forces and stress can be either unintended or intentional. Unintended stresses are caused e.g. by temperature or shrinkage differences or displacement of the supports. The intended stresses are caused by prestressing of the structure. These types of restraint internal forces and stresses can be produced only in statically indeterminate structures.

3. The stresses, due to restraint strains in the cross-section, are produced by non stationary temperature distribution. These stresses also can be either unintended or intentional. Unintended stresses caused by restraint of strains result e.g from temperature gradients, shrinkage differences, plastic deformations and from welding of metallic parts. Stresses due to restraint of strains can be produced both in statically determined structures and statically undetermined structures, but their sum at each cross-section of the structure is zero. These stresses therefore do not develop internal forces in the structure.

Fig. 2 shows the deflections of the two-span beam due to temperature differences in the upper and lower surfaces of the beam and resulting cracking caused by restraint moment. In the same figure the principle of development of stresses due to restraint of strains is presented.

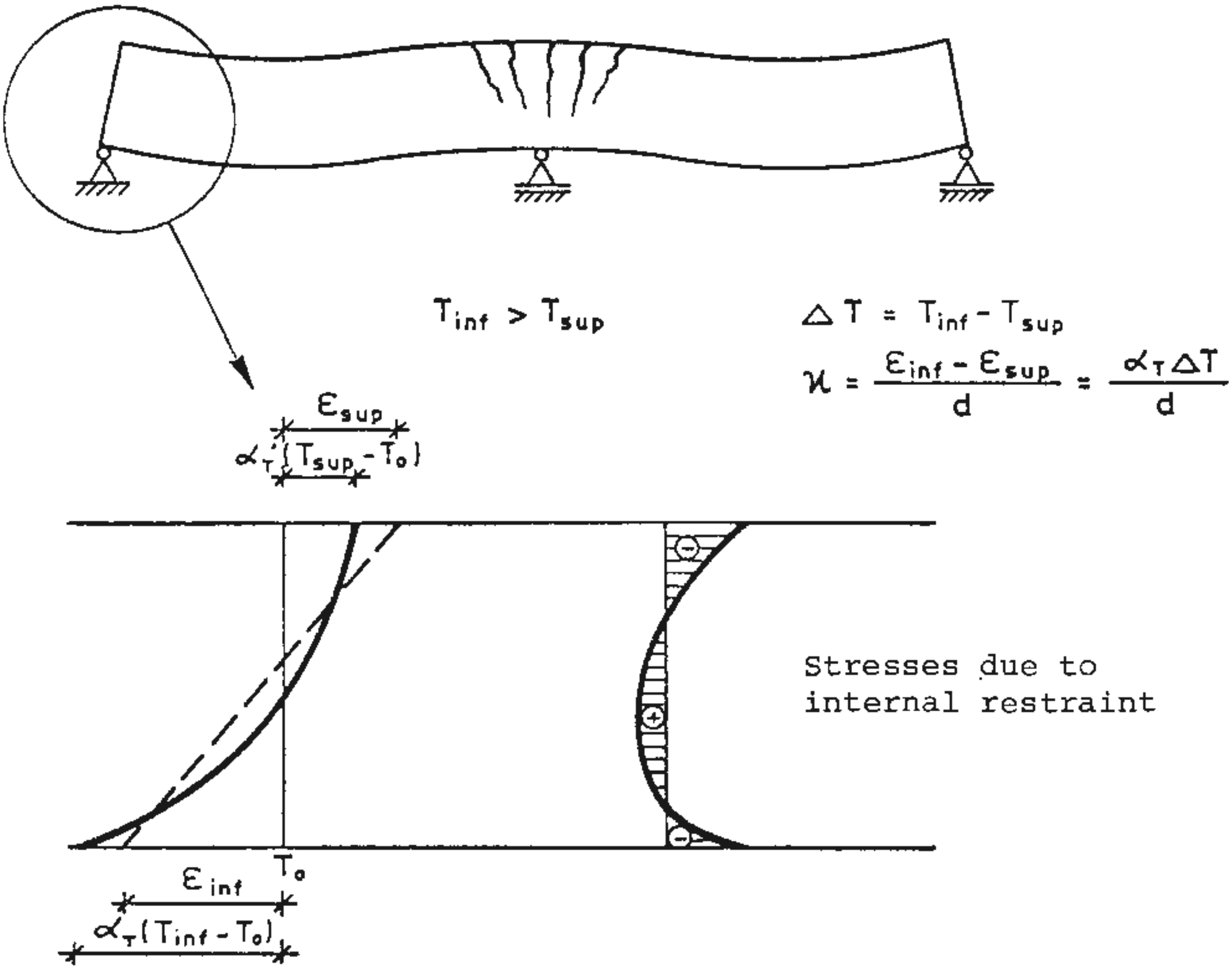


Fig. 2 Principle of development of restraint forces and stresses

Generally the restraint forces in design are taken into account in such a way that the total force in question is calculated, and the amount of reinforcement it requires is determined. This leads in most cases to overdimensioning, since the restraint forces, along with an increase in stress of the structure, are greatly dependent on decreasing stiffness, which in turn is dependent on the amount of reinforcement. The less reinforcement there is, the smaller are the restraint forces in the structure. Reinforcement does not only increase restraint forces but in reality causes their development. With a decrease in the amount of reinforcment the force corresponding to the constant restraint stress diminishes at a relatively slower rate than the reinforcement. For this reason the steel stress increases. There exists a minimum amount of reinforcement with which the steel stress attains to a certain permissible value. In the case of restraint stress greater steel stresses can be allowed, since each increase in stress will then render the structure more flexible and thus diminish restraint forces.

Reinforced concrete structures under restraint stresses have often been viewed from the wrong standpoint. There is no need to determine the amount of steel which would withstand restraint stresses; on the contrary, the width of resulting cracks should be limited to a satisfactory level /10/.

The restraint internal forces are not similar by nature to the internal forces due to external loads. There exists a countless number of solutions fulfilling equilibrium and compatibility conditions. They do not only differ from each other in values of steel stress and crack width but also in values of force quantity. The function of the design is thus to find a solution which is as simple as possible and fulfils the restrictions placed on the crack widths.

4.2 Effect of thermal forces on the behaviour of concrete structures

A statically indeterminate structure which is loaded by temperature gradient is examined. The force deformation dependence, in the crack cross-section, of a reinforced structural member in the force-controlled state of stress differs from the corresponding force deformation dependence in the deformation-controlled state of stress at the time of crack initiation. If the state of stress is force-controlled i.e. external load is continuously increasing, the moment curvature dependence also increases. The degree of increase depends on the amount of steel in the structure and on the bonding properties of reinforcement. A more detailed discussion of the horizontal part of the M-κ curve follows.

If the state of stress is deformation-controlled, i.e. the stress results from prevented deformations, the moment in the crack cross-section decreases subsequent to crack initiation primarily because the tension zone of concrete is no longer acting. The value of the cracking moment is only attained after an increase in curvature. The prerequisite for the action in question is that the reinforcement carries the tensile force transferred from the concrete without yielding. The moment corresponding to a certain deformation or thermal stress can thus be determined unequivocally by taking the formation of cracks into account.

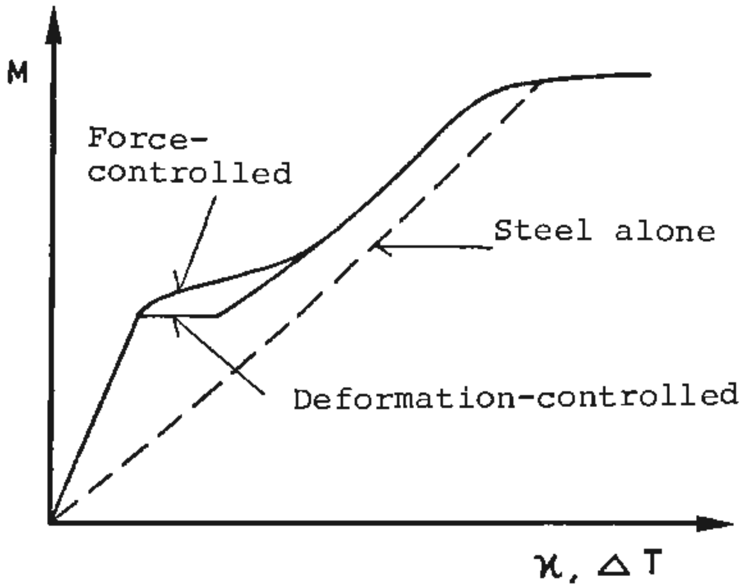


Fig. 3 Difference between curves M - κ or M - ΔT in principle under force-controlled and deformation-controlled load.

The M - κ relationship is examined just prior to and subsequent to the formation of the first crack. It can be assumed that the thermal stress and corresponding deformation, at the time of formation of the first crack, are the same before and immediately after cracking. In both cases the difference in average elongation of the steel will be equally large (Fig. 4).

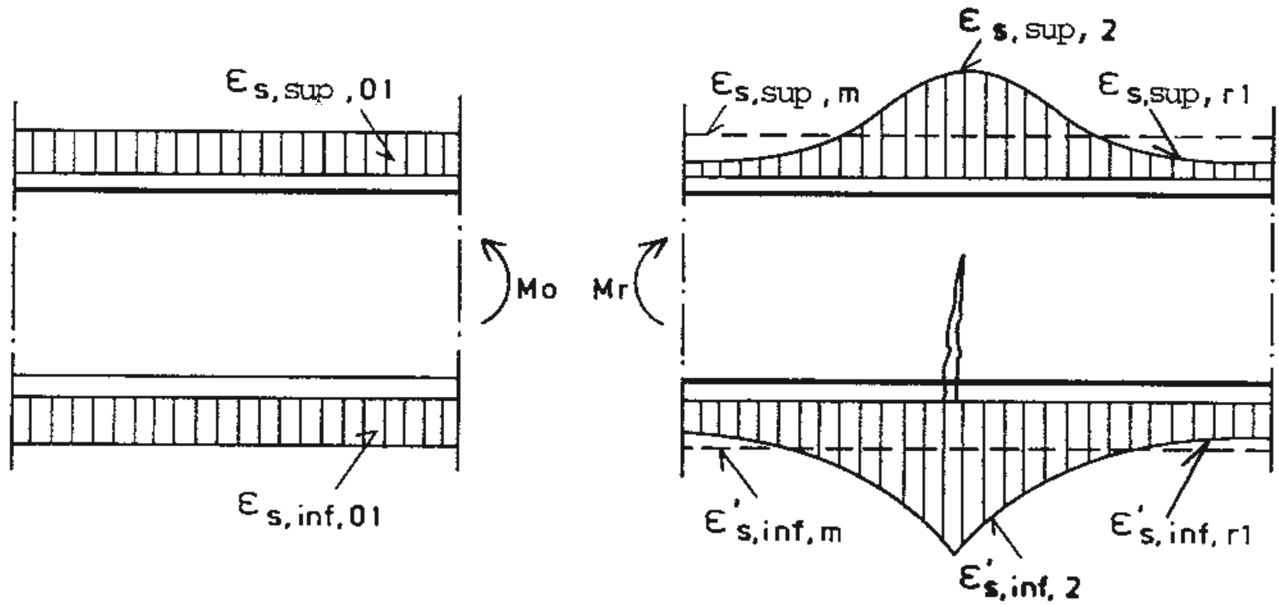


Fig. 4 Elongations of steel prior to and subsequent to crack formation.

$$\varepsilon_{s,inf.01} - \varepsilon_{s,sup.01} = \varepsilon_{s,inf,m} - \varepsilon_{s,sup,m} \quad (6)$$

After cracking, the difference in steel elongation in the uncracked state will be smaller than the difference in average elongation, which difference, on the other hand, is smaller than the difference in elongation at the crack.

$$\varepsilon_{s,inf,r_1} - \varepsilon_{s,sup,r_1} < \varepsilon_{s,inf,m} - \varepsilon_{s,sup,m} < \varepsilon_{s,inf_2} - \varepsilon_{s,sup_2} \quad (7)$$

This means that the difference in elongation prior to cracking is greater than that subsequent to cracking in the uncracked state.

$$\varepsilon_{s,inf,r_1} - \varepsilon_{s,sup,r_1} < \varepsilon_{s,inf,01} - \varepsilon_{s,sup,01} \quad (8)$$

From this, it can be concluded that the moment was greater prior to cracking than immediately afterwards.

$$M_r < M_o \quad (9)$$

Once the structure is subjected to thermal stress another crack cannot be formed prior to an increase in thermal stress to a point where it again corresponds to the cracking force.
This mechanism will recur until the cracking pattern of the entire structure has been stabilized. Only subsequent to this can the value of the restraint force (moment) surpass that of the cracking force (moment) (Fig. 5).

This performance model holds true only if the tensile strength of concrete in the structure is constant. This is not the case in reality, which is why experiments have often proved that the cracking moment slightly increases following crack formation.

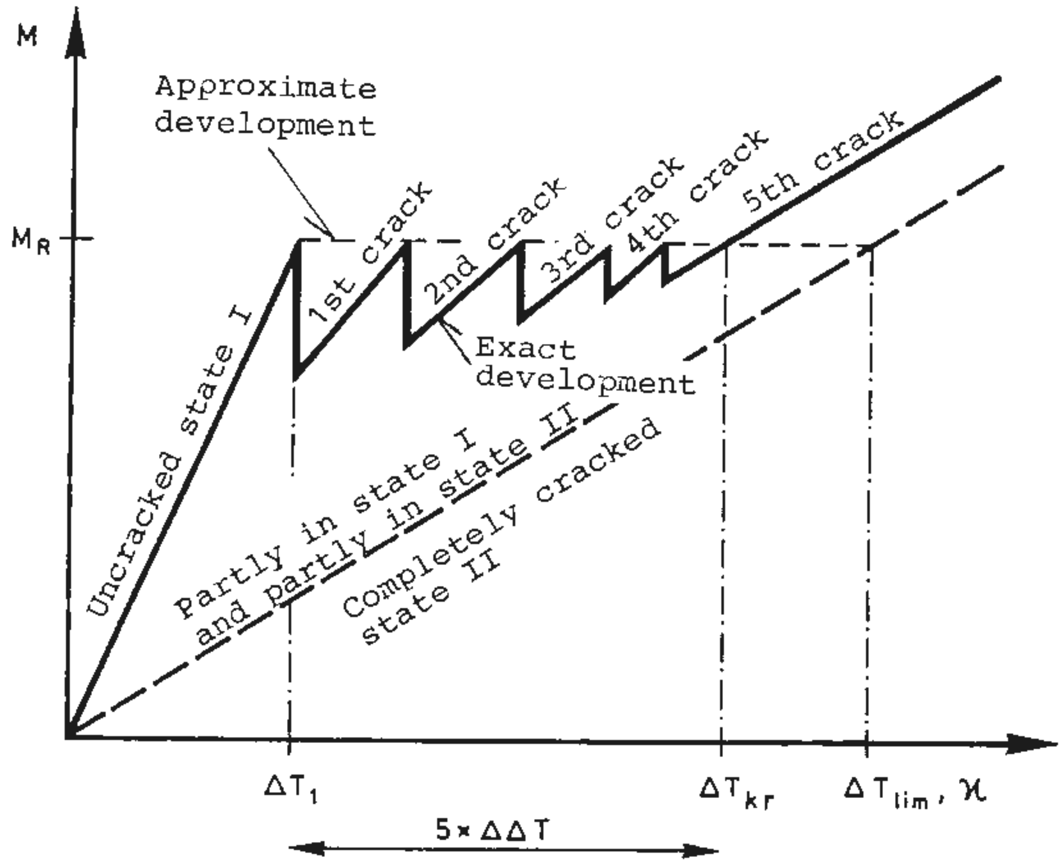


Fig. 5 M - κ curve in the state of restraint stress.

In practice, the temperature is often below 150 $^{\circ}C$ and the corresponding temperature differences $\Delta T < 100$ $^{\circ}C$ and $T_m < 100$ $^{\circ}C$, which, except for creep, do not alter the properties of concrete to a great extent. They cannot produce cracking either as far as the stabilized state.

At usual temperatures thermal stress alone can only cause deflections or cracking in reinforced concrete structures. If an external load is acting, it may also affect the capacity of the structure.

Depending on the factor which has initiated restraint force and on the boundary conditions of the structure, the external and internal restraint forces can act together or separately.

4.3 Effect of thermal stresses on the behaviour of concrete structures

In Fig. 6 the sudden cooling of a slab at one side and the corresponding deformation state are shown. Free deformations ε_o (X,t) are associated with the temperature field T (X,t). Restraint stresses result from strain differences $\Delta\Delta\varepsilon_o$.

Restraint normal force is caused by average deformation values ε_{mo} and the restraint moment by the curvature κ_o.

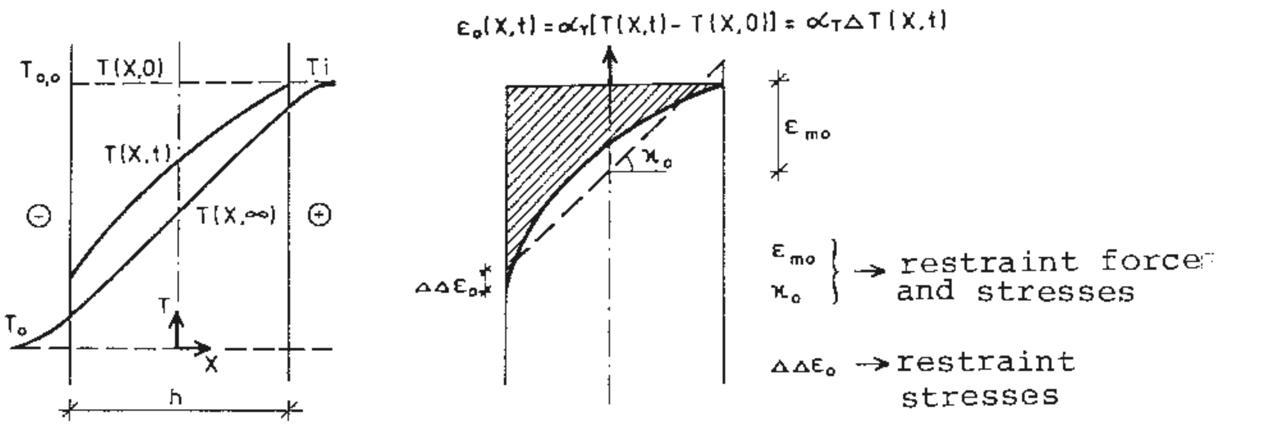


Fig. 6 Deformations associated with thermal stresses.

The restraint stresses are of significance only if absolute tightness is required of the structure. In this case the question primarily concerns the massive structures, in which the heat of hydration can produce dangerous temperature gradients. In the manufacture of precast elements also, there is a risk of cracking in wintertime, if the elements are prematurely removed outside to the cold. Calculations of restraint stresses are of no practical significance if the creep properties of concrete are not known during calculation. Disadvantages of restraint stresses can best be prevented by means of concrete technology. These are e.g. cooling, prevention of drying and cooling. The aim shall be that tensile strength should develop faster than restraint stress.

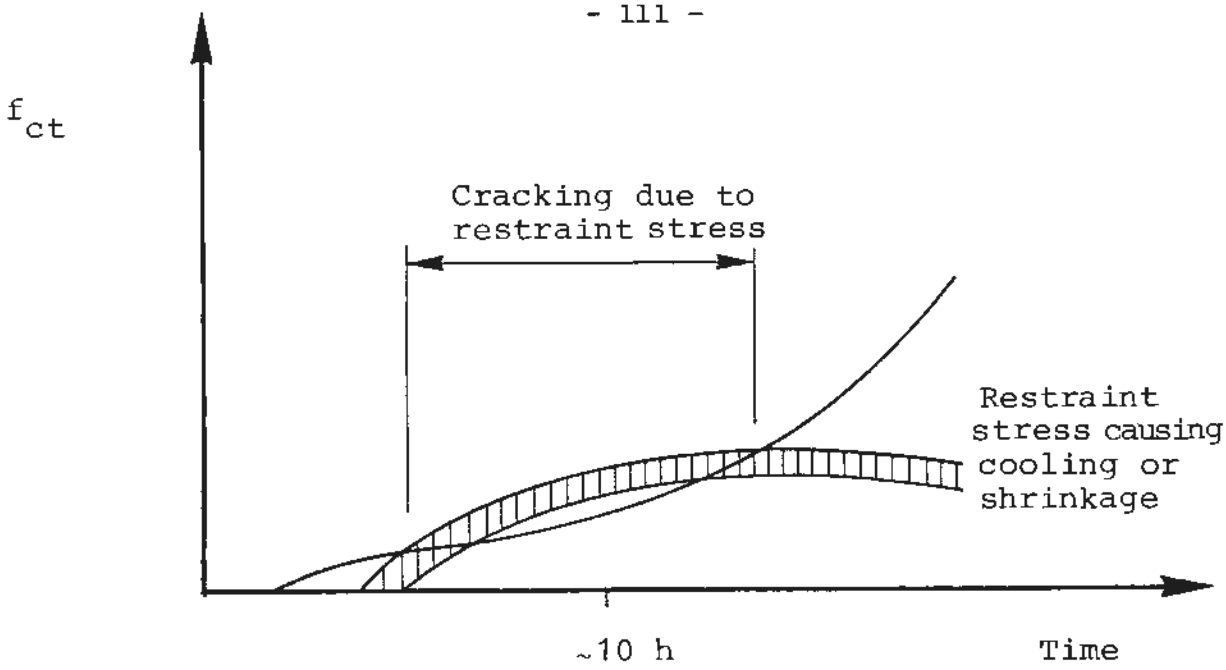


Fig. 7 Formation of hair cracks discernible at an early age of concrete.

If restraint forces and stresses act simultaneously, the restraint stresses have the effect of advancing the crack formation. While cracking progresses the restraint stresses vanish entirely as is often also the case with the restraint forces. It has also been proven that restraint stresses are of no significance to ordinary structural thickness (> 800 mm).

The formation of cracks due to restraint stresses and forces is affected not only by the fairly major variations in tensile strength of concrete but also by the ductility of concrete, the size of indirect stress and the way it acts. The most ordinary cases are /9/

a) rapidly developing, quick-acting

b) " " , long-acting

c) slowly developing, " "

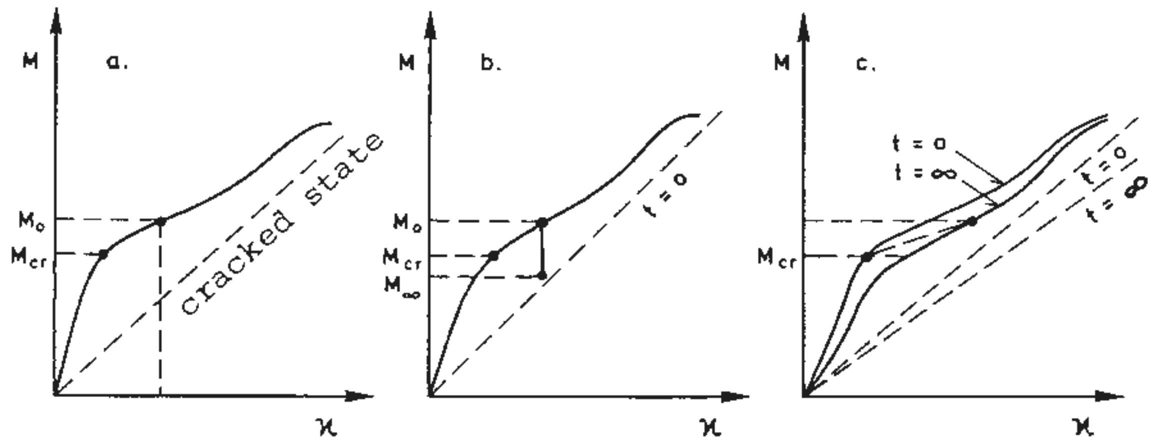


Fig. 8 M - κ curve on the basis of the development of thermal stress and the duration of its action.

4.4 Effect of cracking on restraint forces

The effect of crack formation on thermal moment and bending stiffness can be studied e.g. by means of the fictitious, effective stiffness, the determination of which is possible by means of the rotation of the beam end. The effective stiffness is the stiffness in an uncracked state multiplied by a reducing constant. The constant can also be considered as a reduction factor, by which the calculated thermal moment in the uncracked state must be multiplied in order for it to agree with the moment due to the same thermal stress in the cracked state. Expressed by the moment and curvature the stiffness value is

$$(EI)_{eff,m} = k \cdot E_c I_c = \frac{M_{cr}}{\theta_i} \cdot L \qquad (10)$$

On the other hand the rotation of the beam is

$$\theta_i = \kappa_{r_1} \left(L - \sum^{i} 2\, l_{bm} \right) + \kappa_{r_2 m} \sum^{i} 2\, l_{bm} \qquad (11)$$

and

$$\kappa_{r_1} = \frac{M_{cr}}{E_c I_c} , \qquad (12)$$

from which it is obtained by substituting (10) and (12) for (11) in the equation

$$k_i = \frac{1}{1 + \sum^{i} \frac{2\, l_{bm}}{L} \left(\frac{\kappa_{r_2 m}}{\kappa_{r_1}} - 1 \right)} \qquad (13)$$

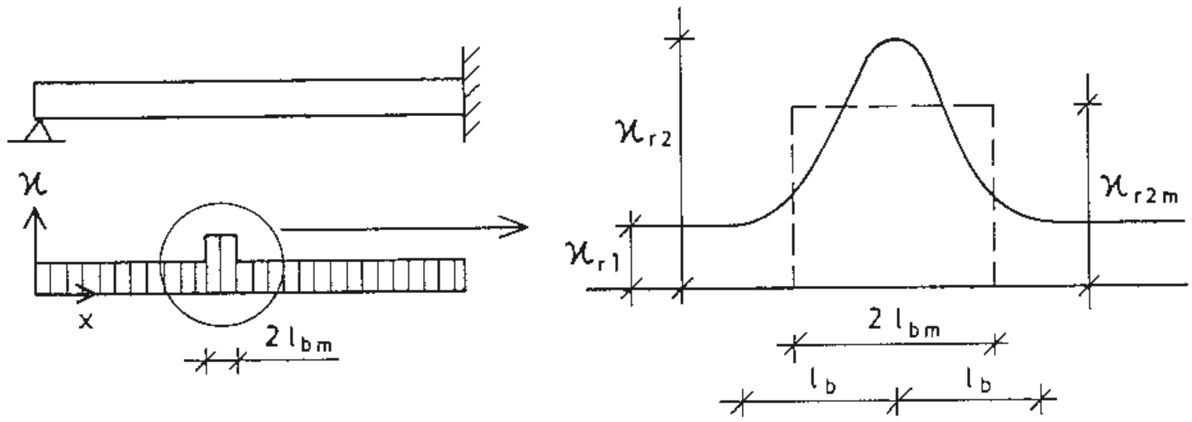


Fig. 9 Curvature distribution in the vicinity of the crack.

The relative value of thermal moment with regard to the cracking moment after the ith crack corresponds to the thermal gradient

$$\Delta T_i = \frac{\kappa_{r1} \cdot d}{k_i \cdot \alpha_t} \tag{14}$$

At the thermal gradient value ΔT the thermal moment is reduced to the value

$$M_{\Delta T_i} = M_{cr} \cdot \frac{k_i}{k_{i-1}} \tag{15}$$

In this calculation method, the difficulty lies in the calculation of bond length since the crack space in this phase has not been stabilized and is still indefinite. It can be concluded, however, from the equation (13) that the formation of an individual crack under bending stress has no particular effect on the course of the M - κ curve (change ∿ 1...5 %), therefore it can be approximated with the horizontal part until the crack pattern has been stabilized. In the case of tensile stress the change in question is greater since the entire cross section cracks.

5. CONSIDERATION OF THERMAL STRESSES IN THE DESIGN OF REINFORCED CONCRETE STRUCTURES ACCORDING TO BUILDING CODES

The design methods for concrete structures most frequently used are based on

- elastic theory
- plasticity theory and
- non-linear theory.

The cross-section state to be used for the determination of forces and the stiffness state particularly for determination of thermal forces depend, as is well known, on the design methods used. Table 1 shows a combination of forces for calculating the effects of external load forces and restraint forces, together with the corresponding design conditions.

Table 1. Calculation principles of forces.

Material/ method	Cross-section state used in calculating forces		Design condition	Maximum stress due to
	external load	thermal load		
Elastic (elastic theory)	uncracked	uncracked	$\sigma_{max} < \sigma_{adm}$	super-position
Plastic (plasticity theory)	entirely plastic	$M_F \rightarrow 0$	$\gamma \cdot M < M_{pl}$	γ_x loading combi-nation
Nonlinear (nonlinear material)	EI = function of load and cross-section	EI = func-tion of thermal force and cross-section	$\gamma \cdot \sigma < \sigma_u$ or $\gamma \cdot \varepsilon < \varepsilon_u$ or $\gamma \cdot M < M_y$	γ_x loading combi-nation

Since the values of thermal forces are dependent on the stiffness of the structure only, it is important to know the actual stiffness of the structure and the factors affecting it. In the case of reinforced concrete structures the stiffness of the structure is greatly affected by

- cracking and
- creep.

The effective stiffness required for calculating the thermal forces also depends on the external loading force which is acting simultaneously.

On the other hand, the external loading forces in the service state are not greatly dependent on cracking and creep. The loading forces are transmitted due to cracking, however, to the stiffer parts of the structure when the load increases. Since the reinforcement is usually designed on the basis of an uncracked state, the forces, once the stiffer parts have cracked, will be transmitted back thus corresponding to the stiffness distribution according to the original assumption, which is equivalent to the structure cracked at field and support. Only in the ultimate limit state can a considerable redistribution of loading force quantities take place due to its plasticity.

The forces in the elastic theory are calculated on the basis of uncracked cross-section. The different loading combinations and effects of deformation can then be summed up.

In the use of plasticity theory and nonlinear theory the addition of forces is not possible.

Considerations of restraint forces in DIN 1045 and in the CEB model code have been compared in Table 2.

Table 2. Consideration of force quantities of different types in different design practice.

Method	Forces of external load	Thermal forces positive effect	Thermal forces negative effect
DIN 1045	Elastic theory unbroken cross-section	may be taken into account	shall be taken into account
		must be calculated with values of cracked cross-section	may be calculated with values of cracked cross-section
		creep may be taken into account	creep may be taken into account
CEB	Elastic theory	restraint forces are taken into account	
	Plasticity theory	can be necessary	
	Nonlinear theory	creep may be taken into account	

It is seen that the cases in which the existing restraint force at the cross-section examined has either a favourable or unfavourable effect on the determining force quantity are separated from each other. Thus when the restraint force acts, with regard to the loading force, in the opposite direction the loading force alone determines the design. When thermal forces are known to act, their effect shall be taken into account only at the values corresponding to the stiffness of the cracked state. If the thermal force acts in the same direction as the loading force this must be taken into account assuming, however, that the structure is in a cracked state. In both cases the effect of creep on the design can be taken into account at a moment in time $t = \infty$. The moment $t = 0$ is, however, often decisive.

In the design of structures under thermal forces in the ultimate limit state the partial safety factor as given in Table 3 can be used when the mode of failure is ductile, forewarning.

Table 3. Partial safety factors of actions of different types.

Method	γ_g and γ_q	γ_f	
DIN 1045 (house building)	1.75	1.0	cracking to be examined
DIN 1075 (bridge and massive structures)	1.75	1.35	
CEB	1.35 1.50	1.0...1.2 0.8...1.0 if cracking examined	favourable effect unfavourable effect

The use of the partial safety factors of the restraint force always presupposes the examination of crack width (the limitation of width, if necessary).

The use of smaller partial safety factors for the thermal force quantities is based on the fact that, when approaching the ultimate limit state, the effect of thermal forces will gradually vanish as the structure becomes plastic. In most cases the rotation capacity of the structure subjected to the bending moment is sufficient for this. If the structure is continuously compressed due to thermal effects, the thermal forces are not supposed to decrease when the load increases. In these cases γ_f should be 1.0...1.5. For the structures affected only by the calculated restraint tensile stress the design in the calculated state of failure is not determining since the restraint stress vanishes when tensile stiffness decreases.

6. DESIGN METHOD FOR CRACK LIMITATION

In many types of structure the thermal forces will become more decisive than the forces caused by external load. In this case, when designing for thermal force only, it is a question of the serviceability limit state design, mostly of the limitation of the crack width to an acceptable level. Cracking is limited by choosing a sufficient amount of reinforcement on the basis of the permissible crack width. Factors which contribute to a reduction in the amount of reinforcement are improving bond and reducing bar size as well as the decreasing strength of concrete.

The design curves for pure thermal force are shown in Fig. 10. The curves are based on the realistic interaction between concrete and reinforcement, in which the behaviour of concrete between the cracks and the bond properties of bars (in the figure the average behaviour of Finnish ribbed reinforcing bar A400H), among others, have been taken into account. The amount of reinforcement and max. bar size required for the stress corresponding to the cracking moment (where the strength properties of concrete, eccentricity, permissible crack width and the cross-sectional values are known) are obtained from the curves.

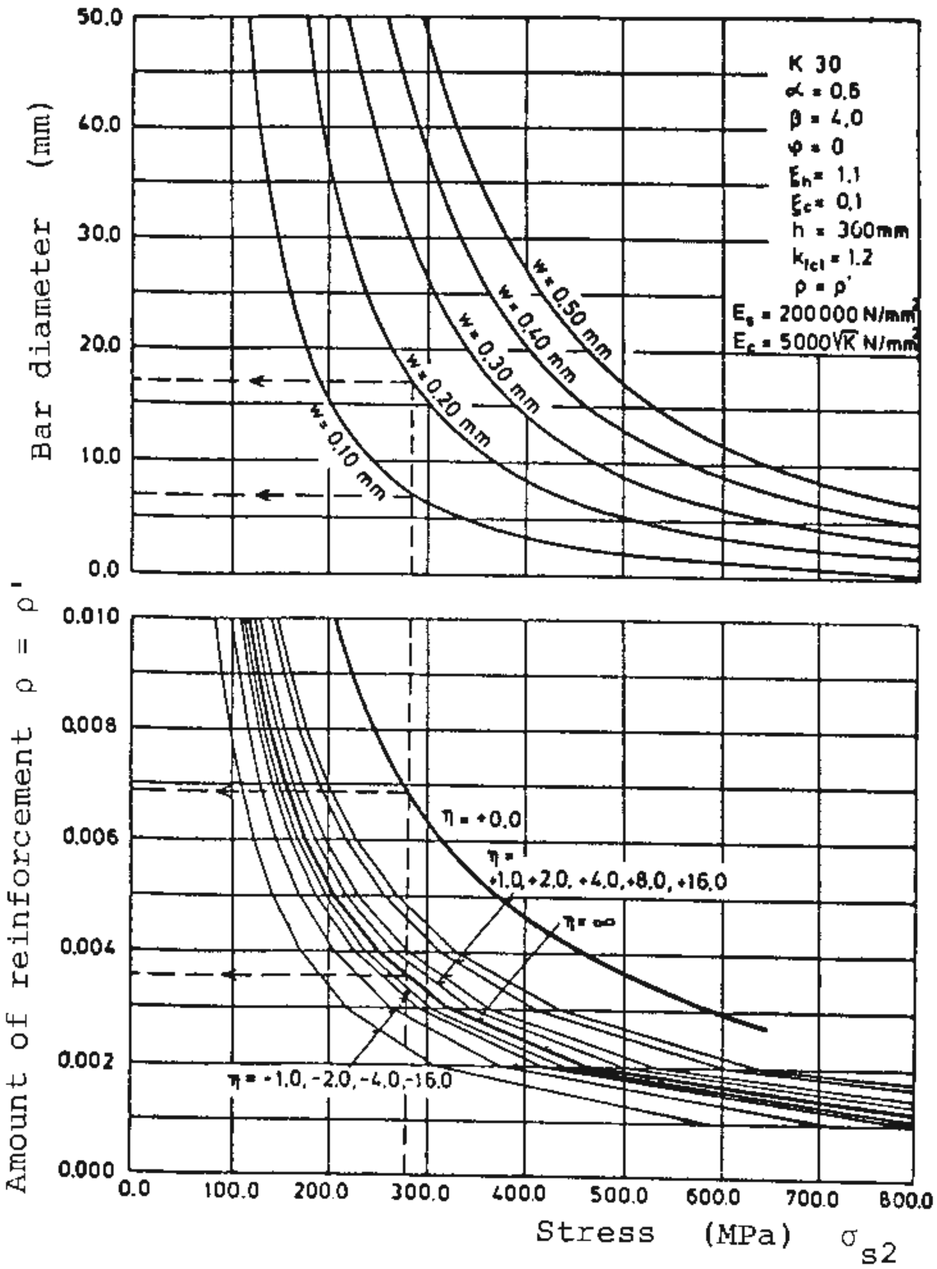


Fig. 10 Example of design curves for pure thermal force /6/.

If the structure is subjected simultaneously to thermal forces and external load, the design task is more complex. Generally through the preliminary design one becomes involved in nonlinear calculation and the use of numerical methods, in which case a solution without the help of a computer is difficult. The principles of most frequently occurring methods of solution are presented below.

The approximate method of effective stiffness is based on the assumption that the effect of cracks on stiffness can be taken into account along the length of the entire structural member by means of a certain reduction factor

$$(EI)_{eff} = k \cdot E_c I_c \qquad (k \leq 1) \tag{16}$$

Factor k is dependent e.g. on the amount of reinforcement at the cross section. Since it is assumed that the structural member possesses a constant stiffness, the force quantities can be calculated on the basis of elastic theory, which, of course, simplifies calculation.

The use of the moment-rotation method requires the preliminary design of the structure, since the M - κ_m relations of the most important cross-sections must then be known. The M - κ_m curves can be determined in different ways, but as realistic a relation as possible will give the most reliable result. It has been proven, however, that the differences in methods do not influence the differences in force quantities to the same degree /10/.

Following the formation of the first cracks the forces must be determined as increments and iteratively so. In the case of the external load of the beam e.g. the support moment can be estimated and the moment distribution determined so that equilibrium is valid. Subsequently, the validity of the boundary conditions is examined, e.g. on the principle of virtual work.

When the restraint and external load are in action there is no significance as to their order since, in any case, in the cracked state the moment distribution must be assumed to be such that the boundary conditions are valid. The moment is a resultant moment, which cannot be classified any longer as a restraint moment and external load moment. The validity of the boundary conditions is tested in the same way as in the case of the external beam load.

This method is also called an integrated method, since the behaviour of concrete, reinforcing bars and bond, particularly after cracking, is described by means of average physical performance models. In spite of considerable numerical work this principle has so far mostly been applied to the design cases which are under the combined thermal and external load.

For materials or composite materials, such as steel and concrete, the physically nonlinear models and cracked model can also be formed using /3/

- individual elements for concrete and steel which are joined together with bond elements
- by dividing concrete and steel to horizontal strip elements and bond elements between them. The behaviour of concrete after cracking at the tension side is then usually considered as a declining part of the curve $\sigma_c - \varepsilon_c$.

These methods inevitably lead to the use of the finite element method.

7. EXPERIMENTAL AND NUMERICAL EXAMINATION OF REINFORCED AND PRESTRESSED CONCRETE BEAMS UNDER THERMAL GRADIENT /5/

7.1 General

In order to study the behaviour of continuous reinforced concrete beams, preliminary tests on continuous reinforced and prestressed concrete beams were carried out so as to check the calculations made by the computer programme ADINA, based on the finite element method taking into consideration the nonlinear behaviour of concrete. The purpose of the research was to clarify the behaviour of the reinforced concrete beams in question when subjected both to temperature stresses and to combined temperature and external mechanical stresses. An attempt was made to draw particular attention to the following matters:

- change of indirect forces when total forces increase
- does the structure under investigation have a sufficient deformation capacity for eliminating the indirect forces in the ultimate state
- effect of prestressing force on the matters in question
- observation of crack formation
- observation of deflections and
- comparison of the behaviour of beams with the values calculated by the finite element method.

7.2 Beam tests

For testing purposes, a total of 12 test beams with a cross-section of 200 x 400 x 7000 mm^3 were made, all similar to one another with regard to their reinforcements and dimensions, except for 4 beams, which were prestressed centrically. The aim was to use three beams out of eight as comparison specimens and to load five beams with a thermal gradient of 80 ^oC and with and external mechanical load.

Thermoelements were inserted into the beams for the purpose of measuring the temperature distribution. Strain gauges were fixed to the reinforcements of some beams for the measurement of steel stress.

In each test the beams were simply supported in such a way that the span was 5000 mm and the length of the cantilever at both ends 1000 mm.

External mechanical load alone was applied to comparison beams P1 to P3. Loading points were symmetrically located at a distance of 2000 mm from the supports. With the point loads resting on the cantilevers, spaced 900 mm from the supports, the rotation of the supports was kept unchanged (=0). External loading of the beams was gradually increased until the beam reached a state of failure.

After each gradual increase in indirect or external load, the rotation at the supports was brought back to zero.

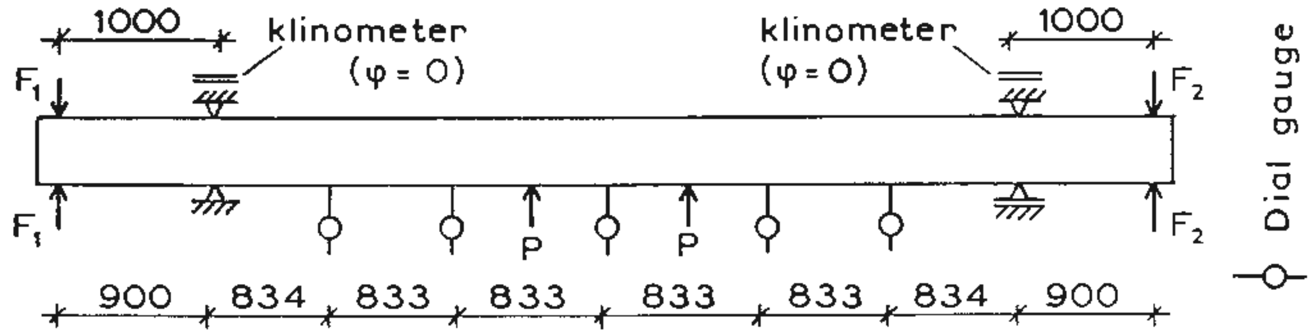


Fig. 11 Loading of the test beams.

Beams PL1 to PL5 were first loaded with a thermal gradient (Fig. 12). The upper surface of the beam was cooled at about +12...14 °C and the lower side of the beam was simultaneously heated to a temperature of about +92...94 °C. The temperature distribution was to be converted into linear form and the temperature difference kept at about 80 °C. The external load was thus increased to failure.

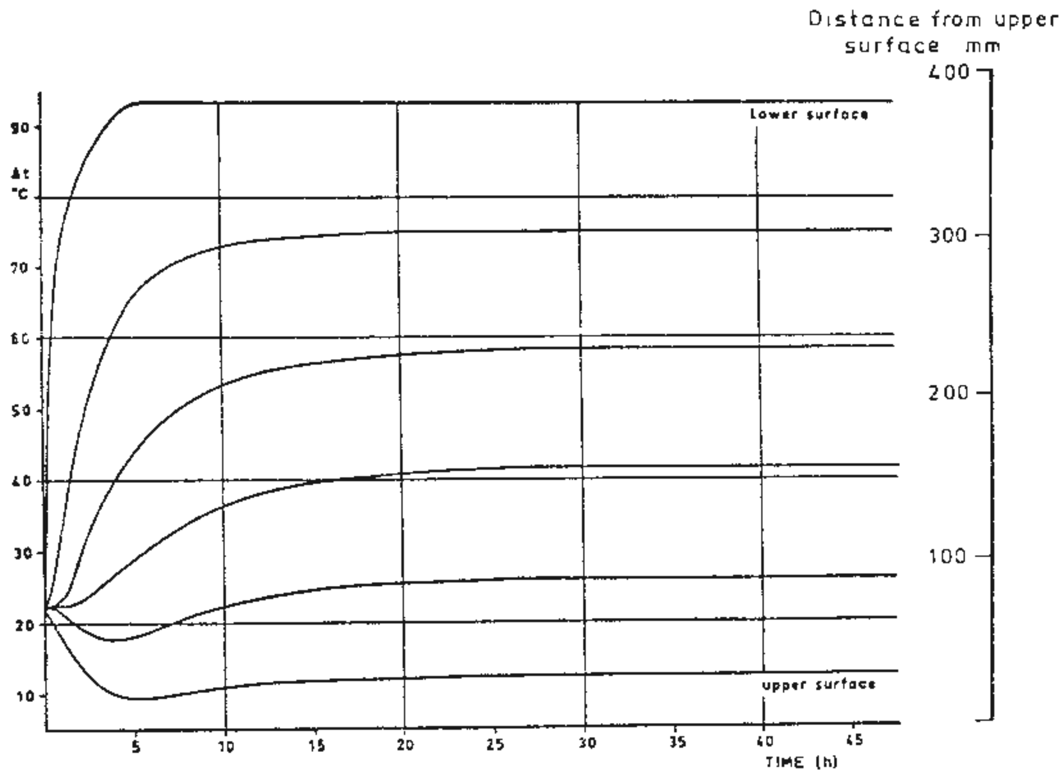


Fig. 12 Change of temperature in test beams.

During thermal loading the support moments varied as presented in Fig. 13.

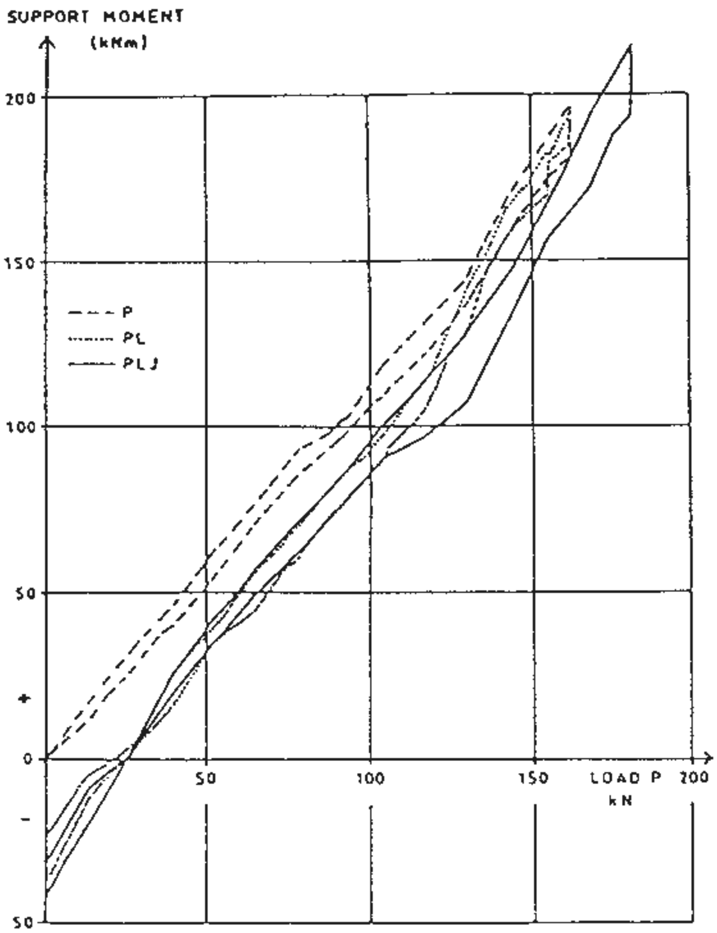


Fig. 13 Support moments during thermal and external load.

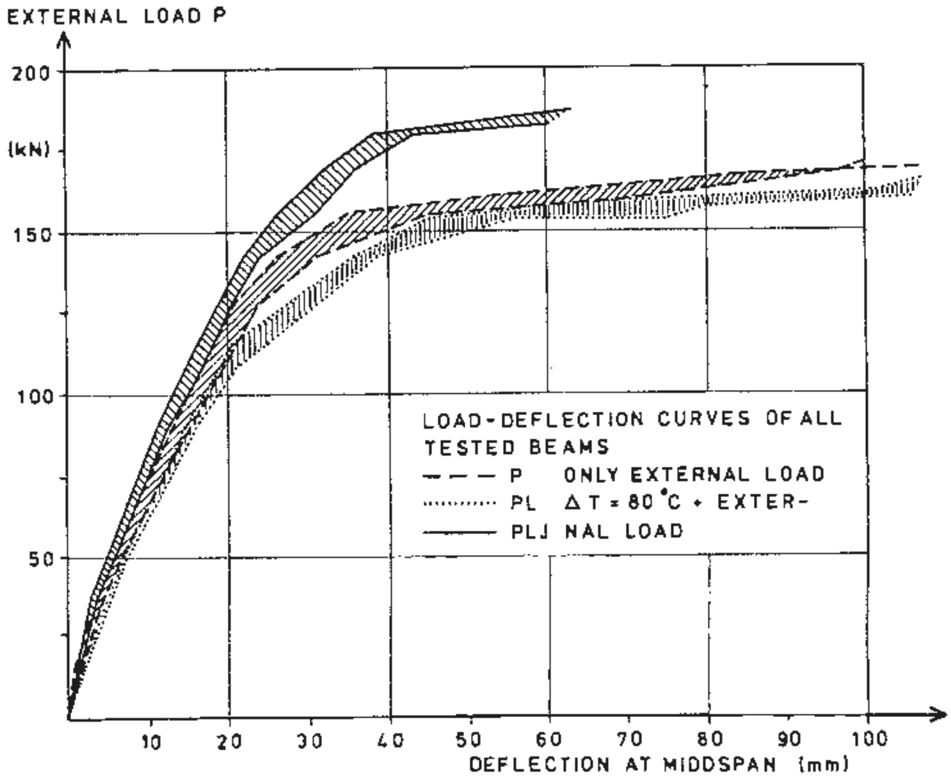
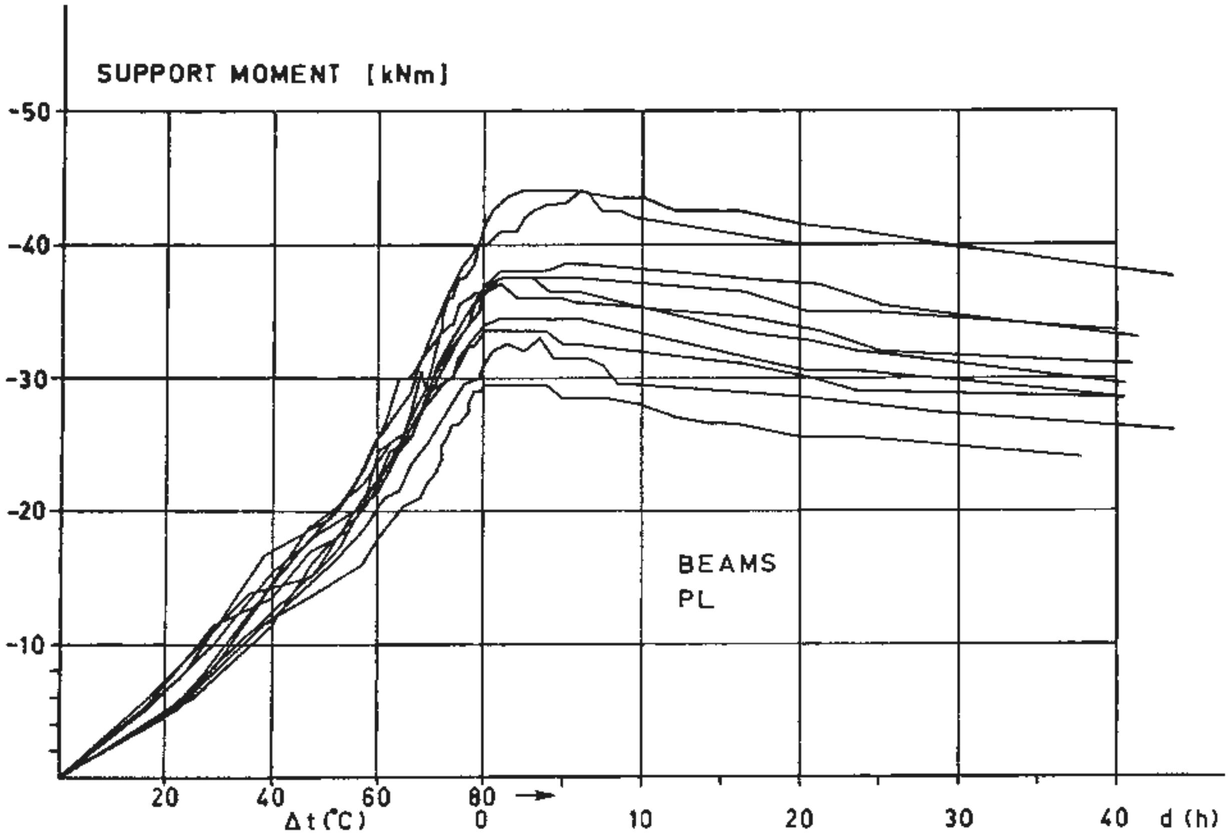


Fig. 14 Load-deflection curves of all beams at the mid-span.

The first cracks on the PL beams were observed at 36...47 °C temperature difference and on PLJ beams at 58...68 °C difference correspondingly.



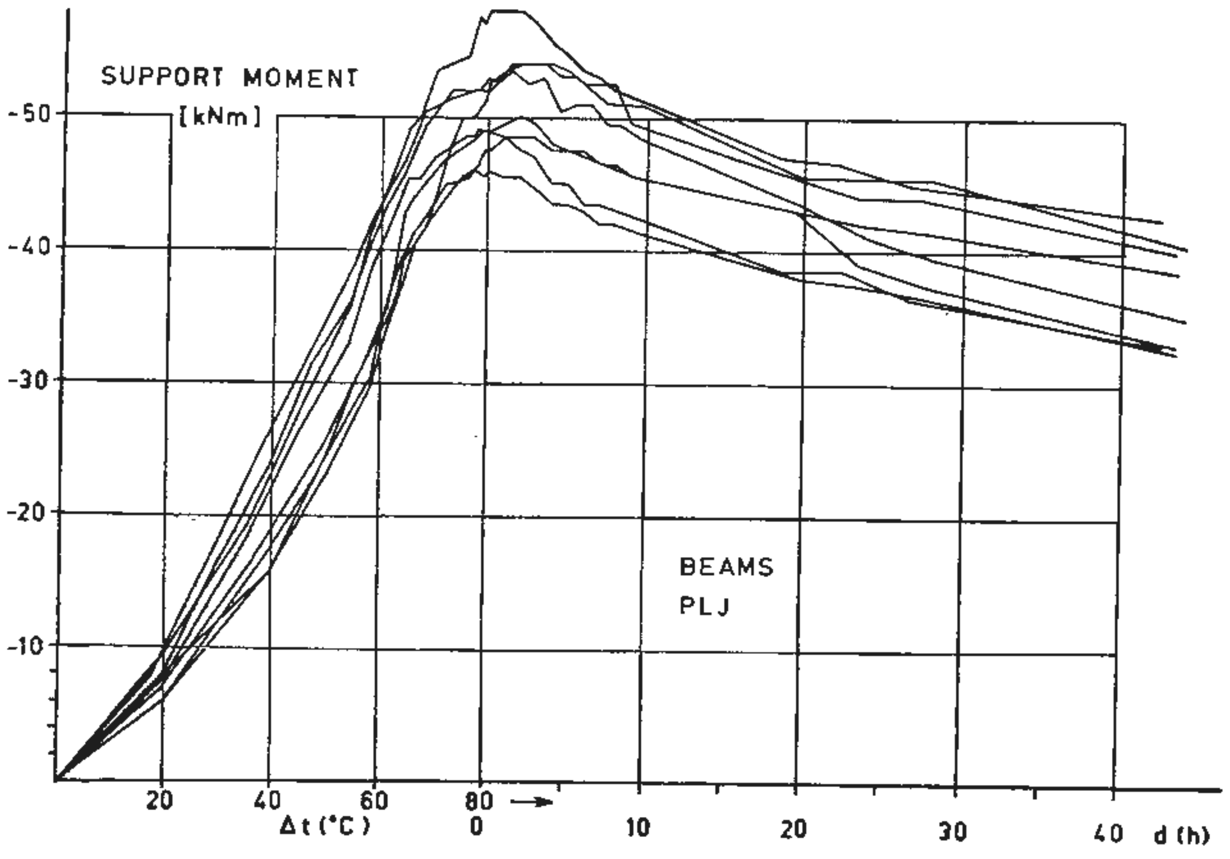


Fig. 15 Change of support moments versus temperature and time.

7.3 Calculations

The calculations have been carried out using the nonlinear computer programmes ADINA (Automatic Dynamic Incremental Nonlinear Analysis) modified by VTT (Technical Research Centre of Finland), and ADINAT based on a finite element method. The latter programme calculated the temperatures at the nodal point of the element mesh, and the former the displacements and stresses at different points in time.

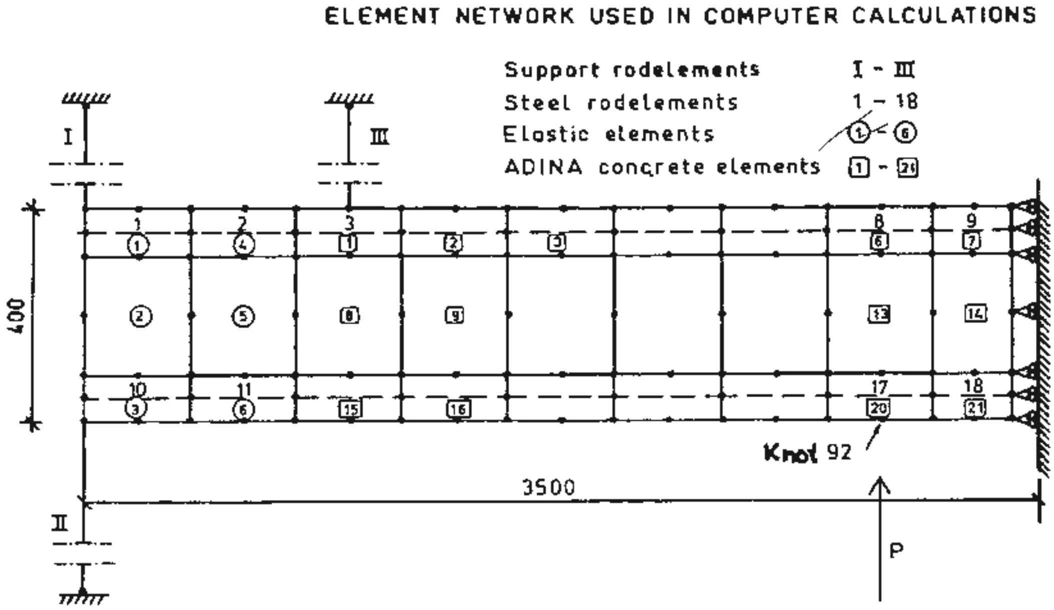


Fig. 16 Calculation model and element mesh.

In the calculations, the first cracks were produced on the upper surface of the beam with a temperature difference of 30 °C. With an increase in temperature more cracks were being produced and deepened. During the final heating stage the entire upper surface of the beam was cracked.

In the static loading phase, formation of diagonal cracks occurred at the support. The cracks on the upper surface due to heating were deepened at the span and pressed together at the supports.

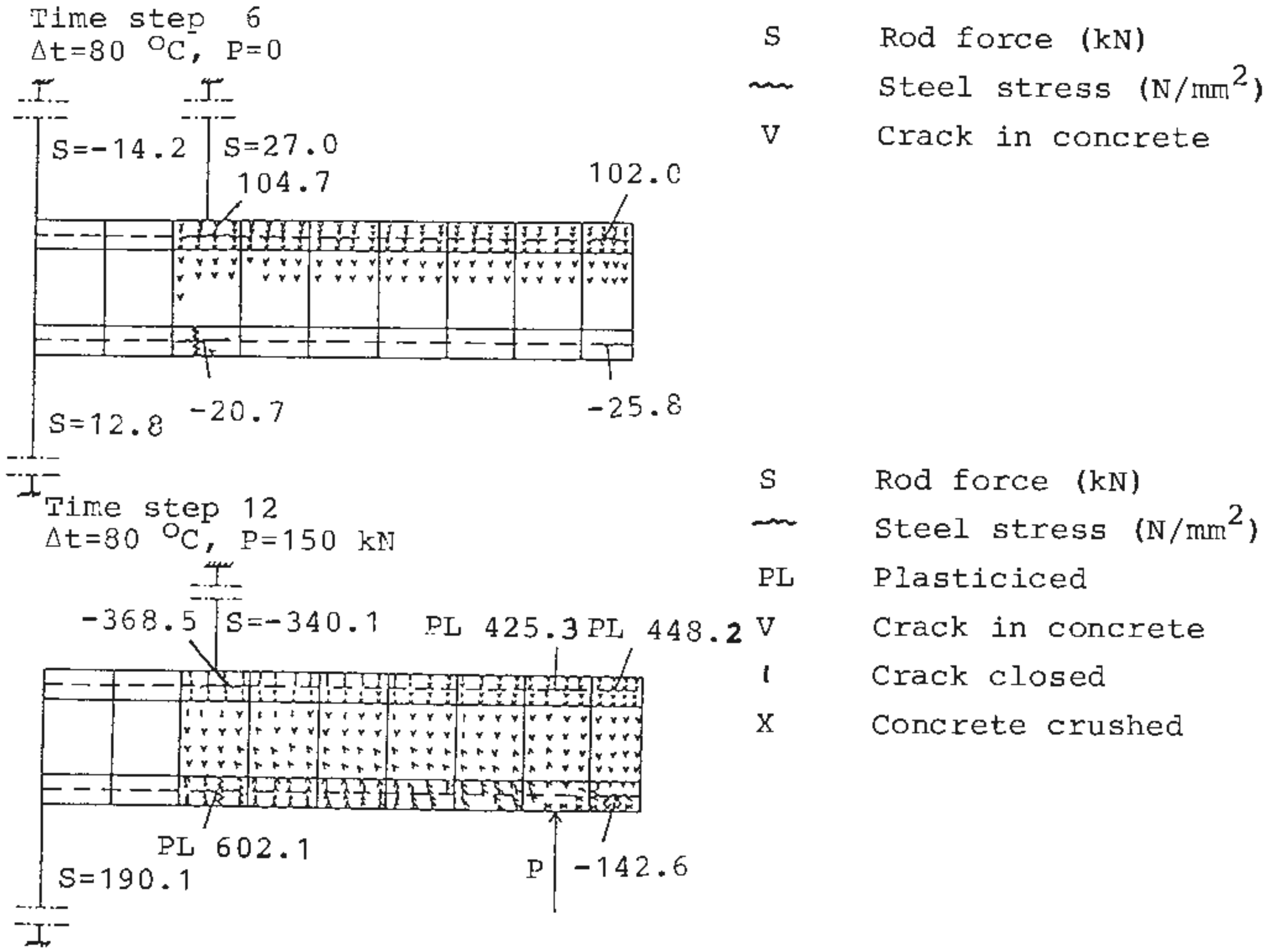


Fig. 17 Calculated crack patterns at T = 80 °C, P = 0 and P = 150 kN.

7.4 Comparison of results

Fig. 18 illustrates the load-deflection curves calculated and established in testing of PL beams.

It is found that the calculated deflections correspond very well to the measured values. However, the calculation model has been slightly stiffer than the actual structure.

The values of the calculated and measured support moments are presented in Fig. 19, which also confirms their rather good compatibility. In the ultimate state only the calculated values are slightly greater.

Cracks on the PL beams were observed a bit later than calculated. The reason for this is probably the underestimating of the modulus of rupture of the concrete.

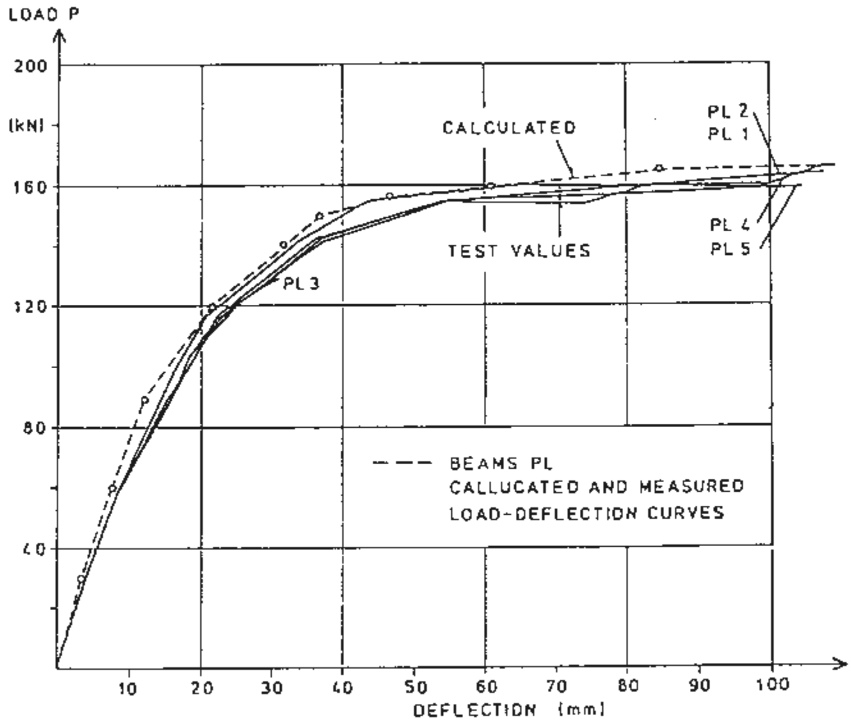


Fig. 18 Load deflection curves of PL beams.

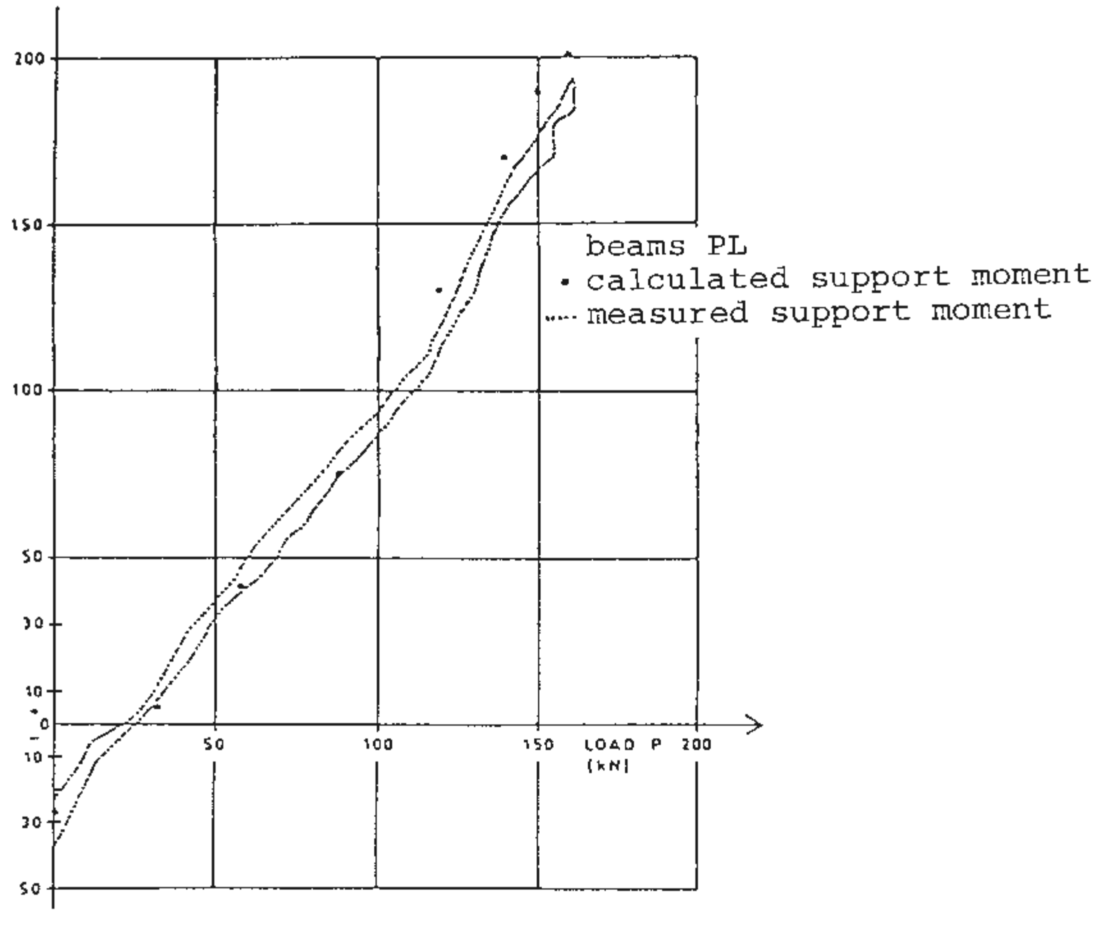


Fig. 19 Support moments of PL beams.

NOTATIONS

P1...P3	Reference or comparison beams in tests.
PL1...PL5	Test beams loaded with thermal load and combined thermal and external load
PLJ1...PLJ4	Prestressed test beams loaded in the same manner as beams PL1...PL5.
E_c	Modulus of elasticity of concrete
I_c	Moment of inertia of concrete cross-section
K	Nominal concrete strength
L	Span
M_o	Moment at time = 0
M_{cr}	Cracking moment
M_∞	Moment at time = ∞
M_f	Restraint moment
Q_1, Q_2	Heat quantities
T	Temperature
T_o	Reference temperature
ΔT	Temperature difference
T_{inf}	Temperature at lower surface
T_{sup}	Temperature at upper surface
c	Specific heat
d	Effective depth of structure
h	Heigh of cross-section
k	Constant
l_b	Bond transfer length
t	time
w	width of crack
α, β	Bond parameters
α_T	Thermal expansion coefficient of concrete
λ	Heat conductivity
ε	Relative strain
κ	Curvature
ρ_k, ρ_c	Cross-sectional parameters
ρ	Density of the material, amount of reinforcement
Θ	Rotation

REFERENCES

1. Alemo, J., The effects of imposed deformations on the behaviour of loaded concrete structures. Lund 1976. Lund Institute of Technology, Division of Structural Mechanics and Concrete Construction, Bulletin 53. 199 p.

2. Bruy, E., Über den Abbau instationärer Temperaturspannungen in Betonkörpern durch Rissbildung. Stuttgart 1973. Otto-Graf-Institut, Schriftenreihe, Heft 56. 179 p.

3. Dinges, D. etc., Untersuchung verschiedener Elementsteifigkeitsmatrizen auf ihre Eignung zur Berechnung von Stahlbetonkonstruktionen. Technische Hochschule Darmstadt, Darmstadt 1982. 174 p.

4. Falkner, H., Zur Frage der Rissbildung durch Eigen- und Zwängspannungen infolge Temperatur in Stahlbetonbauteilen. Berlin 1969. Deutscher Ausschuss für Stahlbeton, Heft 208. 99 p.

5. Jokela, J., Restrained concrete beam under thermal and mechanical bending load. RILEM-ACI Symposium Long Term Observation of Concrete Structures. Budapest, September 1984. Vol 1. 13 p.

6. Jokela, J. & Huovinen, S., Behaviour and design of concrete structures under thermal gradients. Espoo 1984. Technical Research Centre of Finland, Research reports. (In print, Finnish text)

7. Kordina, K., Zur Frage der näherugsweisen Ermittlung von Zwangsschnittgrössen. Symposium Design of Concrete Structures for Creep, Shrinkage and Temperature Changes. Madrid 1970, International Association for Bridge and Structural Engineering. Zurich 1970. P. 441...450.

8. Leonhardt, F. etc., Wasserturm ohne Wärmedämmung, Abminderung von Zwingkräften und Rissebeschränkung. Beton- und Stahlbetonbau 64(1969)6, p. 129...135.

9. Maack, P., Der Einfluss der Zeit auf die Steifigkeit von Stahlbeton-Querschnitten. Dissertation. Technische Universität Braunschweig. Braunschweig 1970. 77 p.

10. Rüsch, H., Die wirklichkeitsnahe Bemessung für lastunabhängige Spannung. In Publikation: Vorträge auf dem Betontag 1965. Deutscher Beton-Verein, Lauterbach 1965. P. 277 - 304.