LIGHTWEIGHT AGGREGATE CONCRETE BEAMS. LOAD-BEARING CAPACITY.

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ABSTRACT

This paper deals with the load-bearing capacity of reinforced beams made of lightweight aggregate concrete with open structure and documents formulas for the moment capacity as well as the shear force capacity.

Keywords: Load-bearing capacity, lightweight aggregate concrete, beam.

1 INTRODUCTION

Components of lightweight aggregate concrete with open structure are very often used in interior walls and in inner walls in facades in low-rise housing.

The project presented in this paper deals with the load-bearing capacity of beams over doors and windows in such walls.

See figure 1 and figure 2.

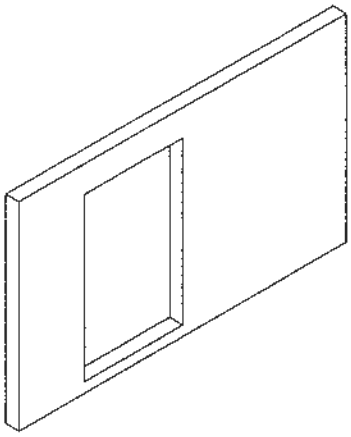


Figure 1. Wall with beam over door.

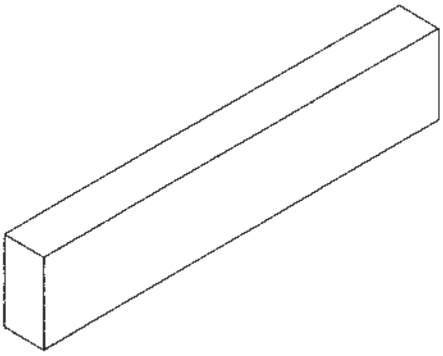


Figure 2. Beam.

EuroCode 2 (EC2) for concrete structures /1/ has formulas for the calculation of the load-bearing capacity of beams. This also applies to part 1-4 of EuroCode 2 for lightweight aggregate concrete with closed structure /2/.

The purpose of the project is to examine these formulas for beams of lightweight aggregate concrete with open structure.
The moment capacity and the shear force capacity are dealt with.

Furthermore the paper deals with formulas for the calculation of the shear force capacity derived from the theory of plasticity.

The presented formulas for the calculation of the load-bearing capacity are verified by results from tests carried out.

2 EXPRESSIONS FOR LOAD-BEARING CAPACITY

2.1 Moment capacity

The expression used for the calculation of the moment capacity (M_{cal}) is

$$M_{cal} = (1 - \tfrac{1}{2} \Phi) \cdot A_s \cdot f_y \cdot d \qquad (1)$$

$$\Phi = A_s \cdot f_y \,/\, (b \cdot d \cdot f_c)$$

where

A_s = area of the reinforcement
b = width of the component
d = effective depth of the cross-section
(distance from the reinforcement to the edge in compression)
f_c = compressive strength of the lightweight concrete
f_y = tensile yield strength of the reinforcement
Φ = degree of reinforcement

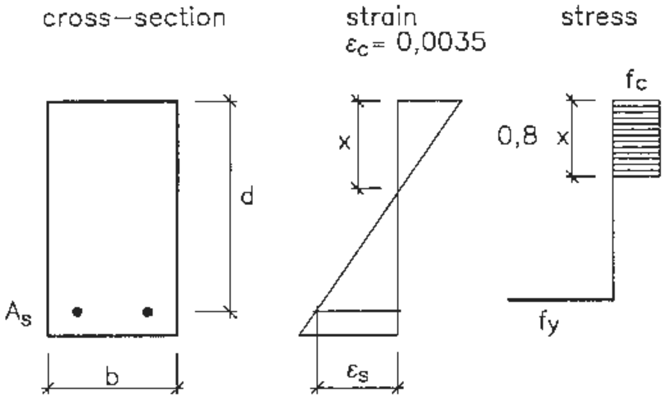


Figure 3. Moment strain and stress distribution in cross-section.

The expression for the moment capacity is established on the basis of the assumptions normally assumed in the calculation of the moment capacity of reinforced "normal weight" concrete.

The principal assumptions are:

plane sections remain plane,

tension stresses in the concrete are ignored,

compressive stresses in the concrete are taken as equivalent to a constant stress equal to the compressive strength on 4/5 of the depth of the compressive zone,

tensile yield stress in the reinforcement, provided that the strain is not taken as greater than the ultimate strain,

corresponding to the assumptions in EuroCode 2.

2.2 Shear force capacity

The shear force capacity determined by

1) the standard method in EuroCode 2,
2) the variable strut inclination method in EuroCode 2 and
3) the shear force capacity determined by the theory of plasticity,

are dealt with.

2.2.1 EuroCode 2 standard method

The shear force capacity of a section with shear reinforcement is

$$V_{cal,stan} = k_c \cdot \tau_u \cdot k_1 \cdot (1,2 + 40 \cdot \varphi) \cdot b \cdot d + k_w \cdot z \cdot A_{st} / s \cdot f_{yt} \qquad (2)$$

$\tau_u = f_{bt} / 8$

$k_1 = 1,6 - d \qquad k_1 \nless 1 \qquad d$ in m

$\varphi = A_s / (b \cdot d)$

$z = \min. \{ 0,9\ d \ ; h_t \}$

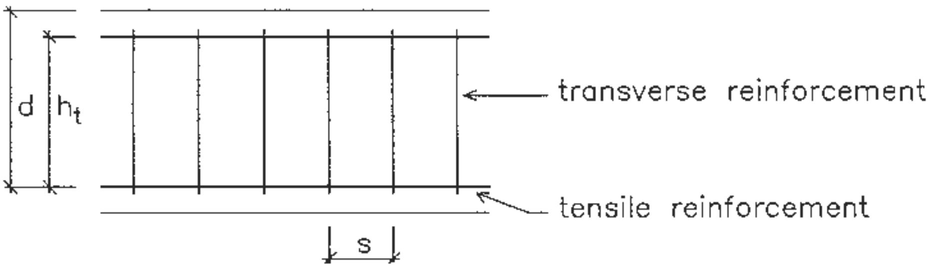


Figure 4.

where

A_s = area of the (tensile) reinforcement
A_{st} = area of the transverse reinforcement
b = width of the beam
d = effective depth of the cross-section
f_{bt} = flexural strength of the lightweight aggregate concrete
f_{yt} = yield strength of the transverse reinforcement
h_t = depth of the transverse reinforcement
k_c = reduction factor
k_w = reduction factor
s = spacing of the transverse reinforcement

k_c and k_w are taken as 1 in EuroCode 2.

In prEN 1520 "Prefabricated Components of Lightweight Aggregate Concrete with Open Structure, /3/, expression (2) is given with $k_c = 1$ and $k_w = 0,8$.

2.2.2 EuroCode 2 variable strut inclination method

The shear force capacity of a beam with shear reinforcement is the lowest of the four values V_1, V_2, V_3, V_4:

$$V_{cal.strut} = \min. \{ V_1; V_2; V_3; V_4 \} \tag{3}$$

$$V_1 = A_{st} / s \cdot z \cdot f_{yt} \cdot \cot(\beta) \tag{4}$$

$$V_2 = b \cdot z \cdot v \cdot f_c / (\cot(\beta) + \tan(\beta)) \tag{5}$$

$$V_3 = A_s \cdot f_y / (0{,}5 \cdot \cot(\beta)) \tag{6}$$

$$V_4 = k_t \cdot A_{st} / s \cdot z \cdot f_{yt} \cdot \cot(\beta) \tag{7}$$

$v = 0{,}7 - f_c / 200$ $\quad f_c$ in N/mm^2

$z = \min. \{ 0{,}9\, d ; h_t \}$

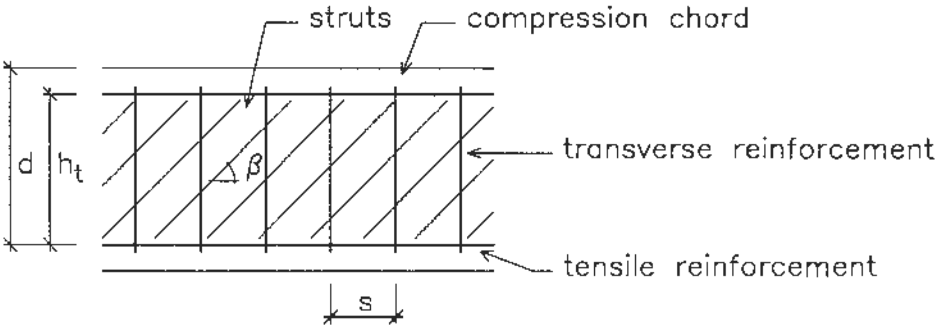


Figure 5.

where

A_s = area of tensile reinforcement
A_{st} = area of transverse reinforcement
b = width of the beam
d = effective depth of the cross-section
f_c = compressive strength of concrete
f_y = yield strength of the tensile reinforcement
f_{yt} = yield strength of the transverse reinforcement
h_t = depth of transverse reinforcement
k_t = reduction factor
s = spacing of transverse reinforcement
β = angle of the concrete struts with the longitudinal axis of the beam

The reduction factor k_t accounts for the (reduced) strength of the welded joints in the transverse reinforcement and may have to be justified.

The angle of the concrete struts β with the longitudinal axis may also have to be justified.

2.2.3 Method of the plastic theory

The expressions, given in /4/, for the upper and lower bound solutions derived from the theory of plasticity, are presented in the following.

For high relative shear spans, $a/z \geq 3$, the lower and upper bound solutions lead to the same equation, and therefore this equation represents the exact solution for the shear force capacity of a beam with transverse reinforcement:

$$V_{cal,pla,o} = \nu_{pla} \cdot f_c \cdot b \cdot z \cdot \sqrt{[\,\omega_{pla} \cdot (1 - \omega_{pla})\,]} \tag{8}$$

$$\omega_{pla} = k_t \cdot A_{st} / s \cdot f_{yt} \; / \; (b \cdot \nu_{pla} \cdot f_c)$$

$$z = \min.\ \{\, 0{,}9\ d\ ;\ h_t \,\}$$

For lower relative shear spans equation (8) is the lower bound for the shear force capacity.

The upper bound for the shear force capacity is

$$V_{cal,pla,1} = \nu_{pla} \cdot f_c \cdot b \cdot z \cdot (\, 0{,}5 \cdot (\, \sqrt{[\, 1 + (a/z)^2\,]} - a/z\,) + \omega_{pla} \cdot a/z\,) \tag{9}$$

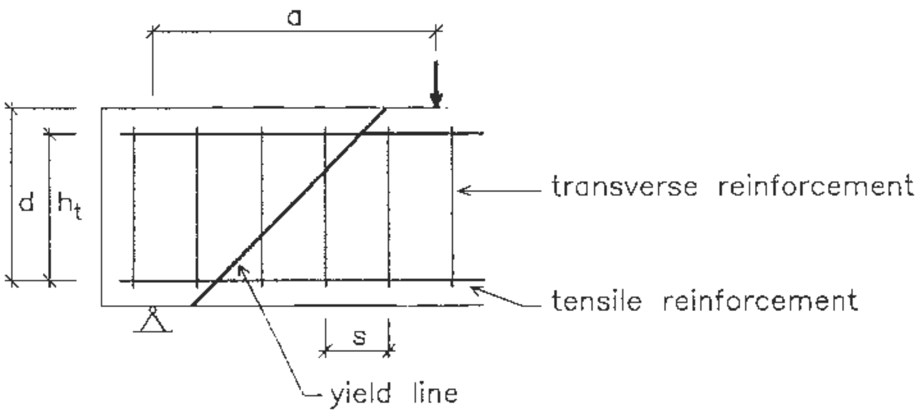


Figure 6.

A_{st} = area of transverse reinforcement
a = shear span
b = width of the beam
d = effective depth of the cross-section
f_c = compressive strength of concrete
f_{yt} = yield strength of the transverse reinforcement
h_t = depth of transverse reinforcement
k_t = reduction factor
s = spacing of transverse reinforcement
ν_{pla} = effectiveness factor
ω_{pla} = degree of transverse reinforcement

The upper bound solution is based on a yield mechanism with a yield line as shown on figure 6. For lower relative shear spans, a/z < 3, the angle of the yield line with the longitudinal axis is determined by: $\tan(\beta) = z/a$.

The lower bound solution is based on concrete struts like the struts shown on figure 5.

The effectiveness factor ν_{pla} has to be justified.

The reduction factor k_t accounts for the (reduced) strength of the welded joints in the transverse reinforcement and may have to be justified.

3 TESTS

3.1 General

Tests for verification of the expressions for the load-bearing capacity were carried out within the following limits of dimensions and materials specifications:

depth of the beams, h = 20 to 42 cm
width of the beams, b = 10 to 24 cm
span (length), l = 1,5 to 3,0 m
compressive strength of lightweight aggregate concrete, f_c = 5 to 20 N/mm^2
density of lightweight aggregate concrete, ρ = 1000 to 1800 kg/m^3
flexural strength of lightweight aggregate concrete, f_{bt} = 1,5 to 3 N/mm^2
reinforcement: hot-rolled deformed steel (ribbed bars) and smooth bars (only stirrups)
diameter of reinforcing bars, 5 to 12 mm
guaranteed value of the tensile yield strength (or 0,2% proof-stress) of the reinforcement, f_{yk} ($f_{0,2}$) = 300 to 550 N/mm^2

The reinforcement of the beams are either a stirrup-reinforcement as shown on Figure 7 or "ladder" reinforcement as shown on Figure 8.

The beams are manufactured at 10 Danish factories and represent the range of the current Danish production.

3.2 Tests with beams to determine the moment capacity and the shear force capacity

The testing was carried out according to DS 434-11 "Performance Test of Beam and Slab Components", /5/.

The components were simply supported at both ends, with one of the supports horizontally movable. The beams were loaded by two transverse loads at the outer quarter points of the span. Some of the beams were loaded by one transverse load at the middle

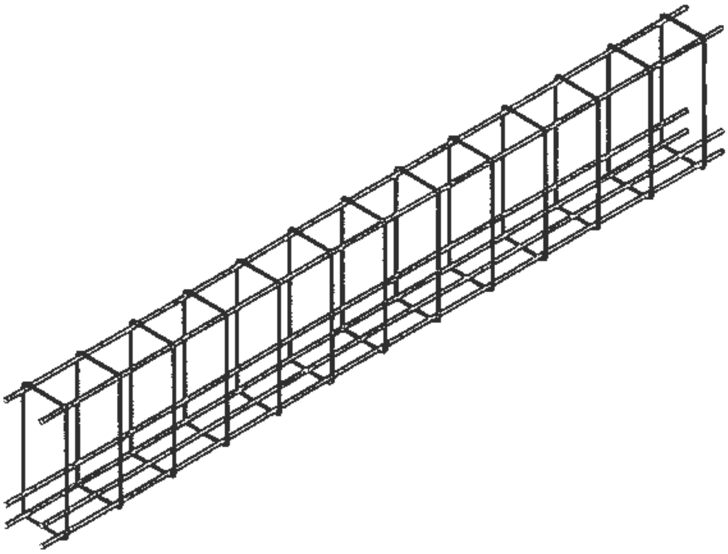


Figure 7. Beam reinforcement with stirrups.

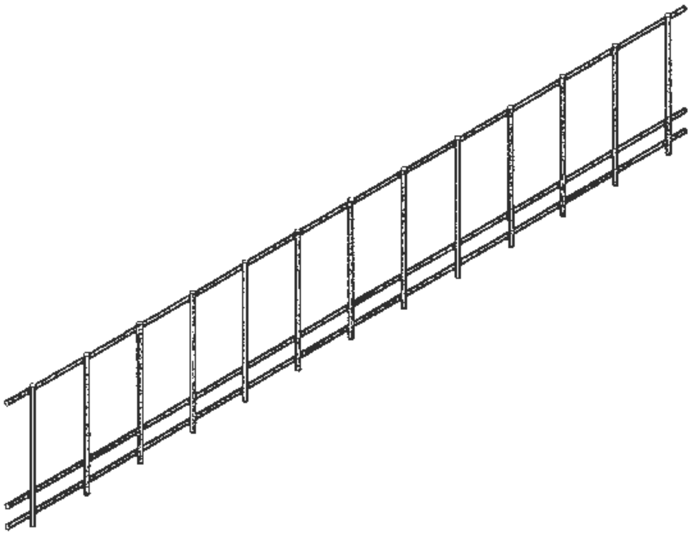


Figure 8. Ladder beam reinforcement.

of the span. In order to ensure a shear failure, some of the beams were loaded by one transverse load at one outer quarter point of the span.

The detailed test results are presented in /6/.

3.3 Testing of materials

Reinforcement

The tensile yield strength of the steel was determined on specimens representative for the reinforcing bars of the tested beams.

Furthermore the strengths of the welded joints were determined for both the ladder reinforcement and the stirrups.

Lightweight aggregate concrete

The compressive strength, the density and the flexural strength were determined.

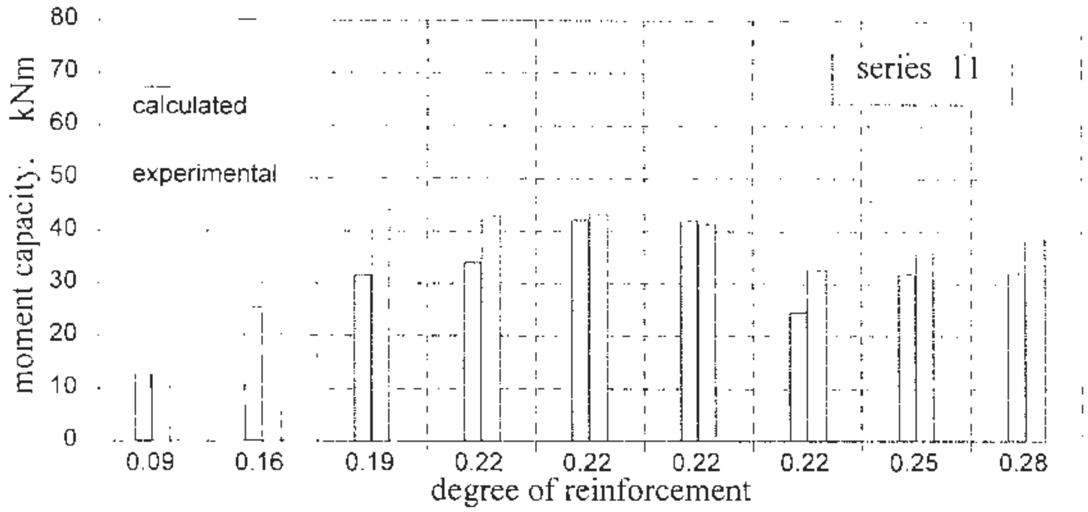
The testing was carried out according to the Danish test standards DS 434, /7/, /8/, /9/.

The test specimens were cut from an additional component produced together with the beams. For the determination of each of the material parameters a set of three specimens were used, and the mean values were used.

Further information and results of the materials testing are presented in /6/ and /10/.

4 EXPERIMENTAL VS. CALCULATED LOAD-BEARING CAPACITIES.

On the basis of the load which the components were capable of carrying in the tests, the experimental moment capacity M_{exp} or the experimental shear force capacity V_{exp} are determined, dependent on the observed failure mode.



length (span), m	2.5	2.5	2.5	2.5	3.1	3.1	2.5	2.5	2.5
depth, cm	30	30	30	30	30	30	30	30	30
width, cm	10	15	15	15	24	24	15	15	15
tensile reinforcement, mm	2 ø 8	4 ø 8	2 ø 12	3 ø 10	6 ø 8	6 ø 8	4 ø 8	2 ø 12	3 ø 10
concrete density, kg/m3	1630	1620	1620	1630	1530	1530	1530	1530	1510
concrete strength, MPa	19	19	19	16	12	12	14	14	13
steel strength, MPa	480	560	610	600	590	590	560	610	600
degree of reinforcement	0.09	0.16	0.19	0.22	0.22	0.22	0.22	0.25	0.28
beam no.	1264	2301	2531	2511	2382	2381	2302	2532	2512

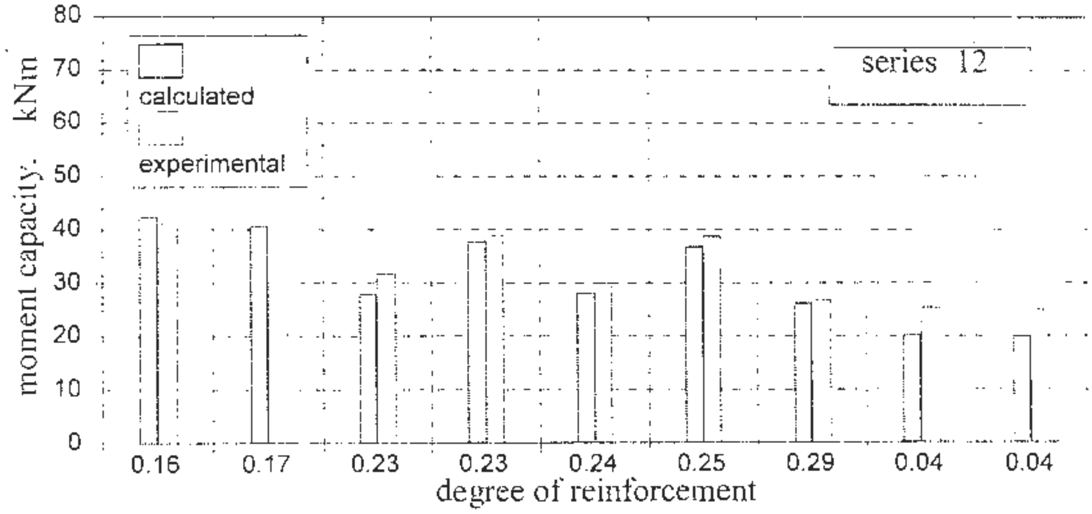
Figure 9. Moment capacity. Series 11.

By use of the expressions in chapter 2 the calculated moment capacity M_{cal} and the calculated shear force capacity V_{cal} are determined based on the actual cross-sectional geometry and the material parameters.

The results of the verification are, in extract, presented below; they are shown in series for the factories one by one. Series with only few results are omitted.

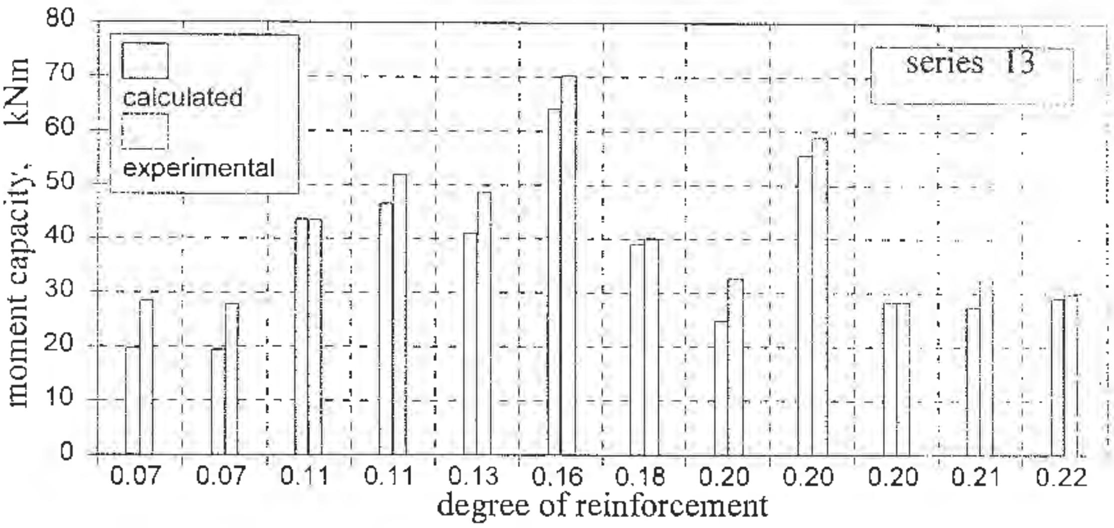
Figures 9 to 11 show the calculated *moment* capacities as well as the experimental ones.

A good agreement can be observed between the experimental moment capacity and the moment determined by the expression in section 2.1 for the moment capacity.



length (span), m	3.1	3.1	1.3	3.1	1.3	2.5	1.3	3.1	3.1
depth, cm	42	42	30	30	30	28	30	42	42
width, cm	15	15	15	20	15	24	15	15	15
tensile reinforcement, mm	4 ø 8	4 ø 8	4 ø 8	6 ø 8	4 ø 8	6 ø 8	4 ø 8	2 ø 8	2 ø 8
concrete density, kg/m3	1540	1510	1540	1550	1510	1460	1450	1610	1610
concrete strength, MPa	13	13	13	15	13	12	10	25	25
steel strength, MPa	602	602	602	564	602	564	564	564	564
degree of reinforcement	0.16	0.17	0.23	0.23	0.24	0.25	0.29	0.04	0.04
beam no.	1394	1395	1331	2201	1332	2212	1315	2262	2272

Figure 10. Moment capacity. Series 12.



length (span), m	3.1	3.1	3.1	3.1	1.7	3.1	3.1	1.3	1.7	2.5	1.3	2.5
depth, cm	42	42	42	43	30	42	30	30	42	30	30	30
width, cm	15	15	15	15	24	15	15	15	15	15	15	15
tensile reinforcemnt, mm	2 ø 8	2 ø 8	4 ø 8	2 ø 12	6 ø 8	6 ø 8	6 ø 8	4 ø 8	6 ø 8	4 ø 8	4 ø 8	2 ø 12
concrete density, kg/m3	1540	1540	1560	1560	1590	1560	1610	1540	1540	1530	1530	1530
concrete strength, MPa	16	16	20	20	21	20	25	16	16	15	15	15
steel strength, MPa	564	564	602	564	564	602	564	564	564	602	602	564
degree of reinforcement	0.07	0.07	0.11	0.11	0.13	0.16	0.18	0.20	0.20	0.20	0.21	0.22
beam no.	2261	2271	1391	1381	2292	1371	2231	2221	2281	1351	1334	1361

Figure 11. Moment capacity. Series 13.

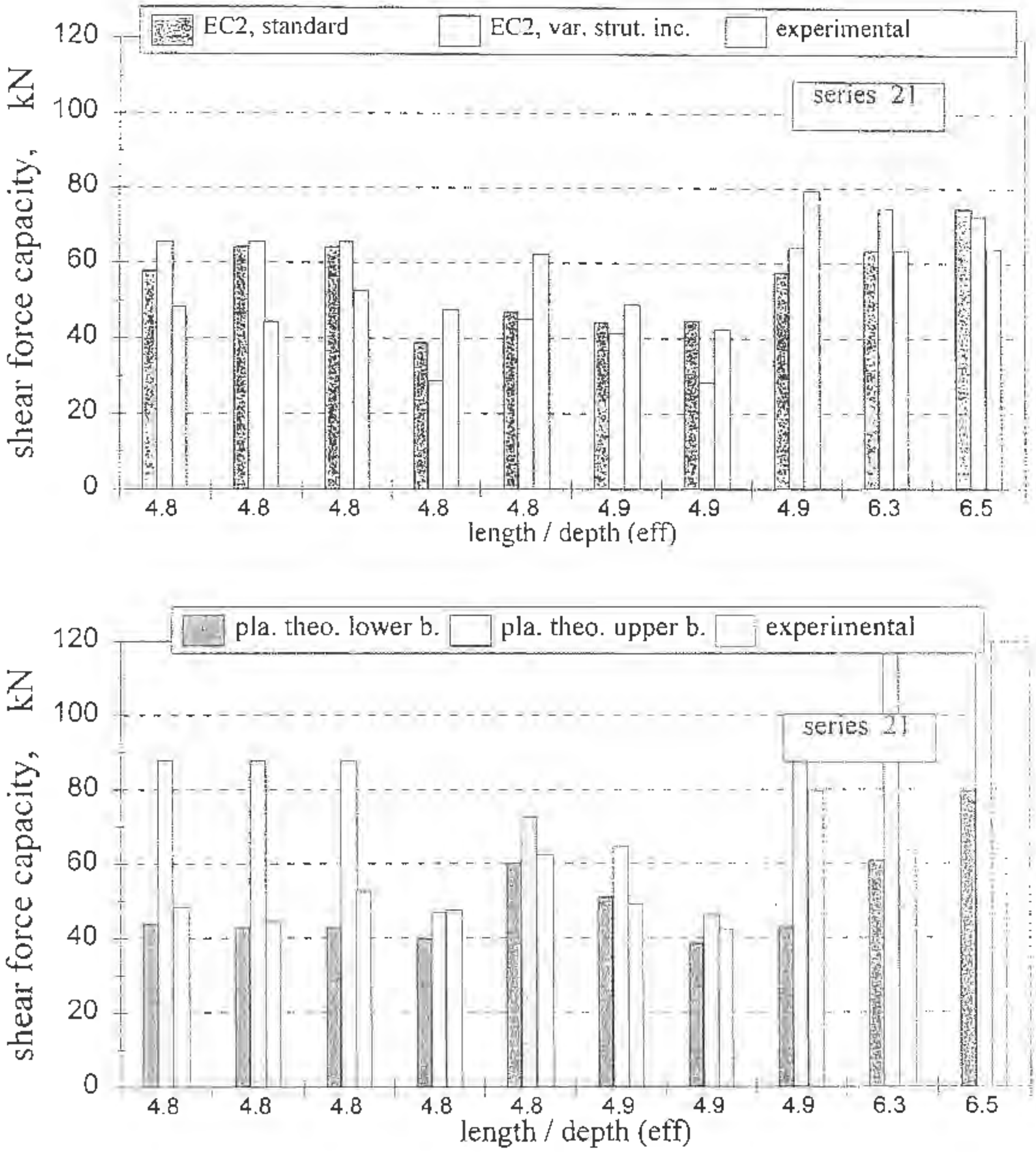
Figures 12 to 15 show the calculated *shear force* capacities determined by the expressions in section 2.2.1 and in section 2.2.2 as well as the experimental ones.

As seen there is a fairly good agreement between the experimental shear force capacity and the shear force capacity determined by the expressions in section 2.2.1 ("EC2, standard") and 2.2.2 ("EC2, var. strut. inc.").

The capacity according to the variable strut inclination method has been calculated with $\cot(\beta) = 1.2$. This low value for $\cot(\beta)$ leads to underestimation of the capacities but the relatively short beams lead to an estimate of an upper limit for $\cot(\beta)$ of about 1.2.

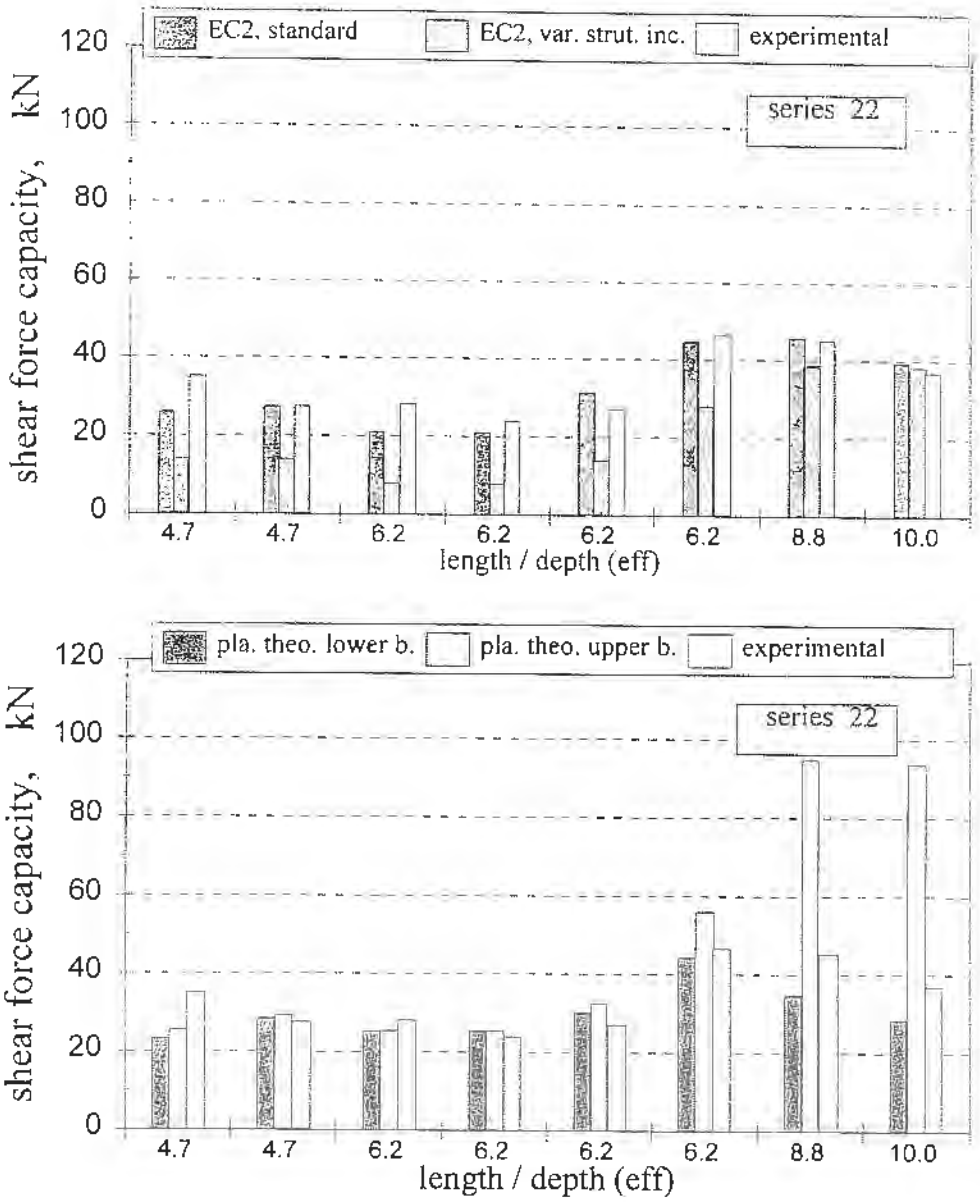
The capacity according to the standard method of EuroCode 2 has been calculated with $k_c = 0.9$ and $k_w = 0.8$ as these values are found to give the best agreement between calculated and experimantal carrying capacity. In /3/ is also applied $k_w = 0.8$ (but $k_c = 1.0$).

Investigations as to how well the two above-mentioned methods give load-bearing capacities corresponding with the performed tests have shown that the standard method gives the best agreement between calculated and experimental load-bearing capacity, but the difference between the two methods is not great.



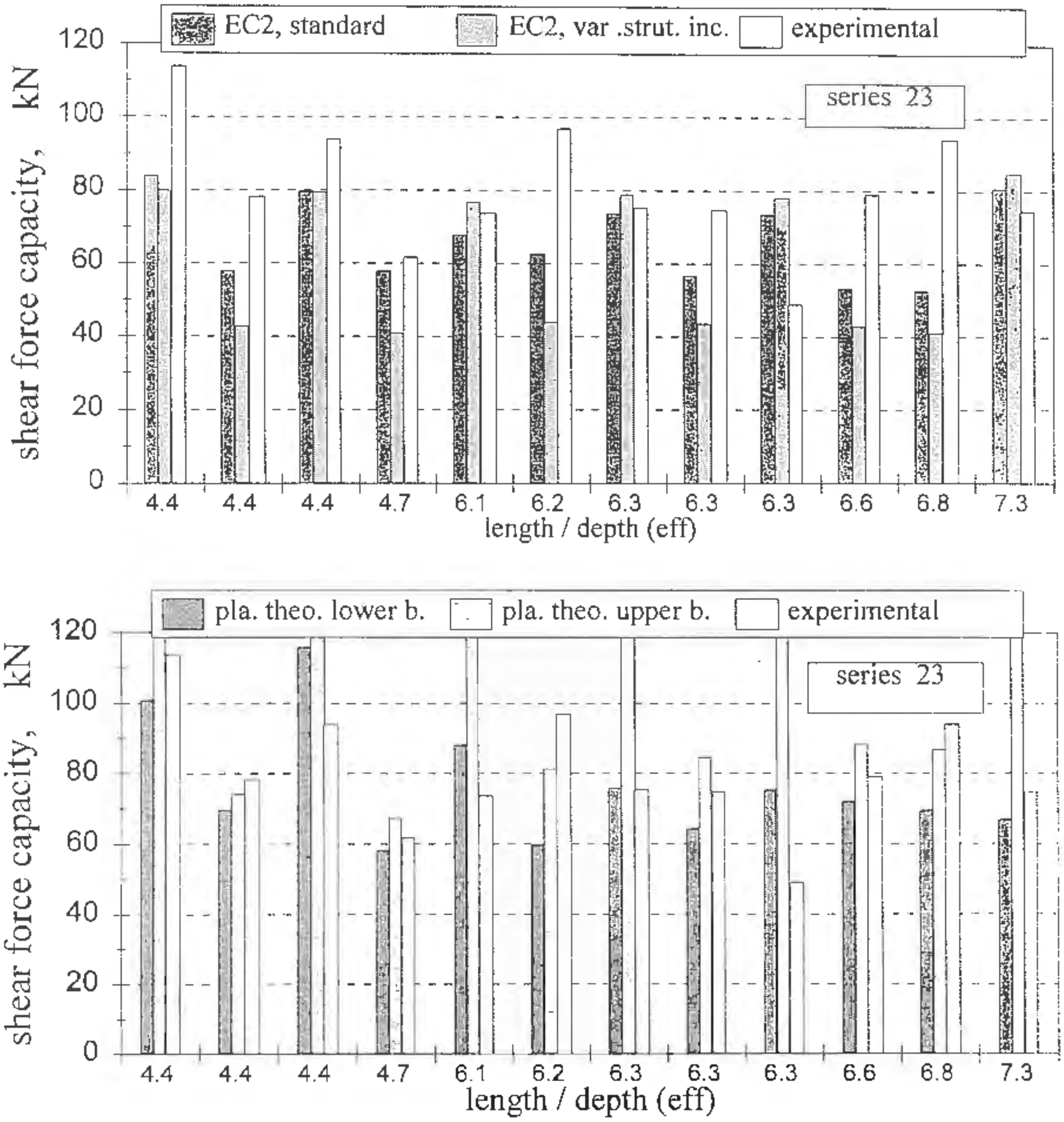
length (span), m	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.7	1.7
depth, cm	30	30	30	30	30	30	30	30	30	30
width, cm	15	15	15	15	10	10	15	15	15	15
concrete density, kg/m3	1460	1530	1530	1460	1650	1660	1530	1460	1530	1620
concrete strength, MPa	10	10	10	10	23	19	10	10	14	19
flexural strength, MPa	2.1	2.9	2.9	2.1	3.4	3.3	2.9	2.1	2.1	3.4
ratio of reinforcement, %	0.40%	0.40%	0.40%	0.52%	0.39%	0.39%	0.53%	0.41%	0.58%	0.59%
degree of transv. reinf.	0.61	0.62	0.62	0.27	0.28	0.31	0.27	0.61	0.51	0.37
type of reinforcement	stirrup	stirrup	stirrup	ladder	stirrup	stirrup	ladder	stirrup	stirrup	stirrup
length / depth (eff)	4.8	4.8	4.8	4.8	4.8	4.9	4.9	4.9	6.3	6.5
beam no.	1321	1322	1342	1317	2501	1241	1318	1341	2542	2541

Figure 12. Shear force capacity. Series 21.



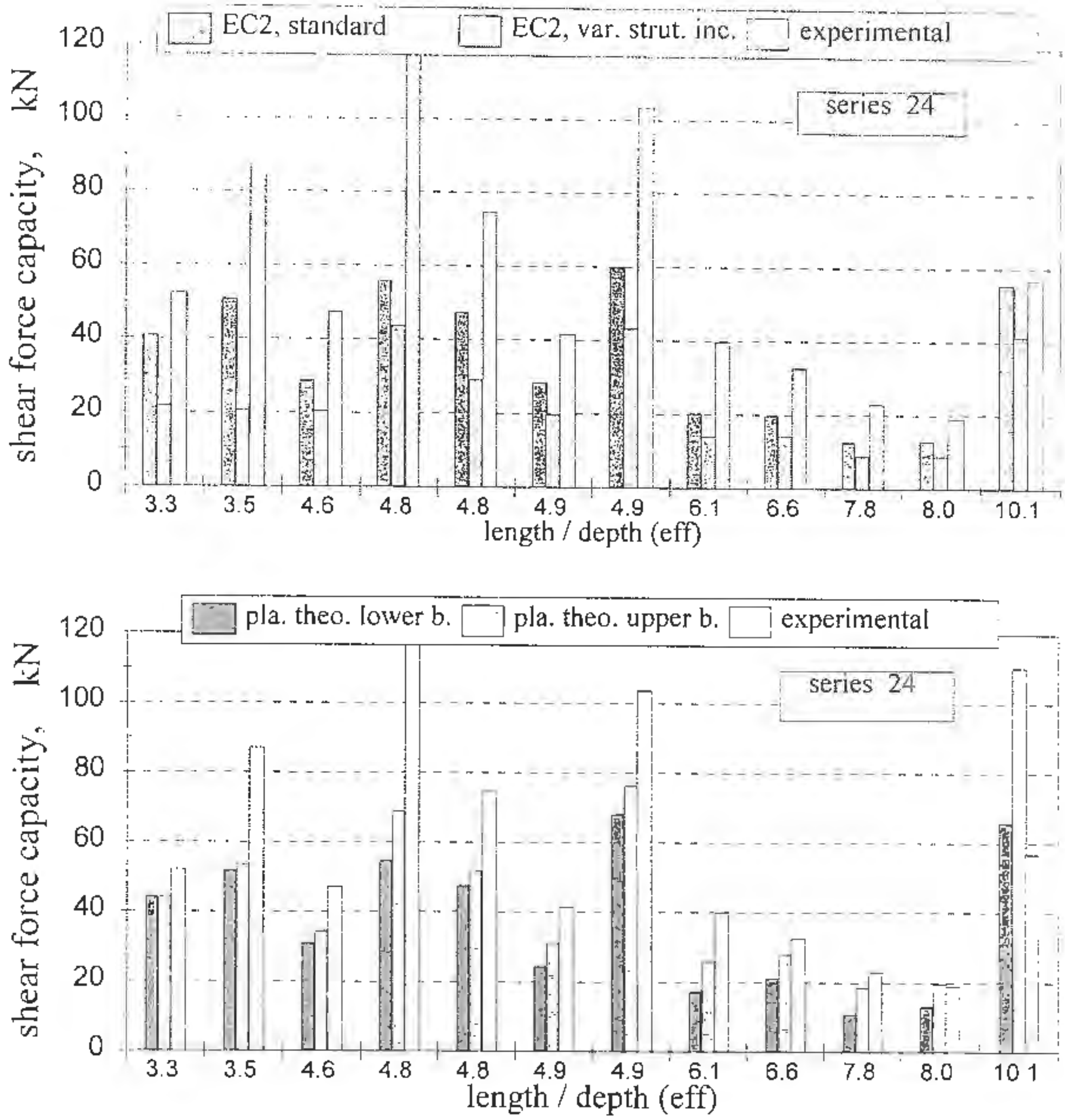
length (span), m	1.3	1.3	1.7	1.7	1.7	1.7	1.7	1.7
depth, cm	30	30	30	30	30	30	22	22
width, cm	10	10	10	10	15	15	15	15
concrete density, kg/m3	1240	1310	1590	1590	1480	1540	1540	1480
concrete strength, MPa	10	14	18	18	10	12	12	10
flexural strength, MPa	2.9	3.1	2.7	2.7	2.3	2.9	2.9	2.3
ratio of reinforcement, %	0.38%	0.38%	0.38%	0.38%	0.25%	0.50%	1.08%	1.22%
degree of transv. reinf.	0.20	0.14	0.06	0.06	0.13	0.22	0.46	0.56
type of reinforcement	ladder	ladder	ladder	ladder	ladder	ladder	ladder	ladder
length / depth (eff)	4.7	4.7	6.2	6.2	6.2	6.2	8.8	10.0
beam no	2351	2352	2722	2721	1552	1542	2712	2711

Figure 13. Shear force capacity. Series 22.



length (span), m	1.7	1.7	1.7	1.8	1.7	1.7	1.7	1.7	1.7	1.7	1.7	2.0
depth, cm	42	42	42	42	31	30	30	31	29	30	29	30
width, cm	24	10	24	10	24	24	24	24	24	24	24	24
concrete density, kg/m3	1440	1590	1480	1580	1480	1500	1440	1440	1440	1480	1480	1500
concrete strength, MPa	11	21	13	16	13	9	11	11	11	13	13	9
flexural strength, MPa	2.0	3.5	1.7	3.5	1.7	2.3	2.0	2.0	2.0	1.7	1.7	2.3
ratio of reinforcement, %	0.25%	0.55%	0.25%	0.55%	0.53%	0.47%	0.54%	0.48%	0.36%	0.50%	0.52%	0.53%
degree of transv. reinf.	0.30	0.21	0.25	0.26	0.35	0.27	0.43	0.24	0.43	0.20	0.20	0.53
type of reinforcement	stirrup	ladder	stirrup	ladder	stirrup	ladder	stirrup	ladder	stirrup	ladder	ladder	stirrup
length / depth (eff)	4.4	4.4	4.4	4.7	6.1	6.2	6.3	6.3	6.3	6.6	6.8	7.3
beam no.	2492	1275	2491	1274	2481	1401	1412	2192	2482	2191	1402	1411

Figure 14. Shear force capacity. Series 23.



length (span), m	1.3	1.3	1.7	1.3	1.3	1.8	1.3	1.7	1.7	1.3	1.3	2.5
depth, cm	42	42	40	30	30	40	30	30	30	20	20	29
width, cm	15	15	7.5	20	15	7.5	20	7.5	7.5	7.5	7.5	24
concrete density, kg/m3	1450	1520	1090	1440	1540	1090	1550	1090	1090	1090	1090	1450
concrete strength, MPa	10	14	12	11	13	8	15	8	12	8	12	12
flexural strength, MPa	2.3	3.4	2.3	2.3	3.0	2.5	2.6	2.5	2.3	2.5	2.3	1.9
ratio of reinforcement, %	0.18%	0.19%	0.36%	0.59%	0.50%	0.36%	0.60%	0.49%	0.53%	0.84%	0.86%	0.52%
degree of transv. reinf	0.14	0.10	0.23	0.31	0.21	0.31	0.22	0.32	0.24	0.32	0.24	0.22
type of reinforcement	ladder	ladder	ladder	ladder	ladder	ladder	ladder	ladder	ladder	ladder	ladder	ladder
length / depth (eff)	3.3	3.5	4.6	4.8	4.8	4.9	4.9	6.1	6.6	7.8	8.0	10.1
beam no.	2242	2241	1709	2135	1314	1701	2134	1691	1699	1681	1689	2211

Figure 15. Shear force capacity. Series 24.

Figures 12 to 15 also show the calculated lower and upper bound values for the shear force capacities determined by the expressions based on the theory of plasticity - in section 2.2.3.

With $\nu_{pla} = 0.25$ there is quite a good agreement between the calculated values and the experimentally found capacity, i.e. the test result lies between the lower bound and the upper bound value, and nearest to the lower bound value.
With larger value for ν_{pla} in too many cases lower bound values are obtained which are larger than the test result.

All the test beams have a length over the support front edge of 200 mm to 400 mm; no specific dependence on the support depth has been found. A 10% reduction may be sensed for the support depth under 300 mm.

The tests comprise both beams with "ladder" reinforcement and beams with stirrup reinforcement. Note that both cases deal with welded reinforcement. Good test results are obtained with both reinforcement types.

The test of the welded connections in the "ladder" reinforcements documented a strength of about 80% of the strength of the transverse steel reinforcement, which leads to a value of k_t in the expressions in sections 2.2.2 and 2.2.3 of 0.8 for the "ladder" reinforcement.

Beams with non-welded reinforcement (with stirrups) have been tested. These tests were not succesfull, due to problems caused by insufficient embedment of the reinforcement.

5 CONCLUSIONS

The agreement between the experimental moment capacity and the moment capacity determined by the expression in chapter 2 is very good.

The agreement between the experimental shear force capacity and the shear force capacity determined by the expressions in chapter 2 is acceptable although the variations are fairly extensive.

The verifications show, however, that the standard method leads to a slightly more correct estimate for the shear force capacity than the other methods.

Based on the investigations carried out, it is concluded that the load-bearing capacity of reinforced beams, made of lightweight aggregate concrete with open structure, can be calculated by means of the presented expressions.

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