SOME ASPECTS OF FORMATION OF CRACKS IN FRC WITH MAIN REINFORCEMENT

Rune Brincker, Associate Professor
Department of Building Technology and Structural Engineering
Aalborg University
Sohngaardsholmsvej 57, DK-9000 Aalborg, Denmark

John Simonsen, Ph.D. Student
Department of Building Technology and Structural Engineering
Aalborg University
Sohngaardsholmsvej 57, DK-9000 Aalborg, Denmark

Will Hansen, Associate Professor
Department of Civil and Environmental Engineering
The University of Michigan
2340 G.G. Brown Building, 2350 Hayward, Ann Arbor, Michigan, USA.

ABSTRACT

In this paper the response of fibre reinforced concrete (FRC) with main reinforcement in pure tension is considered. Test results are presented showing three distinct regimes: a regime of linear elasticity, a regime of yielding at approximately constant stress, and finally, a regime of strain hardening. A simple model is presented which takes into account the debonding between the reinforcement and the fibre reinforced matrix as well as the crack opening relation of the fibre reinforced matrix. The fracture process is described from the un-cracked state and formation of the first crack till the final stage where a large number of cracks have developed and the failure load is reached. The influence of different shapes of the crack opening relation is studied. It is shown, that if the crack opening relation is assumed to consist of a brittle contribution from the cement-based material, and a more ductile contribution from the fibre bridging, a plastic regime will be present in the tensile response. The case of a parabolic crack opening relation defines a brittleness number that describes the transition from formation of unstable discrete cracks to smaller cracks controlled by the softening behaviour of the fibre reinforced matrix. The model is compared with experiments, and reasonable aggreement is achieved.

Key words: Fibre reinforced concrete, Main reinforcement, Crack opening relation, Debonding, Crack development.

1 INTRODUCTION

During the last decade, very strong concrete materials have been developed. Using low water/cement ratios, and densifying the material using small particles like microsilica - the socalled DSP concept - it is possible to make concretes with a compressive strength around 150 MPa.

The main problem using these new materials is their brittleness, i.e. their limited resistance to crack formation and crack extension. Thus, structures built with these materials might develop cracks at unexpectedly low stress levels, and crack widths in these types of structures might be substantially large, leading to larger sensitivity to environmental influences. Hence, for structures built with these new high-strength concretes, cracks might be a more serious problem than for normal-strength concrete.

It is well known, Bache /1/, Tjiptobroto & Hansen /2/, Hansen & Tjiptobroto /3/, that the behaviour of a relatively brittle material such as DSP can be substantially modified by adding discrete fibres in a suitable volume fraction $V_f > 2$ %. The main effect of adding fibres is an improved ductility. The tensile strength is usually not affected much. The improved ductility is due to an enhanced ability to transfer stresses through cracks. Following the Hillerborg concept the stress transfer through a crack can be quantified by the crack opening relation, describing the relation between the crack width and the transferred stress, Hillerborg et al. /4/. The area under the crack opening curve defines the fracture energy G_f of the matrix and is closely related to the ductility of the material, Bache /1/. For a FRC material the stress transfer can be seen as a contribution from the bridging of the particles in the cement-based material (the sub-matrix) and a contribution from the bridging fibres, figure 1. As it appears from figure 1 the contribution from the sub-matrix occurs for relatively small crack widths, only, and that the major contribution to the ductility will typically come from the fibre bridging. Furthermore, the fibre bridging will be highly dependent upon e.g. the fibre geometry, the properties of the interface between fibres and the submatrix, the fibre stiffness and the fibre volume fraction, Li. et al. /5/. In fact, by using high volume fractions of fibres combined with traditional reinforcement it is possible to obtain a ductility and resistance to crack growth that is much larger than for normal strength concrete. In practice, however, situations might arise where the use of a high strength concrete is needed, but where it is not the objective to use high volume fibre fractions, but just to add enough fibres to adjust the ductility of the high strength concrete, so that crack development and crack widths might be controlled. In this case the resulting crack opening relation is dominated by a "spike-like" contribution from the sub-matrix for small crack widths and a more ductile contribution from the fibre bridging for larger crack widths as illustrated in figure 1.

Uniaxial tension tests on conventional reinforced concrete and ferrocement have shown that the response can be divided into 3 stages (see. e.g. Somayaji and Shah /6/ and Naaman /7/). During the first stage the response is elastic and the reinforcement is fully bonded. During the second stage the tensile response is dominated by initial yielding caused by the formation of discrete cracks at a relatively low stress level. The plastic strains developed at this stage in the fracture process, might be large compared to elastic strains and plastic strains developed during the later strain hardening (stage 3), where no further cracks are

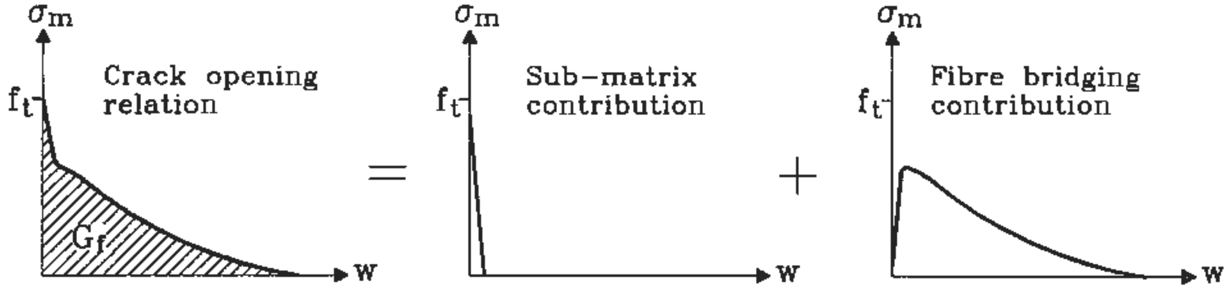

Figure 1: *Illustration of typical crack opening relation for a fibre reinforced matrix.*

formed. In this paper it is shown that a similar response is obtained if moderate fibre fractions are used. However, if larger fibre volume fractions are used the transition from the elastic stage to the strain hardening stage is relatively smooth, i.e. no significant initial yielding occurs.

Thus, the test results confirm that an appropiate model for the tensile response of FRC with main reinforcement has to take into account the ductility of the fibre reinforced matrix. Furthermore, since the opening of the crack requires relative movements between the matrix and the main reinforcement, the model has to take into account the debonding of the main reinforcement from the matrix. In the literature many models have been proposed describing the cracking and the tensile response of concrete members with main reinforcement /4-13/. In most cases however, the models do not take into account the stress transfer through the cracks, or the models do not include debonding between the main reinforcement and the matrix material. Some models include both effects, e.g. Stang and Aarre /8/, in this case however, the model also takes into account other effects such as elastic shear in the concrete and influence of Poisson's ratio.

The idea of the work presented here is to formulate the simplest possible model taking into account debonding as well as the crack opening relation. The model follows the basic ideas presented in the classical paper hy Aveston, Cooper and Kelly /7/, but as an extension of the Aveston model, bridged cracks are considered. The model provides a simple basis for analysing the uniaxial response of a fibre reinforced specimen with main reinforcement. The model gives the solution to the strain contribution due to the cracks formed, and it provides a direct way of analysing the influence of the crack opening relation on the crack development process.

For the case where the crack opening relation is parabolic, the model leads to a brittleness number describing the transition from formation of traction free cracks to smaller cracks controlled by the softening behaviour of the fibre reinforced matrix. For cases where tbe crack opening relation is characterized by a large contribution from the sub-matrix at small crack widths, only, and a smaller contribution from the fibre bridging, the model predicts the above-mentioned initial yielding. A simple analytical expression is given for plastic strain developing during the initial yielding.

The model is compared to experimental results, and it appears to be able to predict observed behaviour qualitatively correct.

2 TEST RESULTS

The test results presented here, are a part of an extensive programme carried out in a co-operation between Michigan University, Ann Arbor, and Aalborg University, Denmark, see Al-Shannaq /15/.

The tension test specimens presented here were made of a fibre reinforced matrix, reinforced with threaded bars. The specimens were rectangular plates of size 300 × 75 × 12 mm, figure 1.a, the cross-section having the area $A = 12 \times 75 = 900$ mm^2. The specimens were tested in a servo controlled 40 kN load frame in displacement control, figure 1.b. The strain was measured with strain gauges and with two clip gauges with a measurement length of 180 mm.

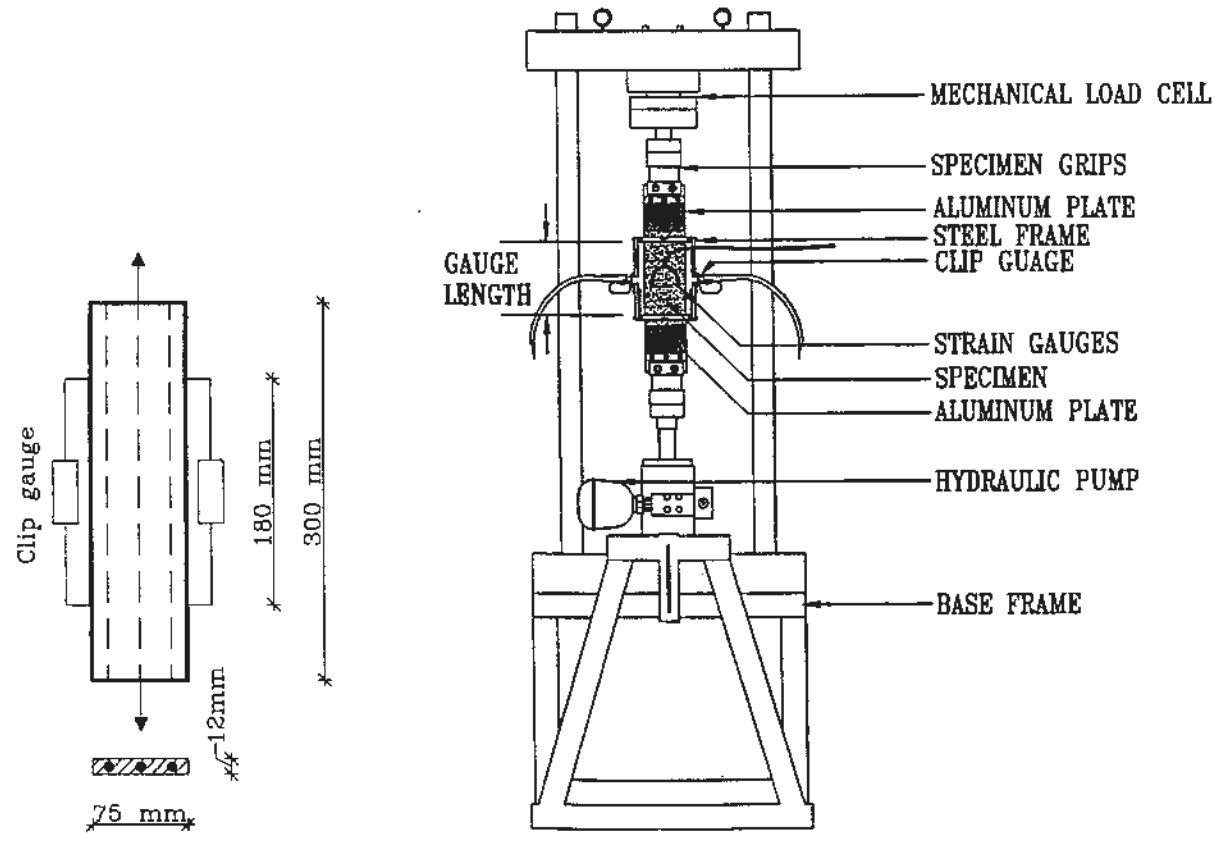

Figure 2: *Specimen and test setup.*

The sub-matrix was a brittle, high strength cement mortar, a so-called DSP material (densified with small particles) made of white Portland cement, microsilica, super-plasticizer, and fine quarts sand aggregates, Bache /1/, Al-Shannaq /15/. The fibre volume fractions of the fibre reinforced matrix were $V_f = 3\%$ or $V_f = 6\%$. The fibres were brass coated steel fibres with a length of 6 mm and a diameter of 0.15 mm. The tensile strength of the fibres was 2950 MPa. The properties of the interface between fibres and the DSP sub-matrix have been studied in detail in Al-Shannaq /15/. Tests showed debonding energies in the range 10 - 50 Nm/m^2, and friction stresses during debonding in the range 4 - 6 MPa. The response of the considered matrices was measured on notched specimens without main reinforcement, the cross-sectional area of the notched cross section being $12 \times 55 = 660$ mm^2. Typical results from these tests are shown in figure 3, where the stress is calculated from the area of the notched cross-section.

The main reinforcement was varied over the reinforcement ratios $\varphi = 1$ %, $\varphi = 3$ %, and $\varphi = 5$ %, 1 % corresponding to 2 bars with an outer diameter 3 mm, 3 % corresponding to 3 bars with an outer diameter 4 mm, and finally 5 % corresponding to 5 bars with an outer diameter 4 mm. The bars had metric threads, the effective area of the bars $A_{ef} = 0.695 A_{nom}$, where the nominal area is determined from the outer diameter. The exact reinforcement ratios based on the effective areas were 1.09 %, 2.91 % and 4.85 %. The failure stress based on the nominal area was 480 MPa for 3 mm bars, and 450 MPa for 4 mm bars, and the nominal Young's modulus (based on the outer diameter) was 135000 MPa. From a practical point of view the used reinforcement ratios might seem to be large. However, since the uniaxial tensile response is considered, the reinforcement ratio should be related to the area of an effective tension zone.

Some typical test results showing stress response to the total strain estimated from the clip gauge measurements are shown in figure 4. As it appears, the response for specimens with $V_f = 6\%$ is close to the response of a material with linear strain hardening, and there is a nearly smooth transition from the elastic regime to the strain hardening regime. For the cases with $V_f = 3\%$ the response is distinctly different. The transition from the elastic regime to the strain hardening regime is no longer smooth as for the case with 6 % fibres. The transition is now governed by initial yielding, i.e. before the strain hardening regime starts, an approximately horizontal regime appears in the response. Thus, a similar response to that of conventionally reinforced concrete is obtained. As it appears, this initial yielding is present for all reinforcement ratios, but the associated plastic strain decreases with increasing reinforcement ratio.

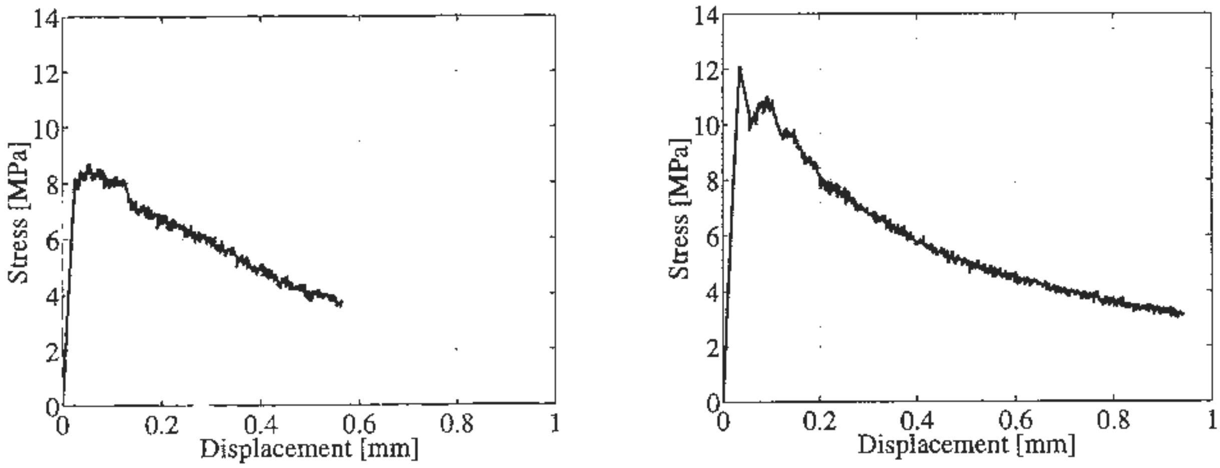

Figure 3: *Measured responses in uniaxial tension for the matrix materials. Left: Response for $V_f = 3\%$. Right: Response for $V_f = 6\%$.*

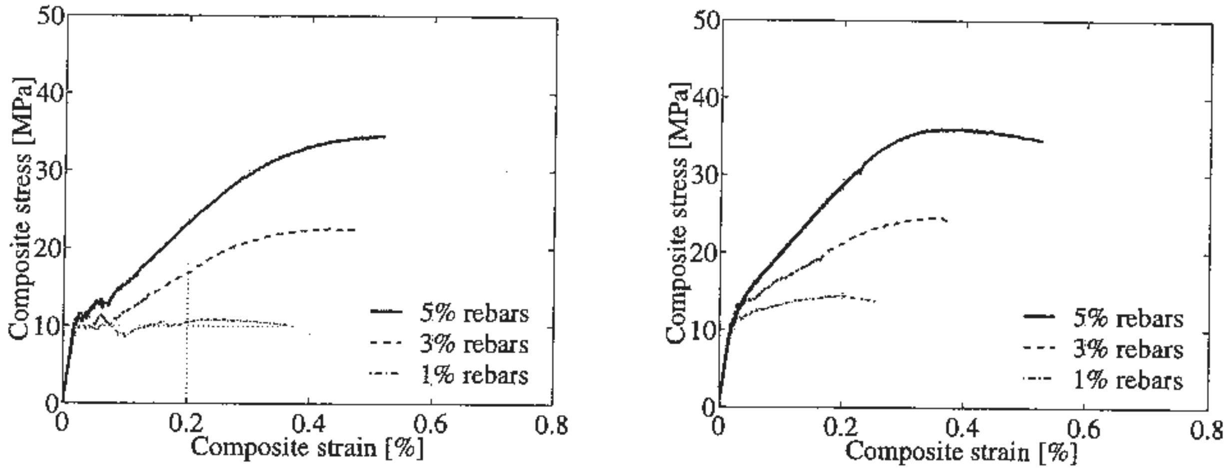

Figure 4: *Meassured responses in uniaxial tension for $V_f = 3\%$ (left) and $V_f = 6\%$ (right).*

3 MODEL FORMULATION

A specimen with cross-sectional area A subjected to uniaxial tension is considered. The specimen consists of a matrix reinforced with m continuous reinforcement bars with radius r. Thus the reinforcement ratio is $\varphi = m\pi r^2/A$. Since the idea of this model is to consider crack development in the case of a relatively tougb matrix, the matrix itself should be considered as a cement-based material (the sub-matrix) reinforced with fibres small enough to allow the fibre reinforced material to be treated as a continuum. Young's moduli of the matrix and the reinforcement are E_m and E_r, respectively, defining the ratio $\alpha = E_r/E_m$.

For a given uniform strain ϵ, the elastic stresses in the reinforcement and the matrix are easily calculated by defining the composite stress $\sigma = (\varphi E_r + (1-\varphi)E_m)\epsilon$ and the composite Young's modulus $E = \varphi E_r + (1-\varphi)E_m$. Now, assuming that the matrix is the weak phase with tensile strength f_t, the critical composite stress (the external stress corresponding to beginning failure of the matrix) is found as

$$\sigma_c = (1 + [\alpha - 1]\varphi)f_t = \frac{E}{E_m} f_t \tag{1}$$

At this state, the reinforcement stress is $\sigma_r = \alpha f_t$.

Now, let us assume, that a crack starts opening uniformly over the cross-section. This leads to a change of the state of stress around the crack. A simple model of the stress redistribution is given by assuming plane deformation and that the reinforcement is debonded over a zone with depth a, the shear stress in the rebar matrix interface being constantly equal to τ_f, figure 5.a. The reduction of the matrix stress in the crack is denoted s_m. Thus, the matrix stress in the crack is $\sigma_m = f_t - s_m$, and by requiring equilibrium, the corresponding stress in the reinforcement is obtained:

$$\sigma_r^m = \alpha f_t + \frac{1-\varphi}{\varphi} s_m \tag{2}$$

Now consider a piece of reinforcement around the crack. At the crack, the stress is given by eq. (2), and at the depth a the stress is given by αf_t as explained above. Thus, by considering equilibrium of the reinforcement, the length of the debonded zone is expressed by:

$$a = \frac{(1-\varphi)r}{2\tau_f \varphi} s_m \tag{3}$$

By assuming plane deformation in the reinforcement as well as in the matrix material, the stress disturbance introduced by the crack will be a linear deviation from the initially constant stress field along the debonded zone. Thus, the unknown crack opening is easily obtained by integrating the difference of strain in the reinforcement and the matrix:

$$\begin{aligned} w &= 2\int_0^a (\epsilon_r - \epsilon_m) dx \\ &= \left(\frac{1-\varphi}{\varphi E_r} + \frac{1}{E_m} \right) a s_m \\ &= \frac{E}{E_r E_m \varphi} a s_m \end{aligned} \tag{4}$$

Finally, by using (3) the following expression is obtained:

$$w = \frac{E(1-\varphi)r}{2\tau_f E_r E_m \varphi^2} s_m^2 \tag{5}$$

This expression will be denoted the equilibrium curve since it describes possible combinations of w and σ_m that satisfy the equilibrium of the specimen. Thus, intersection points between the equilibrium curve and the crack opening relation are model solutions satisfying both the equilibrium and the constitutive behaviour of the matrix, figure 5.b. However, it follows from (5) that the equilibrium curve is a horizontal parabola centred at $\sigma_m = f_t$ at the vertical axis, i.e. the derivative of the equilibrium curve approaches infinity for $w \to 0$. Therefore, due to the matrix resistance to crack growth, the crack is not allowed to open at initial cracking, and hence the state $\sigma_m = f_t$ and $w = 0$ is stable. Thus, if the matrix is considered as a homogeneous material the model predicts that infinitesimal small uniformly distributed cracks have to be formed throughout the specimen and hereby that the matrix will behave as an ideal plastic material. A similar behaviour is obtained for CRC (Compact Reinforced Composite) materials, Bache /1/, but not for the specimens tested in this study, cf. figure 4.

Based on the above observations, the idea of the model is that the matrix material away from the crack can sustain stresses larger than f_t. The rational argument for this assumption is that the strength of the matrix material will vary throughout the specimen, and hence the first cracks will form in areas with low tensile strength. First, assume that there is no upper limit for the tensile strength away from the crack and that only one crack will be formed. In this case the stress distribution at a given composite stress σ will still be as illustrated in figure 5.a, when f_t is replaced by $\sigma E_m/E$, c.f. (1). Therefore, the expression for the equilibrium curve is identical to (5) when $s_m = \sigma E_m/E - \sigma_m$ is used. Thus, the crack development can be obtained by successively displace the equilibrium curve vertically

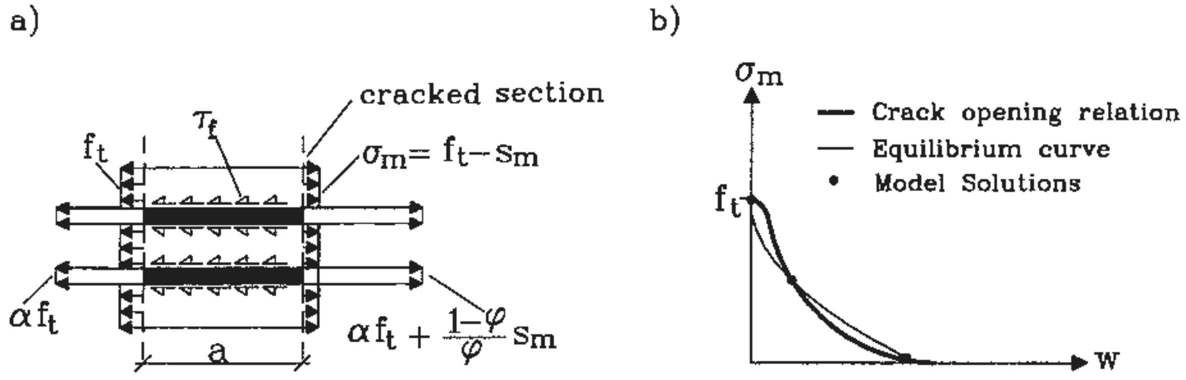

Figure 5: *Illustration of 5.a: the stress distribution at initial cracking and 5.b: the model solutions at initial cracking.*

in the (w, σ_m) diagram. As long as the slope of the equilibrium curve at the first intersection point is steeper than the slope of the crack opening relation the cracking process is stable. However, if the slopes become identical the crack is free to open uncontrolled to the next model solution, and hence a plateau will be present at the stress strain curve. Figure 6 shows two cases of uncontrolled crack development. Figure 6.a shows the case of a relatively weak reinforcement while figure 6.b shows the case of a relatively strong reinforcement. It is noticed, that for the case of a strong reinforcement, the crack must pass through a larger set of stable states before reaching an unstable state. Furthermore, the increase Δw of the crack width at uncontrolled crack development becomes larger in the case of a weak reinforcement. Thus, increasing the amount of reinforcement will help stabilizing existing cracks and help reducing the crack widths as it would be expected. As it appears from (5), the same effect is achieved by decreasing the radius r of the reinforcement or by increasing the friction stress τ_f. In practice, however, the task will often be to adjust the crack opening relation of the matrix. As it appears from figure 6 the model predicts that the slopes of the crack opening relation and the equilibrium curve plays a significant role for the crack development. Thus, in order to obtain an effective crack controlling matrix, the model snggests that emphasis should be put on designing a matrix having a crack opening relation with a moderate slope.

The above formulation provides the most simple form of the proposed model. However, by using this form, the model predicts that the matrix stresses away from the crack becomes unrealisticly high, even at low composite stresses. In reality, new cracks can be formed along the specimen until a certain composite stress $\sigma = \sigma_0$ is reached, where the whole reinforcement matrix interface will be debonded. Thus for values of σ higher than σ_0 the length of the debonding zone for individual cracks cannot extense as assnmed by the above formulation. A simple way of simulating this process is to introduce a maximum tensile stress $f_{\max}$ for the matrix, defining $\sigma_0 = f_{max} E / E_m$, and by assuming that all cracks will be formed at σ_c, cf. (1), with the crack spacing $2a_0$. As long as $\sigma < \sigma_0$ the length of the debonded zone is lower than a_0, and the crack development process will be as described above. However, when $\sigma = \sigma_0$ the length of the debonded zone becomes equal to a_0. Thus, for composite stresses larger than σ_0, the debonding length for the individual cracks is a_0.

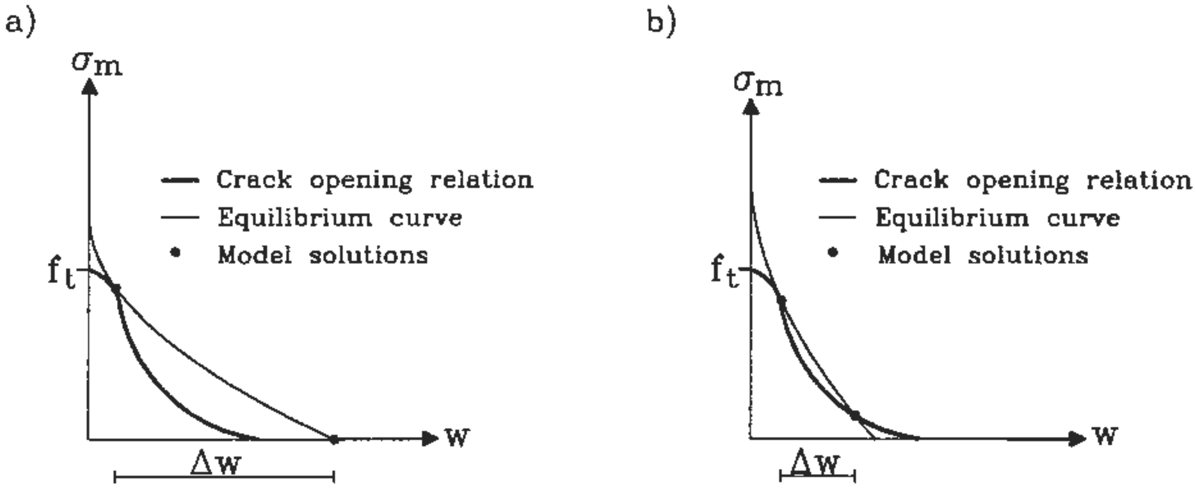

Figure 6: *Illustration of uncontrolled crack development for the case of 6.a: a weak reinforcement and 6.b: a relatively strong reinforcement.*

In this case the equilibrium curve no longer will be parabolic, but linear:

$$w = \frac{2a_0}{E_r\varphi}\sigma - \frac{2\tau_f E a_0^2}{E_m E_r(1-\varphi)r} - \frac{2a_0 E}{E_m E_r \varphi}\sigma_m \tag{6}$$

Since the expression for this equibrium curve is on the form $w(\sigma_m) = C_1(\sigma) + C_2 + C_3\sigma_m$, the crack development in this case too, can be found by by successively displacing tbis curve vertically in the (w, σ_m) diagram. Thus, the conclusions regarding the influence of the reinforcement and the stress-crack width relation, respectively, on the crack development still hold.

Furthermore, the final debonding length a_0 is determined from the stress level $\sigma = f_{max}E/E_m$. Therefore, both a_0 and the predicted crack widths will increase wit larger $f_{\max}$. Thus, the simple model without any limitation of the matrix stresses provides an upper-bound solution in estimating the crack width.

Finally, in the cases of a weak reinforcement or a brittle matrix, uncontrolled crack development will occur almost immediately after the crack formation, leading to a large stress decrease s_m and hence an instantaneous large length a of the debonding zone. The refined model generally assumes that the crack spacing is larger than $2a$. However, in reality the neighboring crack can just as well be formed at a distance a from the crack. Thus, for the above cases the model is expected to underestimate the strain contribution due to cracking.

4 SOME THEORETICAL SOLUTIONS

The simple model proposed in the preceding sections provides a basis for definition of a brittleness number B_r for the reinforced material. Assume that the crack opening relation for the matrix material is parabolic like the equilibrium curve, figure 7.a. In this case, if and only if the area G_r under the equilibrium curve is less than the fracture energy G_f of the matrix, all crack opening states will be stable. Thus, the stability criterion is $G_r < G_f$ defining the brittleness number $B_r = G_r/G_f$ and the stability criterion $B_r < 1$. The defined brittleness number is easily obtained:

$$B_r = \frac{1}{6}\frac{E(1-\varphi)rf_t^3}{\tau_f E_r E_m G_f \varphi^2} = \frac{1}{6}\frac{E(1-\varphi)f_t}{\tau_f E_r \varphi} B_0 \tag{7}$$

where $B_0 = f_t^2 r/(E_m G_f)$ is the traditional brittleness number for homogeneous materials /1/, where the reinforcement radius r is used as the characteristic length.

The value of the brittleness number might be taken as a guideline for the type of crack formation in the reinforced material. If B_r is well below one, stable cracks are expected resulting in a well distributed crack pattern. On the other hand, if B_r is well above one, unstable cracks are expected to form resulting in formation of larger discrete cracks.

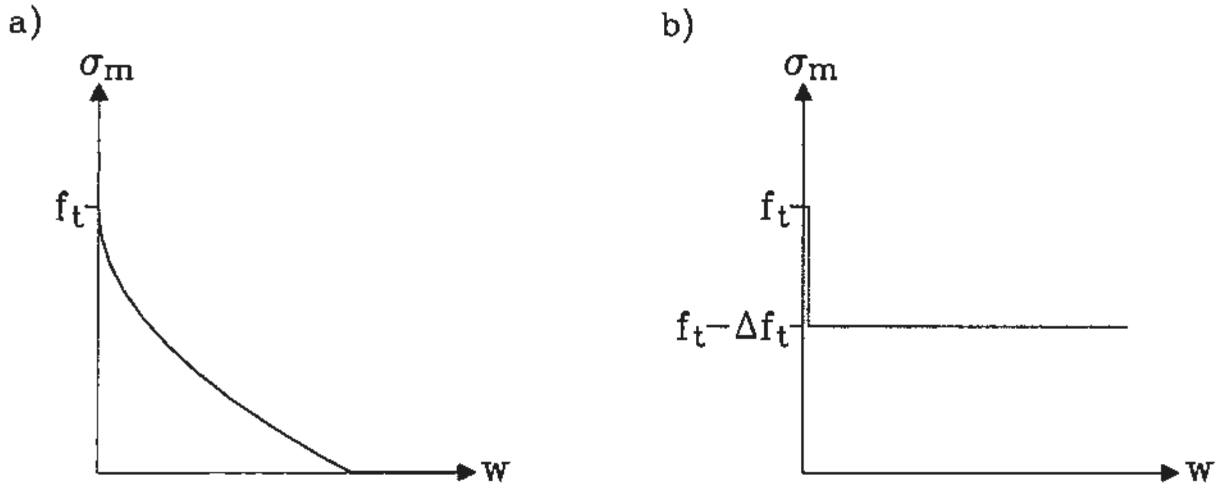

Figure 7: *Idealized crack opening relations. 7.a: Parabolic crack opening relation. 7.b: Constant crack opening relation with an initial spike due to the sub-matrix contribution*

Sometimes the shape of the crack opening relation is far from being parabolic, and an analysis based on the brittleness number defined above might be misleading. An important case is when the matrix is lightly or moderately fibre reinforced. In this case, the first part of the crack opening relation is governed by crack formation in the sub-matrix resulting in a sharp peak at the beginning of the crack opening relation. Because of their brittle nature, it is difficult to measure these effects experimentally, but the tendency has been observed, see e.g. Li et al. /9/ and figure 3. For this case therefore, the crack opening relation might be idealized as a delta spike at $w = 0$ followed by a constant level, figure 7.b.

Due to the spike with depth Δf_t uncontrolled crack development will occur immediately after the critical composite stress σ_c has been reached. The finite crack opening caused by the spike, will result in an initial plastic strain $\Delta\epsilon$, figure 8. Assuming that the cracks form at the distance $2a$, where a is given by (3), the stress distribution around the cracks at initial cracking will be as illustrated in figure 5 when $s_m = \Delta f_t$ is used. Thus, since the stress deviation from the initially constant stress field is linear along the debonded zone, the initial plastic strain is found as:

$$\Delta\epsilon = \int_0^a \frac{\epsilon_r - \epsilon_c}{E_r} dx = \frac{1-\varphi}{2\varphi E_r} \Delta f_t \tag{8}$$

As stated earlier, the distance beween the cracks is in the range from a to $2a$. Thus, the expression given above underestimates the plastic strain.

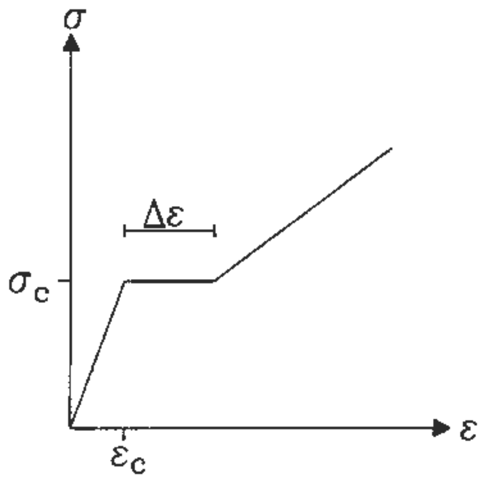

Figure 8: *Definition of plastic strain $\Delta\epsilon$ due to initial yielding.*

5 MODEL VALIDATION

The simulated responses using the model presented in the preceding sections are shown in figure 10.

By assuming that the softening behaviour of the matrix material can be divided into 3 stages, the crack opening relations were estimated from figure 3. The first regime was fitted by a linear descending function. The second stage was fitted by a linear ascending function, and finally, the third stage was fitted by a function of the form $\sigma = f_t/(1 + (w/w_0)^p)$ as proposed by Stang and Aarre /8/. The argument for dividing the softening behaviour into three stages is that the sub-matrix is brittle. Thus, at small crack widths, stage 1, the major contribution to the stress transfer comes from the sub-matrix. For larger crack widths, stages 2 and 3, the stress transfer comes from fibre bridging, only. At stage 2 all the fibres are not fully debonded from the sub-matrix, and hence the fibre bridging stress will be an increasing function of the crack width. During stage 3 all the fibres are fully debonded, and the fibre bridging curve is a decreasing function since the fibres are pulled out from the sub-matrix. It should be noticed that for the case of $V_f = 3\%$ a smooth behaviour is obtained for the softening behaviour. However, from figure 3 it is seen that the tensile strength is significantly lower than the one obtained for $V_f = 6\%$. This large difference is unexpected since the fibre amount usually affects the tensile strength only to a small degree. Therefore, due to the brittleness of the sub-matrix, it is believed that some initial cracks have been introduced in making the notches. Therefore, the estimated crack opening relation for $V_f = 3\%$ has been supplied by a spike at $w = 0$. The estimated crack opening relations are shown in figure 9. However, since the crack opening relations were measured on notched specimens, and since cracks will form at the weakest points along the specimen, the measured crack opening stresses should be reduced. Thus, in the simulations the estimated crack opening relations were reduced by a constant factor. For the case of $V_f = 3\%$ the values $f_t = 9.5$ MPa and $f_{max} = 10$ MPa were used, while the values $f_t = 11$ MPa and $f_{max} = 12$ MPa were used for the case of $V_f = 6\%$. Finally, for the shear resistance in the rebar matrix interface the value $\tau_f = 5$ MPa was used.

As it appears from figure 10, the model is able to predict a response qualitatively close to the response measured in the experiments. The model predicts the right tendencies concerning critical stress, initial yielding and strain hardening as well as final failure load. However, some deviations are obtained. First, for the case of $V_f = 3\%$ the model has a tendency to underestimate the strains. This is true for the initial plastic strain as well as the plastic strain developed during strain hardening. Partly this might be due to the simple assumption that the distance beween cracks is twice the calculated debonding length. As mentioned earlier, this leads to an underestimation of the strain contribution due to cracking. Further, for the case of $V_f = 6\%$ the model seems to overestimate the strain at initial yielding. This overestimation might be due to the assumption that all the cracks are formed simultaneously. Finally, relatively large discrepancies occur for large stress levels and the failure loads are overestimated. This is believed to be due to the the simple model of the reinforcement response. In the model the reinforcement was modelled as being linear elastic material up to the failure load. This is a rough approximation close to failure where yielding and necking might substantially decrease the reinforcement stiffness. If the stiffness is decreased in the model, this will lead to larger crack widths, and this again will lead to smaller matrix stresses tending to reduce both the stiffness and the final failure load estimated by the model. However, from a practical point of view these differences are of minor importance, since they occur at stresses close to failure, and hence at stress levels well above the serviceability limit.

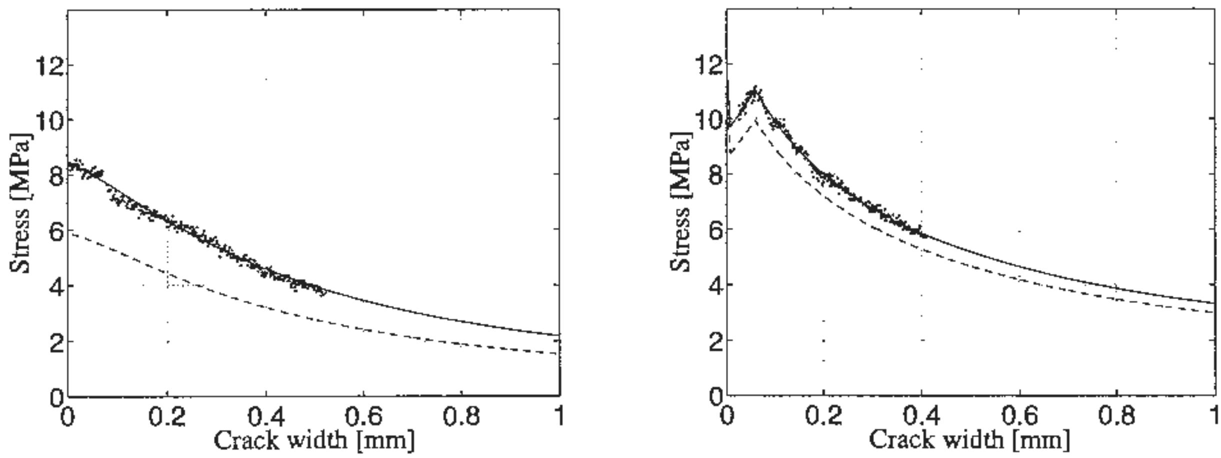

Figure 9: *Measured (. .), estimated (—) and simulated (- - - -) crack opening relations for $V_f = 3\%$ (left) and $V_f = 6\%$ (right).*

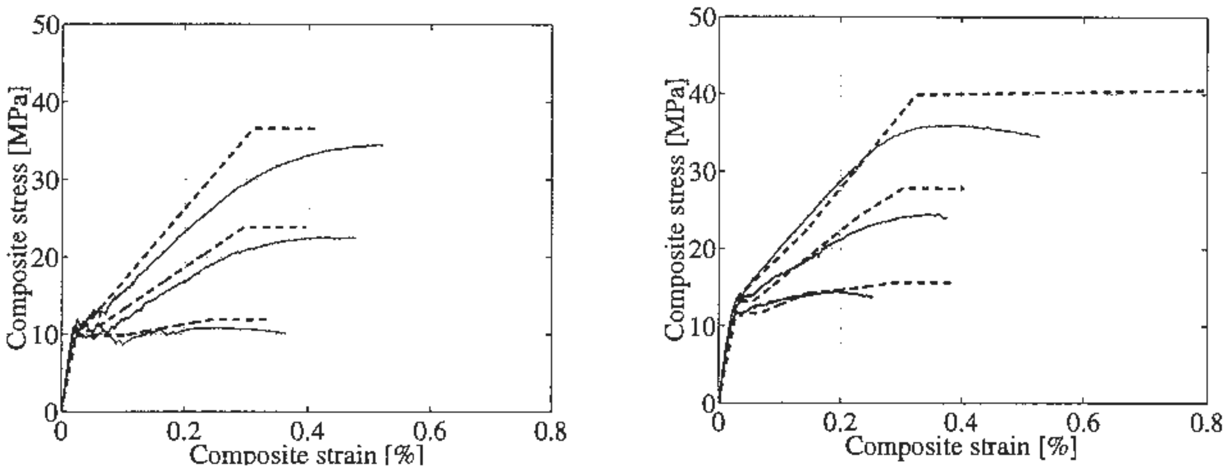

Figure 10: *Measured (—), and simulated (- - - -) responses for $V_f = 3\%$ (left) and $V_f = 6\%$ (right).*

6 CONCLUSIONS

In this paper test results have been presented showing that there is a transition from a relatively smooth behaviour when high fibre volume fractions are used together with main reinforcement to a behaviour with initial yielding - plastic strain development at approximately constant stress preceding the hardening regime - at moderate fibre contents.

A model has been presented that takes into account the shape of the crack opening relation as well as debonding of the reinforcement around the crack. The model is extremely simple, and in its most simple form, it gives a suitable approximation only for the material response around the point of first cracking. Therefore, an extension of the model is proposed, taking into account multiple cracking of the specimen.

The presented model provides a direct way of analysing the influence of the shape of the crack opening relation. In general the model suggests that emphasis should be put on designing a matrix having a crack opening relation with a moderate slope. Furthermore, two cases of idealised crack opening relations are considered. One case is when the crack opening relation is parabolic. For this case, a brittleness number is defined describing the transition from a response without initial yielding dominated only by multiple cracking and strain hardening (high fibre volume fractions) to a response with formation of larger discrete cracks (lower fibre volume fractions). The other case considered is when the initial part of the crack opening relation is dominated by the contribution from the sub-matrix. Knowing the strength contribution from the sub-matrix and the fibre reinforcement, a simple analytical solution provides an estimate of the plastic strains developed during initial yielding.

The model compares well with the experimental results and predicts the right tendencies concerning critical stress, initial yielding and strain hardening as well as final failure load. However, some deviations are obtained. These deviations might be explained by some of the simplifications introduced in the modelling.

7 ACKNOWLEDGEMENTS

Financial support from the Danish Technical Research Council is gratefully acknowledged. Thanks are due to Mr. Nicholas Flint and Mr. Henning Andersen for their help in preparing the specimens and in carrying out the tests.

8 REFERENCES

/1/ Bache, H.H.: *Compact Reinforced Concrete Composite, Basic Principles*, CBL Report No. 41, Aalborg Portland, Denmark, 1987, 87 pages.

/2/ Tjiptobroto, P. and W. Hansen: *Tensile Strain Hardening and Multiple Cracking in High-Performance Cement-Based Composites Containing Discontinuous Fibers*, ACI Materials Journal, Vol. 90, No. 1, January-February 1993.

/3/ Hansen, W. and P. Tjiptobroto: *An Energy Based Model for Predicting the Elastic Tensile Strain Capacity of High Performance Fiber Reinforced Composites (FRC)*, Nordic Concrete Research, Publication No 10, 1991.

/4/ Hillerborg, A., M. Modéer and P.E. Petersson: *Analysis of Crack Formation and Crack Growth in Concrete by means of Fracture Mechanics and Finite Elements*, Cement and Concrete Res., Vol. 6, 1976.

/5/ Li, V.C., H. Stang and H. Krenchel: *Micromechanics of Crack Bridging in Fibre-Reinforced Concrete*, Materials and Structures, 1993, No. 26.

/6/ Somayaji, S. and S.P. Shah: *Bond Stress versus Slip Relationship and Cracking Response of Tension Members*, ACI Journal, May-June 1981.

/7/ Naaman, A.E.: *Design Predictions of Crack Widths in Ferrocement*, ACI Publication SP-61, 1979.

/8/ Stang, H. and T. Aarre: *Evaluation of Crack Width in FRC with Conventional Reinforcement*, Cement and Concrete Composites, Vol. 14, 1992.

/9/ Aveston, J, G.A. Cooper and A. Kelly: *Single and Multiple Fracture*, Proceedings from Conference on the Properties of Fibre Composites, National Physical Laboratory, November 4th, 1971.

/10/ Bartoletti, L.: *First Cracking Load of Concrete Subjected to Direct Tension*, ACI Materials Journal, Vol 88 No 1, January-February 1991.

/11/ Yang, S. and J. Chen: *Bond Slip and Crack Width on Tension Members*, ACI Structural Journal, Vol 85 No 4, July-August 1991.

/12/ Hegemier, G.A., H. Murakami and L.J. Hageman: *On Tension-Stiffening in Reinforced Concrete*, Journal of Structural Engineering, Vol. 116, No. 11, November 1990.

/13/ Massicotte, B., A.E. Elwi and J.G. MacGregor: *Tension-Stiffening Model for Planar Reinforced Concrete Members*, Journal of Structural Engineering, Vol. 116, No. 11, November 1990.

/14/ Gupta, A.K. and S.R. Maestrini: *Tension-Stiffness Model for Reinforced Concrete Bars*, Journal of Structural Engineering, Vol. 116, No. 3, March 1990.

/15/ Al-Shannaq, M.J.: *Tensile Behaviour of Fiber Reinforced DSP*, Ph.D.-Thesis, Dept. of Civil Engineering, University of Michigan, 1995, 211 pages.