IN-PLANE SHEAR JOINT CAPACITY OF PRECAST LIGHTWEIGHT AGGREGATE CONCRETE ELEMENTS

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ABSTRACT

The paper establishes and documents formulas for in-plane shear capacity between precast elements of lightweight aggregate concrete with open structure. The joints investigated are rough or toothed and have all been precracked prior to the testing in order to obtain realistic test results. The paper documents the shear force capacity for the joint strength between the most common joint types between precast LAC roof and floor elements used in Scandinavia.

Keywords: Shear, Joint, Precast, Lighweight, Concrete, CEN

1. INTRODUCTION

Precast elements of normalweight concrete (NC), autoclaved, aerated concrete (AAC) and lightweight aggregate concrete with open structure (LAC) are used extensively in Northern Europe. This have recently resulted in european product standards for LAC-elements /1/ and AAC-elements which also presents a number of design rules for the element.

An essential part of the use of such elements is, however, the combination of those elements into larger structural parts as e.g. roofs or floors, in which the wind loads can be transferred through in-plane shear forces. This obviously requires transfer of in-plane shear in the joints, which for NC-elements is described in the European Code (ENV 1992-1-3) /2/ for structures of precast NC-elements, but which is not yet covered by any European Code for AAC or LAC elements structures.

This paper will focus on the in-plane shear capacity of rough and toothed joints between two LAC-elements (see Figure 1) and present a new set of formulas for estimating the capacity, which is based on the use of plasticity theory and efficiency factors. The hypothesis assumed is that these joints behave in a similar way as the joints between the NC-elements. The theoretical formulas and the ENV-formulas will be verified by comparison to test results.

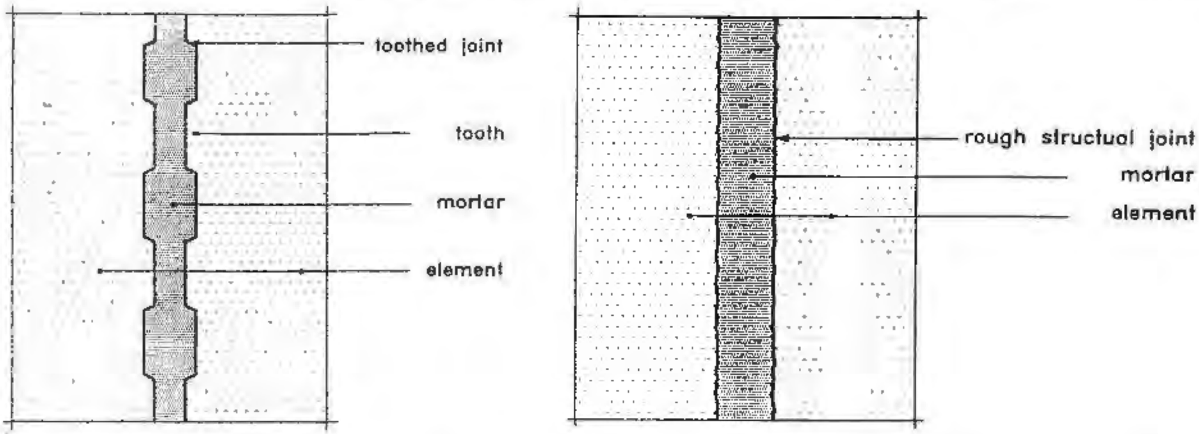

Figure 1. Geometry of toothed joint (left) and rough joint (right) seen from above.

2. DESIGN RULES IN EUROPEAN CODES

The shear capacity V_u of the joint between two precast elements is determined according to Eurocode 2's part for precast elements ENV1992-1-3 /2/ for normalweight concrete with a closed structure. The formulas for a joint with a reinforcement placed perpendicular to the joint are

$$V_u = \tau_u * A_j \tag{1}$$

$$\tau_u = K_T * \tau + \mu * (\sigma_N + \Phi * f_c) < 0.2 * f_c \tag{2}$$

$$\Phi = (A_s * f_y) / (A_j * f_c) \tag{3}$$

where

V_u is the joint shear-capacity

A_j is the joint area effective in the shear transfer. (The tooth area in a toothed joint, but the whole joint area in a rough joint).

A_s is the reinforcement area

f_c	is the compressive strength of concrete in the joint or in the element, whichever is the lowest
f_y	is the yield strength of the reinforcement
K_T	is a shear coefficient
τ	is a shear strength parameter
μ	is the coefficient of shear friction
σ_N	is the compressive stress perpendicular to the joint

This type of formula is used by many codes, although often with slightly different K_T and μ coefficients. The coefficients in ENV1992-1-3 are shown in Table 1.

Type of Joint	K_T	μ
Toothed	2.0 (0.5)	0.9
Rough	1.8 (0.0)	0.7

Table 1. Coefficients in EVN1992-1-3 for precast NC-elements. Values in brackets are valid for cracked joints.

The shear strength parameter is related to the tensile strength f_t as follows

$$\tau = 0.25 * f_t \tag{4}$$

The tensile strength is, however, rarely measured and must thus be found from the flexural strength f_{ft} (which is tested more often) as

$$f_t = 0.5 * f_{ft} \tag{5}$$

These formulas are expected to be valid for elements of lightweight aggregate concrete with an open structure. The conservativeness of the formulas and the coefficients will, however, be verified later in this paper through comparison to theoretical estimation and test results.

3. TESTS

At the Institute of Building Design (IBD) at the Technical University of Denmark a number of tests were carried out with precast LAC multilayer elements where the very open structure in the very light center layer (density 600kg/m^3) ensured a rough joint. Other tests were carried out with LAC-elements with a monolithic cross-section (density 1750kg/m^3), where the side of the elements were profiled for a toothed joint.

The two types of elements and joints are representative for the roof and floor elements currently produced in Denmark, but the concrete densities also represent the extremes in densities in the LAC-elements and should thus be a good basis for evaluating the joint shear capacities.

The elements are produced at two factories (Dansk Leca A/S and A/S Fibo) and are joined in IBD's laboratory. The joints were precracked at the ends of the joints in order to lead to a more realistisk performance.

The test have been carried out with two different testing benches. The first bench is shown on Figure 3 and is used for testing the elements with reinforcement placed at the element ends, perpendicular to the shear joint's direction. The second bench is shown on Figure 4 and is used for testing joints without any reinforcement, but with some amount of compression (σ_N) perpendicular to the joint.

The test series are described in Table 2 and Figure 2.

Series	Element type	Joint	σ_N	Φ	Testing arrangement
1	Multilayer	Rough	0	>0	1
2	Multilayer	Rough	>0	0	2
3	Multilayer	Rough	0	>0	1
4	Monolithic	Toothed	0	>0	1

Table 2. Overview of test series.

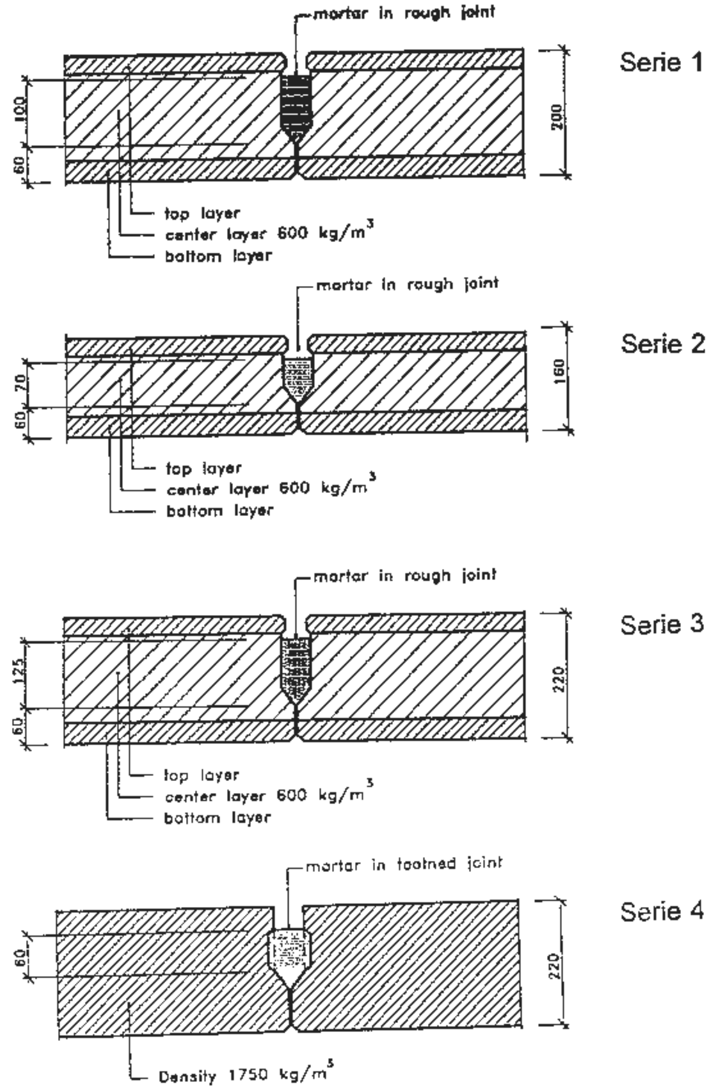

Figure 2. Cross-section in series 1 (top) to 4 (bottom).

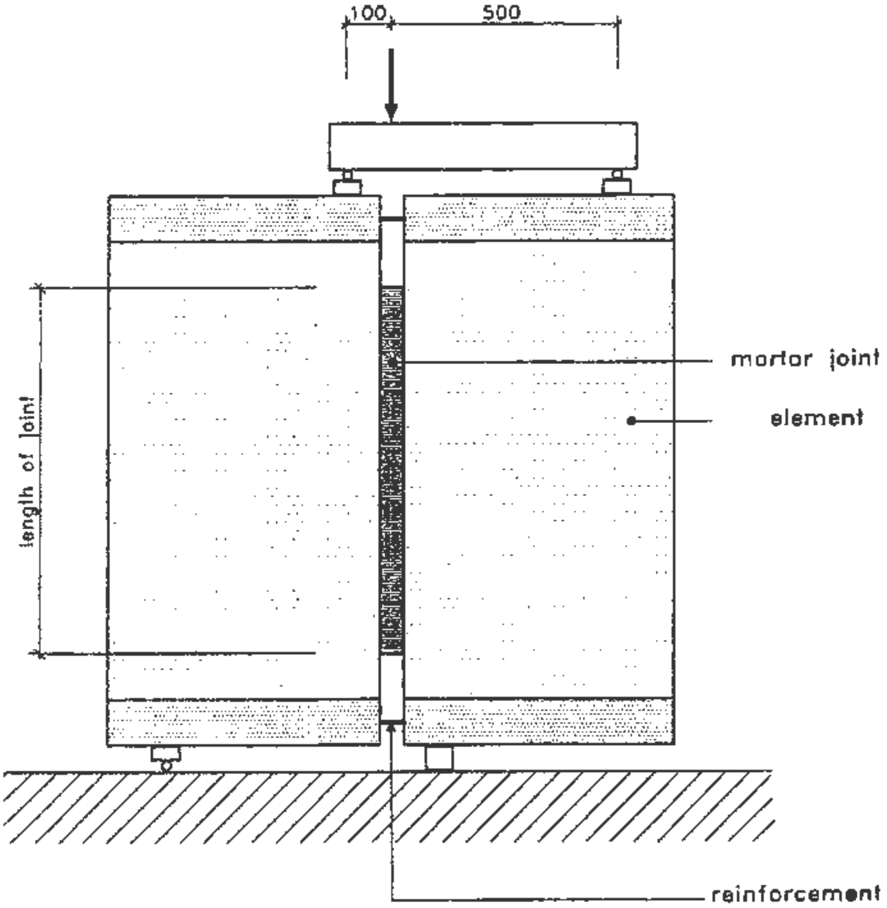

Figure 3. Testing arrangement 1 in series 1,3,4.

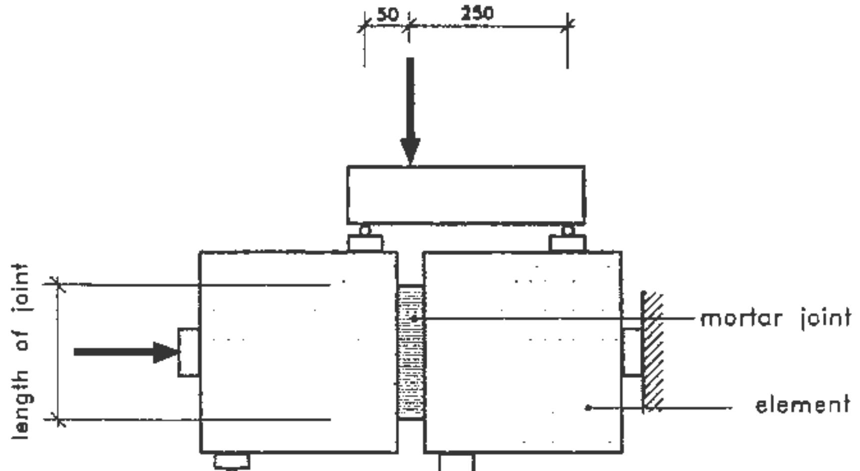

Figure 4. Testing arragement 2 in series 2.

No	A_j mm^2	f_c MPa	A_s mm^2	A_s*f_y kN	Φ	V_{exp} kN	Δ mm	τ_{exp} MPa	τ_{exp}/f_c
2	80000	3.0	56	32.6	0.14	48.0	11	0.59	0.20
3	89700	3.0	157	76.9	0.28	72.5	15	0.81	0.27
5	96000	3.0	101	61.3	0.21	65.1	17	0.68	0.23
6	79000	3.0	101	61.3	0.25	52.7	2	0.66	0.22
7	78300	3.0	157	76.9	0.33	65.4	7	0.83	0.28
8	75525	3.0	113	65.3	0.29	68.9	24	0.92	0.31

Table 3. Test series 1. Joint length app.800mm and joint depth app.100mm.

No	A_j mm^2	f_c MPa	σ_N MPa	σ_N/f_c	V_{exp} kN	τ_{exp} MPa	τ_{exp}/f_c
1	12210	3.0	0.29	0.10	6.4	0.52	0.17
2	12180	3.0	0.44	0.15	9.5	0.78	0.26
3	14350	3.0	0.50	0.17	11.0	0.77	0.26
4	13520	3.0	0.66	0.22	11.0	0.81	0.27
5	13760	3.0	0.78	0.26	13.2	0.96	0.32
6	13120	3.0	0.95	0.32	14.6	1.11	0.37
7	13760	3.0	1.04	0.35	15.5	1.13	0.38
8	13000	3.0	1.24	0.41	14.7	1.13	0.38
9	14020	3.0	1.16	0.39	15.6	1.11	0.37

Table 4. Test series 2. Joint length app.180mm and joint depth app.70mm.

No	A_j mm^2	f_c MPa	A_s mm^2	A_s*f_y kN	Φ	V_{exp} kN	Δ mm	τ_{exp} MPa	τ_{exp}/f_c
1	98800	3.7	56	29.7	0.08	43.7	4.5	0.44	0.12
2	99400	3.7	56	29.7	0.08	43.0	4.4	0.43	0.12
3	94700	3.7	101	60.0	0.17	52.0	8.5	0.55	0.15
4	98500	3.7	101	60.0	0.17	54.4	7.1	0.55	0.15
5	97200	3.7	157	79.0	0.22	62.8	1.6	0.65	0.18
6	97500	3.7	157	79.0	0.22	63.8	2.6	0.65	0.18
7	98600	3.7	226	104.0	0.29	67.4	2.4	0.68	0.19
8	95800	3.7	226	104.0	0.30	60.5	2.7	0.63	0.17
9	99300	3.7	282	133.7	0.37	72.9	2.2	0.73	0.20
10	100100	3.7	282	133.7	0.36	67.5	2.2	0.67	0.18

Table 5. Test series 3. Joint length app.800mm and joint depth app.125mm.

No	A_j mm^2	f_c MPa	A_s mm^2	A_s*f_y kN	Φ	V_{exp} kN	Δ mm	τ_{exp} MPa	τ_{exp}/f_c
1	21600	21.5	101	52.7	0.11	72.5	0.5	3.4	0.16
2	21600	21.5	101	52.7	0.11	93.9	0.5	4.3	0.20
3	21600	21.5	201	105.4	0.23	150.7	0.4	7.0	0.32
4	21600	21.5	201	105.4	0.23	142.2	2.4	6.6	0.31
5	21600	21.5	226	212.9	0.46	157.3	0.6	7.3	0.34
6	21600	21.5	226	212.9	0.46	133.6	6.0	6.2	0.29
7	21600	18.15	101	52.7	0.13	70.9	3.0	3.3	0.18
9	21600	18.15	201	105.4	0.27	116.8	0.9	5.4	0.30
10	21600	18.15	201	105.4	0.27	112.6	0.3	5.2	0.29

Table 6. Test series 4. Joint length app.720mm and joint depth app.60mm.

The important parameters are listed in Tables 3 to 6 where

V_{exp}	is the experimental shear capacity
τ_{exp}	is the experimental shear stress ($=V_{exp}/A_j$)
Δ	is the shear deformation at maximum shear force
A_j	is the joint area, equal to joint length*joint height in rough joints (series 1,2,3) but equal to 0.5*joint length*joint height in toothed joints (series 4).
f_c	is the compressive strength of the concrete in the elements, except in series 4,no.7 to 10 where it denotes the compressive strength of the mortar in the joint (which is weaker than the concrete in those tests).

The yield strength of the reinforcement was measured on the bars used in the test specimens. The yield strength has been tested on 3 bars of each diameter (6,8,10 and 12mm) and varies from 460 to 600 MPa.

The compressive strength of the concrete were measured on Ø100mm cylinders, extracted from the specimens with a height equal to the height of the cross-section, but not exceeding 200mm. Earlier investigations /6/ have shown that the height of Ø100mm cylinders can vary between 100mm and 200mm without any influence on the compressive strength. The strength measured on cylinders from the multilayer elements represent the weaker middle layer. The compressive strength of the mortar in the joint was measured on seperately cast cylinders.

The shear deformations were measured in each test as a function of the shear load and showed large deformation capacities as shown on Figure 5 and in Tables 3, 5 and 6. In series 2 a normal compressive stress σ_N is applied on the joint instead of a reinforcement perpendicular to the joint.

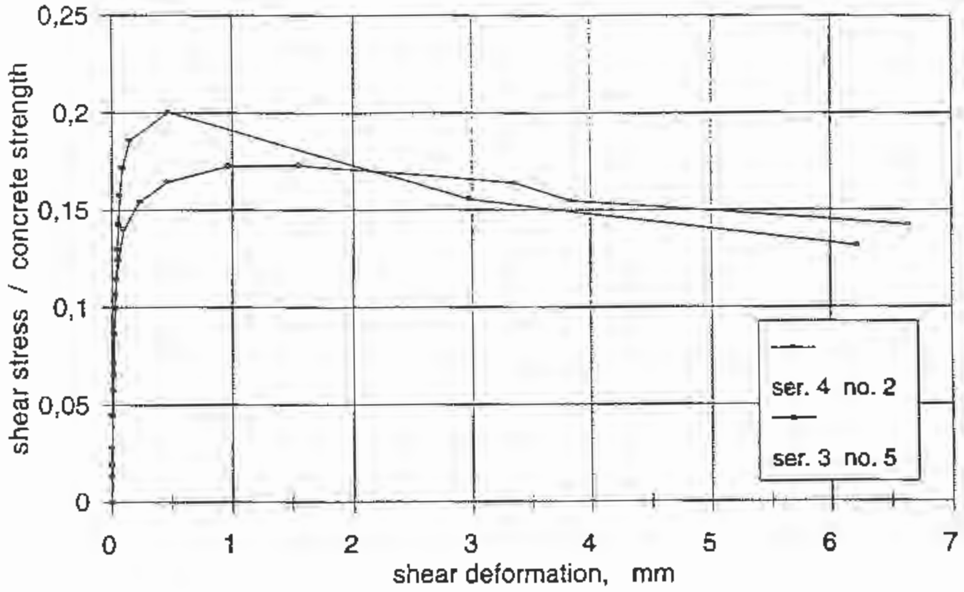

Figure 5. Load deformation curves.

Figure 6. Rough side of multilayer element prior to joining.

The joint between the elements are either toothed or rough (as shown on Figure 1). The rough surface is ensured by the quite open structure of the lighter concrete (density (600kg/m^3), which gives a quite rough surface as shown on Figure 6. The joint failures in the tests were usually placed in the interface between the mortar and the elements as shown in Figures 7 and 8.

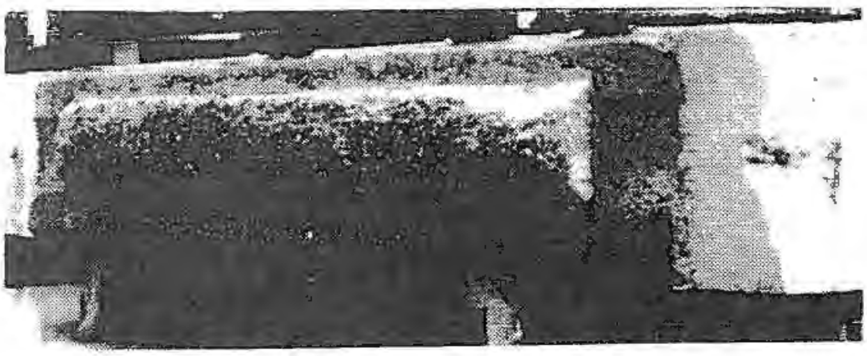

Figure 7. Fracture surface in rough joint.

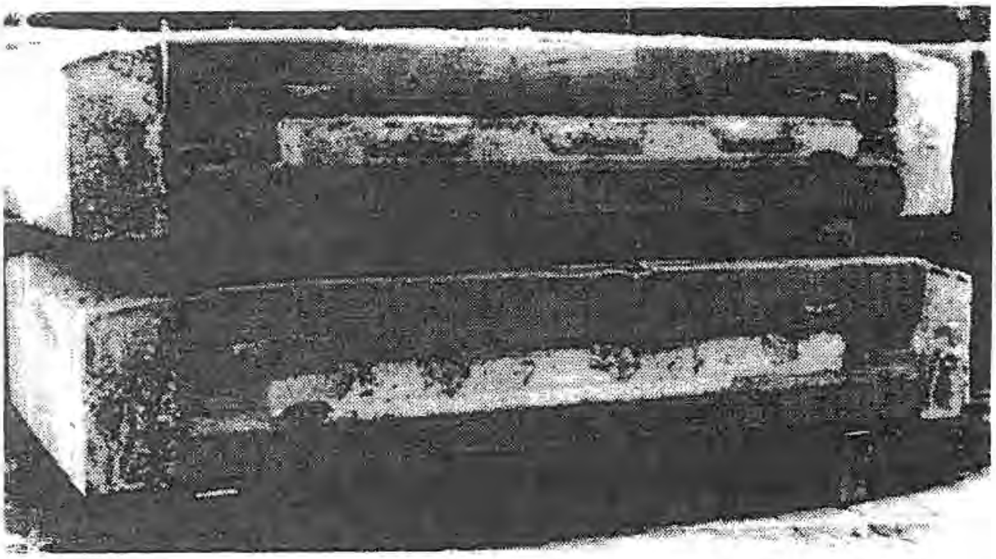

Figure 8. Fracture surface in toothed joint.

The tensile strength of the concrete in the elements has not been measured in the actual elements, but data from the same element productions in the same period /5/ shows the flexural strength f_{ft} to be fairly constant:

f_{ft}/f_c = 0.21 for the multilayer elements in Table 3,4 and 5
f_{ft}/f_c = 0.14 for the monolitic elements in Table 6

These values will be used later for the comparison between the European codes predictions of shear capacity and the experimental capacities.

4. THEORETICAL ESTIMATIONS

4.1 General

A theoretical estimation of the in-plane shear capacity of the joint can be obtained through the use of plasticity theory. These estimates can be upper limit solutions as well as lower limit solutions /3/,/4/.

The use of upper and lower bound theorems based on ideal-plastic material models and assumed failure modes will enable an estimate of the load carrying capacities, but will usually deviate significantly from the test results.The estimate can be much improved through the introduction of an efficiency factor, which describes e.g. to which extent the whole joint is effective.

4.2 Upper limit analysis

The use of an upper limit analysis is based on the bound limit theorem: "The load found from the equation of work for an arbitrary, geometrically admissible failure figure is greater than or equal to the yield load".

A geometrically admissible failure figure is a deformation field that satisfies the compatibility conditions and the geometrical boundary conditions. Coulombs failure hypothesis is assumed for the concrete and the mortar. The tensile strength of the concrete (mortar) normal to the joint is neglected due to the precracking.

Formulas for the shear capacity of the joint are derived by assuming plane stress field as well as plane strain field. Upper limit analysis leads to these formulas for the shear capacity of a joint:

$$\tau_u/f_c = 0.5*\nu \qquad \text{for } 0.5*\nu \leq \Phi \qquad (6)$$

$$\tau_u/f_c = \nu*\delta*(1 - \sin(\phi))/(2*\cos(\phi)) + \Phi*\tan(\phi) \qquad \text{for } \nu*\delta*(1 - \sin(\phi)) / 2 \leq \Phi \qquad (7)$$

$$\tau_u/f_c = \sqrt{[\Phi*(\nu*\delta - \Phi)]} \qquad \text{for } \Phi \leq \nu*\delta*(1 - \sin(\phi)) / 2 \qquad (8)$$

of which the lowest estimate represents the capacity and where

τ_u	is the shear strength in the joint
ν	is an efficiency factor, which accounts for the simplification of the stress-strain curve of the concrete and mortar.
δ	is a joint efficiency factor, describing the weakness of the joint, related to a monolithic structure
ϕ	is the internal angle of friction in the concrete

4.3 Lower limit analysis

The use of a lower limit analysis is based on the lower bound theorem: "The load belonging to a safe and statically permissible stress field is smaller than or equal to the yield load".

A safe stress field is a stress field in which the yield strengths are not exceeded. A statically permissable stress field is a stress field which satisfies the equilibrium conditions and the statical boundary conditions. The following failure modes are considered:

1: The reinforcement is assumed to yield before the concrete is crushed.
2: The concrete is assumed to be crushed before the reinforcement yields.
3: The concrete is assumed to be crushed and the reinforcement to yield at the same time.

This leads to the formulas

$$\tau_u/f_c = \nu*\delta*\Phi*\cot(\alpha) \qquad \text{for } 0 \leq \Phi \qquad (9)$$
$$\tau_u/f_c = 0.5*\nu \qquad \text{for } 0.5*\nu \leq \Phi \qquad (10)$$
$$\tau_u/f_c = \sqrt{[\Phi*(\nu - \Phi)]} \qquad \text{for } 0 \leq \Phi \leq 0.5*\nu \qquad (11)$$

of which the lowest estimate represents the capacity and where

α is the crack angle (the angle of the compressive struts in the joint mortar with the longitudinal direction of the joint).

5. VERIFICATION OF ESTIMATIONS

The theoretical estimations and the codes formulas are compared to the test results in the following.

The efficiency factors (ν and δ), the crack angle (α) and the angle for friction (ϕ) for the theoretical estimations are established on the basis of the test results.

Table 7 shows the values for these parameters, which leads to fairly conservative estimates. The authors have considered these values acceptable as the major part of the test results are above or even well above the estimated curve.

Joint	Density kg/m^3	ϕ	ν	δ	α
Toothed	1750	35°	0.6	0.65	20°
Rough	600	25°	0.4	0.55	18°

Table 7. Parameters in the formulas in upper and lower limit solutions.

The teoretical shear capacities for the rough joints between the multilayer elements and the experimental shear capacities are shown on Figure 9 as functions of the "equivalent" degree of reinforcement ($\Phi + \sigma_N/f_c$). (Series 2 has $\sigma_N > 0$ and $\Phi = 0$, whereas series 1 and 3 have $\sigma_N = 0$ and $\Phi > 0$).

The shear capacity for a rough joint according to the European code prEN1992-1-3 is estimated as shown on Figure 9.

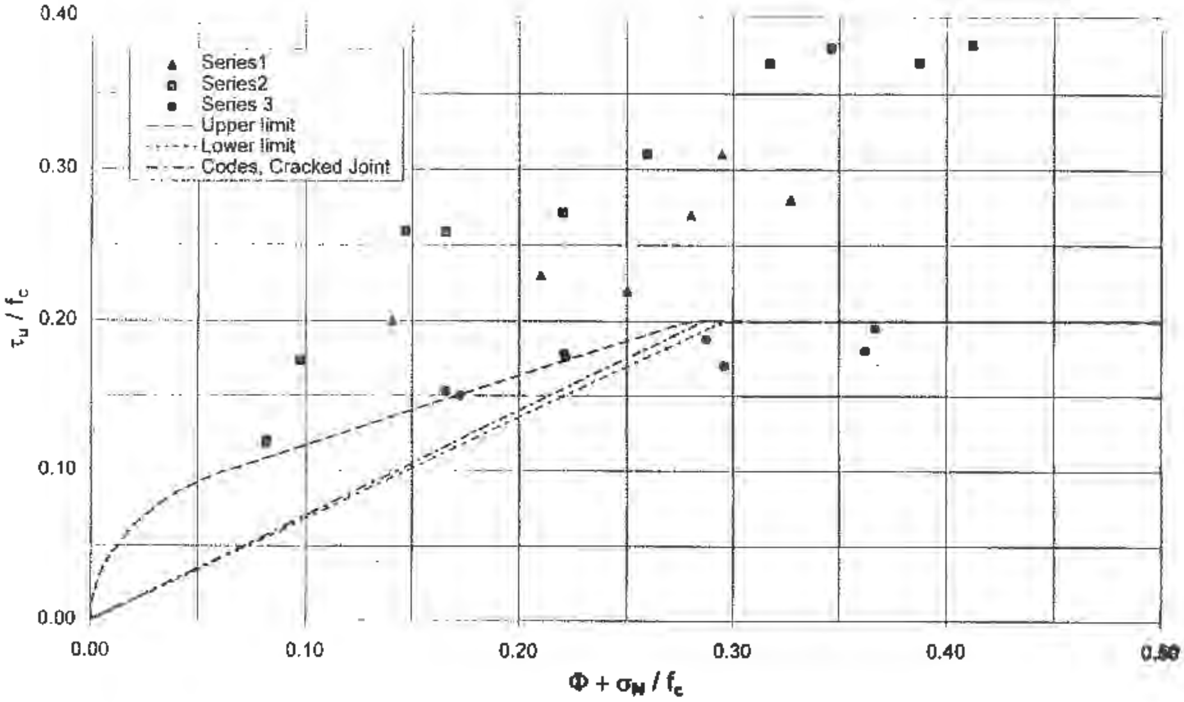

Figure 9. Rough joint between the multilayer elements (concrete density in center layer is app. 600kg/m^3): Experimental shear capacities and estimated capacities.

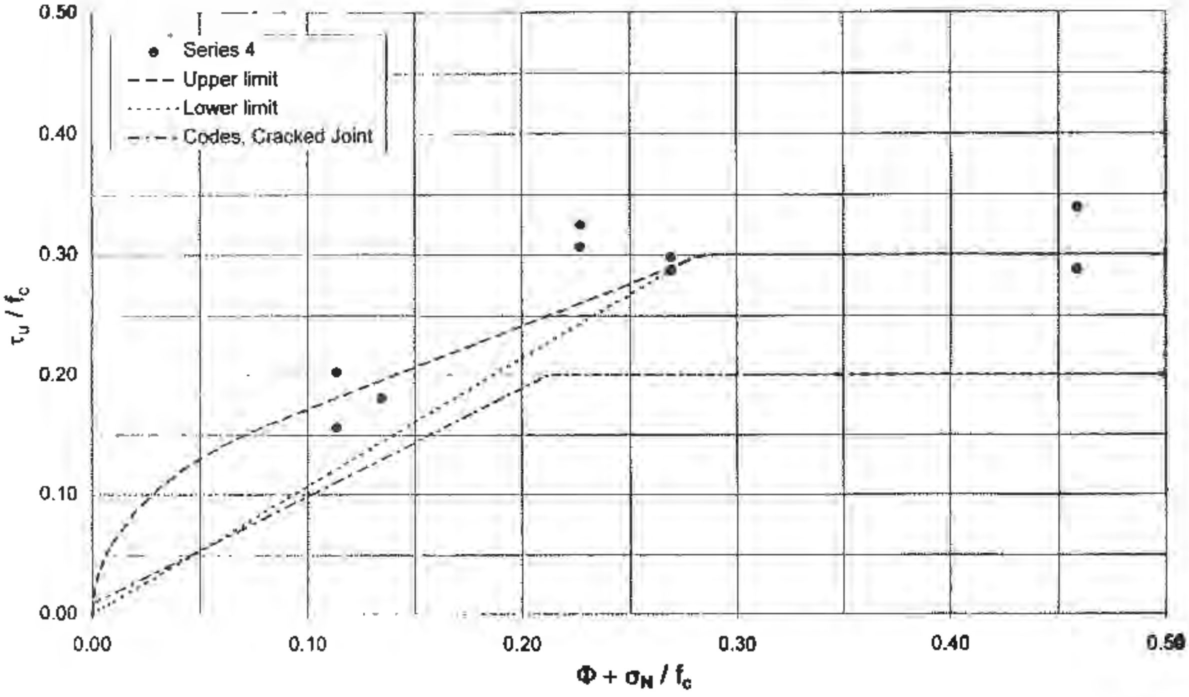

Figure 10. Toothed joint between monolitic elements (concrete density is 1750 kg/m^3): Experimental shear capacities and estimated capacities.

Figure 9 shows that the upper and lower limit solutions can be used to estimate the joint shear capacities in a fairly conservative way.

Similar analysis can be made of the toothed joints in series 4 ($\sigma_N=0$ and $\Phi>0$) as shown on Figure 10.

Figure 10 shows a similar consistency and conservativeness of the teoretical estimations as on Figure 9.

The Figure 9 and 10 shows that the codes estimate is fairly conservative, just as it verifies our assumption that the joints between elements of lightweight aggregate concrete with an open structure behaves in the same way as between elements of normalweight concrete with a closed structure.

6. CONCLUSIONS

The comparisons between the tests and the European code estimations of shear capacities have shown that the code rules for normalweight concrete when used on lightweight aggregate concrete with open structure are quite conservative. However, it has also shown that the joint between LAC-elements behaves like joint between normalweight concrete elements.

A theoretical estimation, based on the plasticity theory and the use of efficiency factors, has verified a more optimal set of calculation rules for the shear joint strength, which will allow a higher capacity than the codes.

These conclusions are valid for rough and toothed joints with reinforcement placed at an angle of 90° to the joint. Some further work needs, however, still to be done on smooth joints and on joints with other concrete densities.

7. ACKNOWLEDGEMENT

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9. NOTATION

A_j	Joint area effective in the shear transfer (The tooth area in a toothed joint, but the whole joint area in a rough joint)
A_s	Reinforcement area
f_c	Compressive strength of the concrete or the joint mortar, whichever is the lowest
f_{ft}	Flexural strength of the concrete
f_t	Tensile strength of the concrete
f_y	Yield strength of the reinforcement
K_T	Shear coefficient according to Table 1
P	Compressive force perpendicular to the joint
V_{exp}	Experimental shear capacity of joint
V_u	Shear capacity of joint.
α	Crack angle
δ	Joint efficiency factor
Δ	Shear deformation of joint at maximum shear force
Φ	Degree of reinforcement
ϕ	Internal angle of friction in the concrete
σ_N	Compressive stress perpendicular to joint
τ	Shear parameter
τ_{exp}	Experimental shear capacity of the joint
τ_u	Shear strength
ν	Efficiency factor
μ	Coefficient of shear friction according to Table 1