

## MOMENT CAPACITY AND SHEAR FORCE CAPACITY OF LIGHTWEIGHT AGGREGATE CONCRETE FLOOR COMPONENTS



Henning Larsen, Assoc. Prof. M.Sc.  
Institute of Building Design, Technical University of Denmark  
DK 2800 Lyngby, Denmark.  
Tel: +45 45 93 44 31      Fax: +45 45 93 44 30



N.U. Ingholt, Director, M.Sc.  
Ingholt Consult ApS,  
Christian X's Alle 168  
DK 2800 Lyngby, Denmark  
Tel: +45 45 88 06 33      Fax: +45 45 88 06 84



Per Goltermann, M.Sc. Civ.Eng. Ph.D.  
G.M. Idorn Consult, RH&H A/S,  
Bredevej 2  
DK 2830 Virum, Denmark  
Tel: +45 45 98 67 30      Fax: +45 45 98 69 32

### ABSTRACT

The load-bearing capacity of reinforced floor components, made of lightweight aggregate concrete with open structure, is determined - in Denmark - by full-scale testing. The components are solid components as well as three-layer components.

This paper presents formulas for the calculation of the moment capacity and shear force capacity of such components, based on the strength of the materials used.

The formulas are verified by comparison of the calculated capacities based on experimental material parameters with the experimental load bearing capacities.

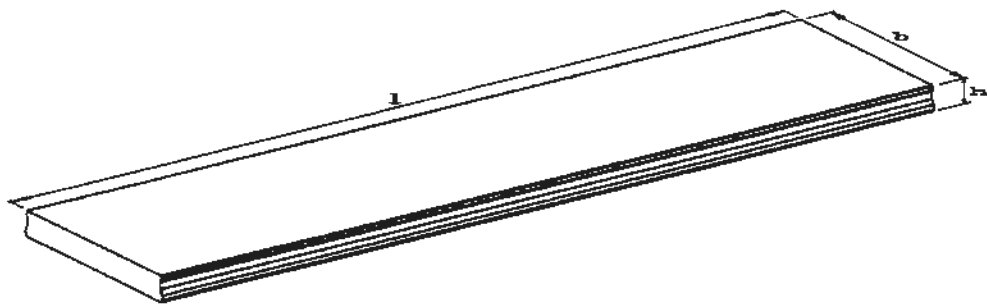
**Keywords:** floor components, lightweight aggregate concrete, load-bearing capacity, moment capacity, shear force capacity.

## 1. INTRODUCTION

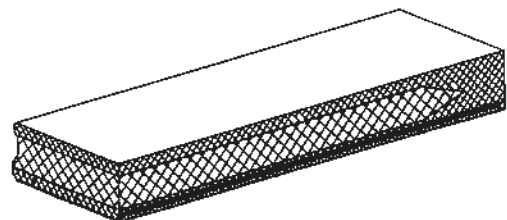
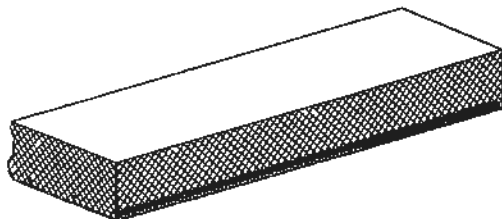
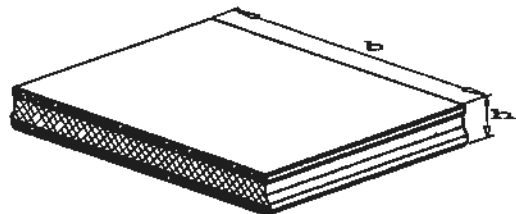
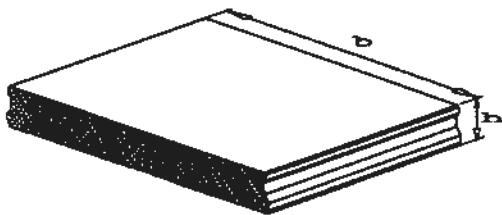
The load-bearing capacity of reinforced floor components, made of lightweight aggregate concrete with open structure, shall be determined through full-scale testing - according to DS 420 "Code of Practice for the Structural Use of Lightweight Concrete", /1/.

In this paper results are presented from a project initiated in order to establish and document formulas for the calculation of the moment capacity and shear force capacity, based on the cross-section and the strength of the materials used.

The components tested in the project are solid components ("homogeneous") as well as three-layer components ("sandwich"-type). Figure 1.



a) component



b) details of solid component

c) details of three-layer component

Figure 1. Floor slab components

For the three-layer components the density of the middle layer is lower than the one of the outer layers. - This type of component is cast "solid" in outer-layer-concrete over a short length at the ends, figure 1c.

The components are cast in forms. The concrete is compacted by a roller or a plate vibrator. Finally the component is steam cured. The production of the components is described in /2/.

The presented formulas for the calculation of the load-bearing capacity are verified by the results from full-scale tests with components.

## 2. EXPRESSIONS FOR LOAD-BEARING CAPACITY

### 2.1 Moment capacity

The expression used for the calculation of the moment capacity ( $M_{cal}$ ) is

$$M_{cal} = (1 - \frac{1}{2} \Phi) \cdot A_s \cdot f_y \cdot d \quad (1)$$

where

$$\begin{aligned} \Phi &= A_s \cdot f_y / (b \cdot d \cdot f_c) \\ A_s &= \text{area of the reinforcement} \\ f_y &= \text{tensile yield strength or 0,2\% proof stress of the reinforcement} \\ b &= \text{width of the component} \\ d &= \text{effective depth of the cross-section} \\ &\quad (\text{distance from the reinforcement to the edge in compression}) \\ f_c &= \text{compressive strength of the lightweight concrete} \end{aligned}$$

For three-layer components  $f_c$  is the compressive strength of the concrete in the outer layer in the side in compression.

The expression for the moment capacity is established on the basis of the assumptions normally assumed in the calculation of the moment capacity of reinforced "normal weight" concrete.

The principal assumptions are:

- plane sections remain plane,
- tension stresses in the concrete are ignored,
- compressive stresses in the concrete are taken as equivalent to a constant stress equal to the compressive strength on 4/5 of the depth of the compressive zone,
- tensile yield stress in the reinforcement, provided that the strain is not taken as greater than the ultimate strain.

For three-layer components the compression zone is limited to the outer layer in the side of compression.

## 2.2 Shear force capacity

The expression used for the calculation of the shear force capacity ( $V_{cal}$ ) is

$$V_{cal} = \tau_u \cdot k \cdot (1,2 + 40 \phi) \cdot b \cdot d \quad (2)$$

where

$$k = 1,6 - d; \quad d \text{ in m}$$

$$\phi = A_s / (b \cdot d)$$

$$\tau_u = 0,125 \cdot f_{bt}$$

$$f_{bt} = \text{bending tensile strength of the lightweight concrete}$$

$$b = \text{width of the component}$$

$$d = \text{effective depth of the cross-section}$$

$$A_s = \text{area of the (tensile) reinforcement}$$

$k$  shall not be taken as less than 1.

For three-layer components  $f_{bt}$  is taken as the bending tensile strength of the lightweight concrete in the middle layer.

The expression of the shear force capacity originates from prENV 1992-1-4, Eurocode No. 2, Part 1-4, The Use of Lightweight Aggregate Concrete with Closed Structure, 1993, /3/.

Furthermore the expression is proposed in prEN 1520, Prefabricated Components of Lightweight Aggregate Concrete with Open Structure, 1994, /4/.

## 3. TESTS

Tests for verification of the expressions for the load-bearing capacity of the component were carried out within the following limits of dimensions and materials specifications:

### *solid components*

depth of the components,  $h = 120$  to  $260$  mm

width of the components,  $b = 0,6$  to  $1,2$  m

span (length), up to  $6,7$  m

compressive strength of lightweight concrete,  $f_c = 14$  to  $27$  N/mm<sup>2</sup>

density of lightweight concrete,  $\rho = 1550$  to  $1850$  kg/m<sup>3</sup>

bending tensile strength of lightweight concrete,  $f_{bt} = 2,2$  to  $3,3$  N/mm<sup>2</sup>

reinforcement: hot-rolled deformed steel (ribbed bars)

number of reinforcing bars,  $6$  to  $12$

diameter of reinforcing bars,  $8$  to  $12$  mm

guaranteed value of the tensile yield strength (or 0,2% proof-stress) of the reinforcement,  $f_{yk} (f_{0,2}) = 410$  to  $550 \text{ N/mm}^2$

*three-layer components*

depth of the components,  $h = 120$  to  $280 \text{ mm}$

width of the components,  $b = 0,6$  to  $1,2 \text{ m}$

span (length), up to  $6,5 \text{ m}$

compressive strength of lightweight concrete in the outer layers,  
 $f_c = 10$  to  $24 \text{ N/mm}^2$

compressive strength of lightweight concrete in the middle layer,  
 $f_c = 2,1$  to  $4,0 \text{ N/mm}^2$

density of lightweight concrete in the outer layers,  
 $\rho = 1330$  to  $1640 \text{ kg/m}^3$

density of lightweight concrete in the middle layer,  
 $\rho = 550$  to  $660 \text{ kg/m}^3$

bending tensile strength of lightweight concrete in the middle layer,  
 $f_{bt} = 0,6$  to  $1,1 \text{ N/mm}^2$

reinforcement: hot-rolled deformed steel (ribbed bars)

number of reinforcing bars: 6

diameter of reinforcing bars, 6 to 12 mm

guaranteed value of the tensile yield strength (or 0,2% proof-stress) of the reinforcement,  $f_{yk} (f_{0,2}) = 410$  to  $550 \text{ N/mm}^2$

### **3.1 Tests with components to determine the moment capacity and the shear force capacity**

Full-scale testing was carried out according to DS 434-11 "Performance Test of Beam and Slab Components", /5/.

The components were simply supported at both ends, with one of the supports horizontally movable. Supports extended over the full width of the component. The components were loaded by two transverse line-loads at the outer quarter points of the span.

Some of the components were loaded by one transverse line-load at one outer fifth point of the span in order to ensure a shear failure.

The components were produced by three Danish producers, and the tests were performed at the particular factories in 1992 and 1993, under supervision by the authors.

The detailed test results are presented in /6/ and /7/.

### 3.2 Testing of materials

#### *Reinforcement*

The ultimate tensile strength and the yield strength (or the 0,2 % proof-strength) were determined on specimens representative for the reinforcing bars of the tested components.

#### *Lightweight aggregate concrete*

The compressive strength, the density and the bending tensile strength were determined.

The testing was carried out according to the Danish test standards DS 434, /8/, /9/, /10/.

The test specimens were cut from an additional component produced immediately after each component for full-scale testing. This component had a length of 1,5 to 2,0 m; width and depth were identical to those of the test component.

For the determination of each of the material parameters a set of three specimens were used, and the mean values were used.

The compressive strength of the lightweight concrete in the outer layer in the three-layer components was determined by use of special specimens, since the layers are too thin to cut standard specimens; these specimens were of dimensions 100 mm · 100 mm · 20 mm.

For each test of compressive strength two specimens were tested as a pair, upright placed in the testing machine with a free distance of 60 mm.

Otherwise the testing procedure fully complied with the standard.

The materials testing was carried out at the particular factory producing the primary components, under supervision by the authors.

Further information and results of the materials testing is presented in /11/.

## 4. EXPERIMENTAL VS. CALCULATED LOAD-BEARING CAPACITIES.

On the basis of the load which the components were capable of carrying in the tests, the experimental moment capacity  $M_{exp}$  and the experimental shear force capacity  $V_{exp}$  are determined.

By use of the expressions in chapter 2 the calculated moment capacity  $M_{cal}$  and the calculated shear force capacity  $V_{cal}$  are determined based on the actual cross-sectional geometry and the material parameters.

Figure 2 and 3 shows the calculated moment capacities as well as the experimental ones. Figure 4 and 5 shows the calculated shear force capacities as well as the experimental ones.

Figure 2. Moment capacities. Solid components.

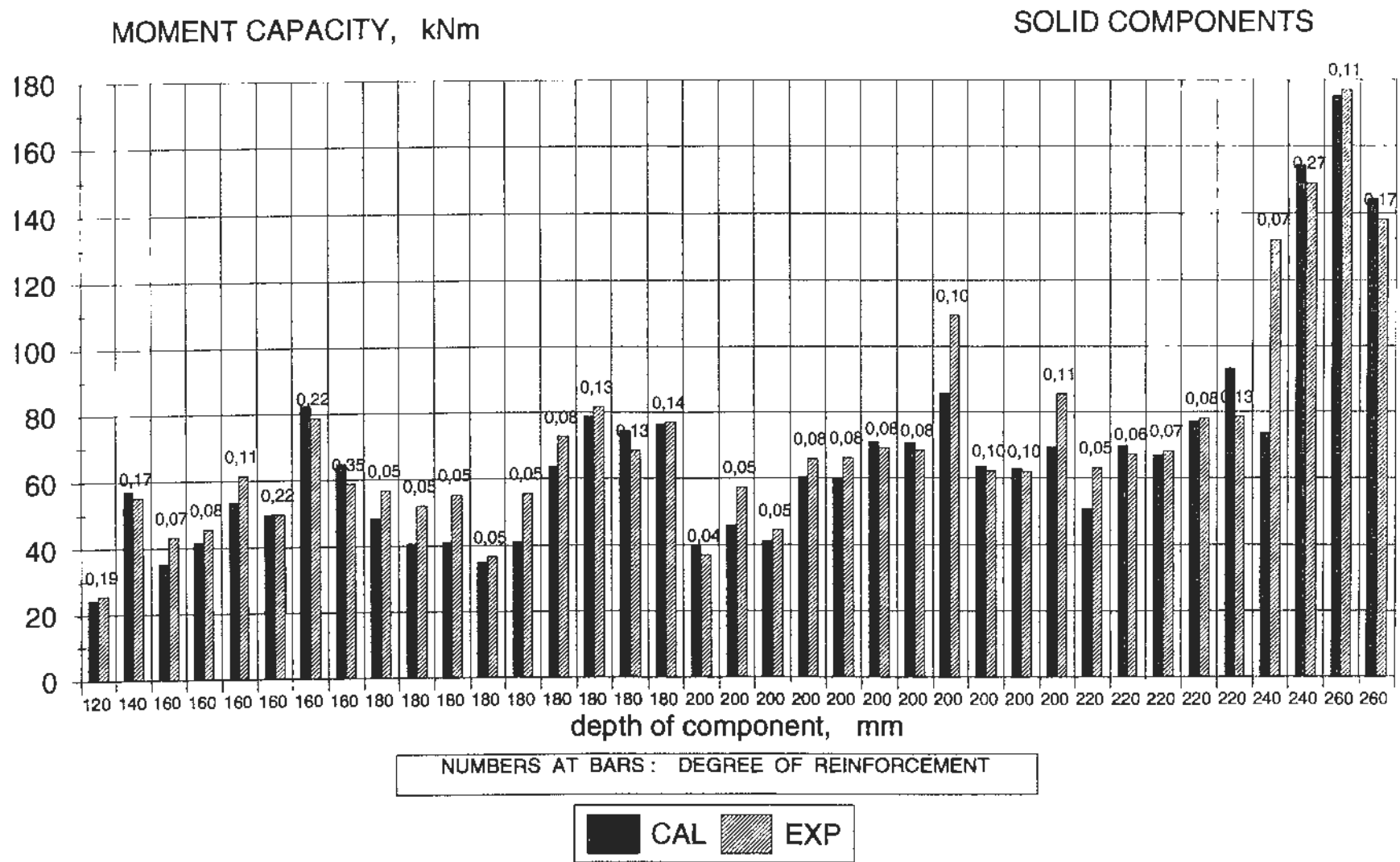


Figure 3. Moment capacities. Three-layer components.

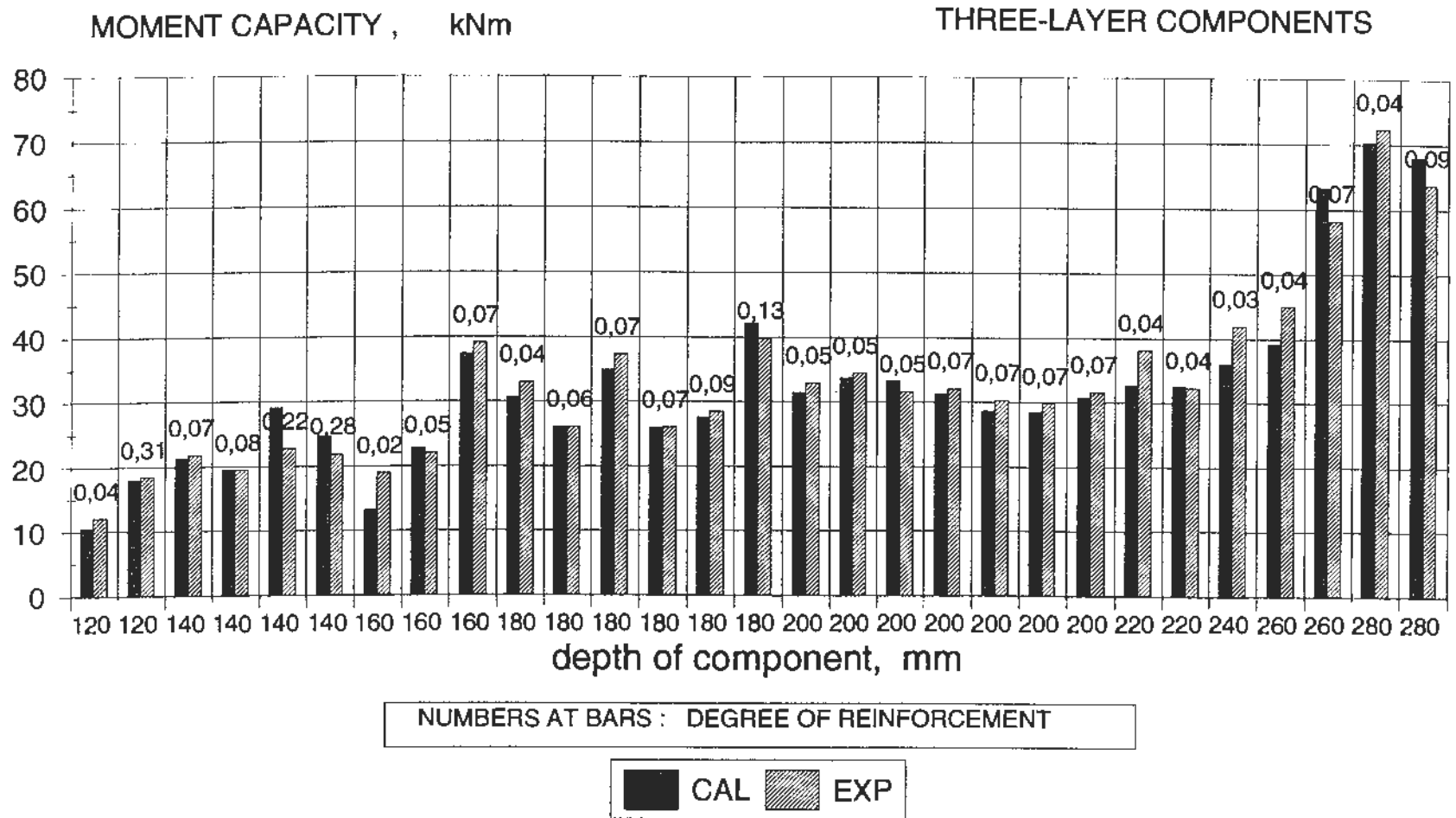




Figure 4. Shear force capacities. Solid components.

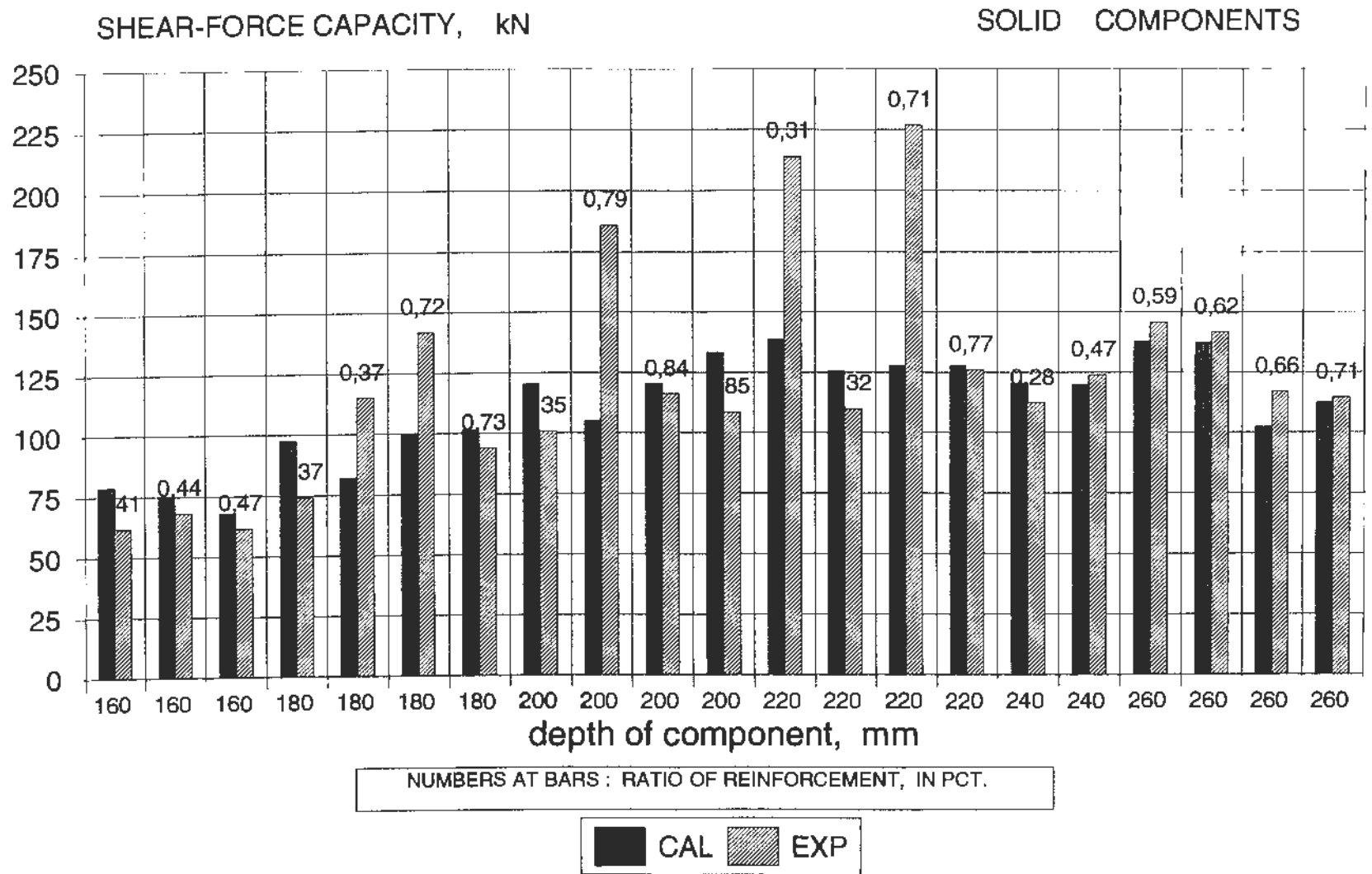
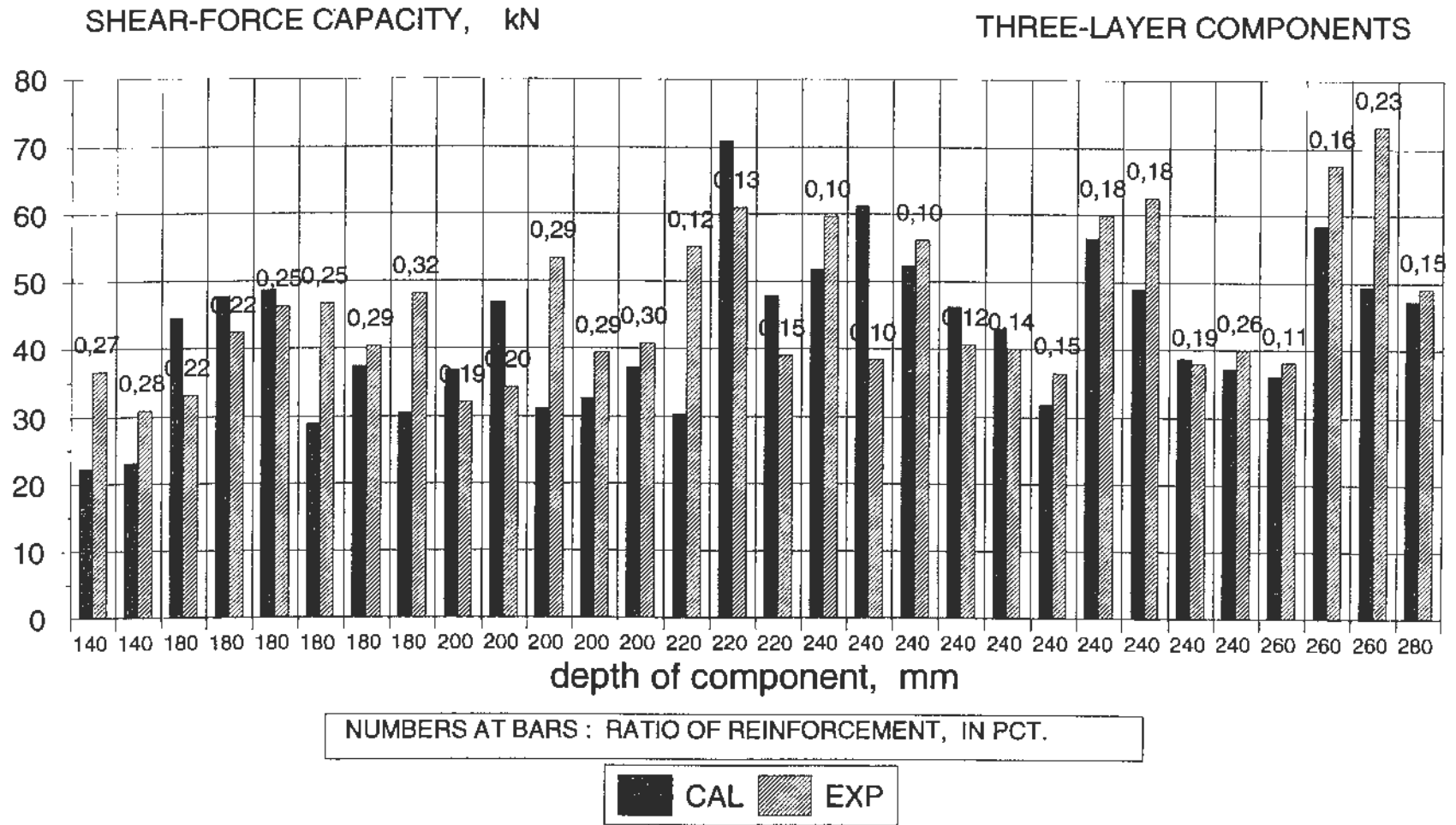


Figure 5. Shear force capacities. Three-layer components.



In figure 6 is shown results from other investigations of the moment capacity of three-layer components, /12/, /13/. These tests were carried out at the Technical University of Denmark and at the factories involved.

A good agreement can be observed between the experimental moment capacity and the moment determined by the expression in section 2.1 for the moment capacity, - for solid components as well as three-layer components.

The mean value of the ratio between calculated and experimental moment capacities is 0,94 for solid components and 0,98 for three-layer components.

It should be noticed that components, for which the moment capacity is decisive for the load-bearing capacity, a mode of failure is observed conforming to a bending failure.

Agreement between the experimental shear force capacity and the shear force determined by the expression in section 2.2 for the shear force capacity is not similar satisfying.

It is found especially that the variations are fairly extensive for this type of failure. And for some of the components the calculated capacity differs to considerable extent from the experimental capacity, but for these components the calculated values of shear force capacity are conservative.

The mean value of the ratio between calculated and experimental shear force capacities is 0,98 for solid components and 0,95 for three-layer components.

## **5. CONCLUSIONS**

The agreement between the experimental moment capacity and the moment capacity determined by the expression in section 2.1 is very good.

The agreement between the experimental shear force capacity and the shear force capacity determined by the expression in section 2.2 is not similar satisfying. It is found especially that the variations are fairly extensive.

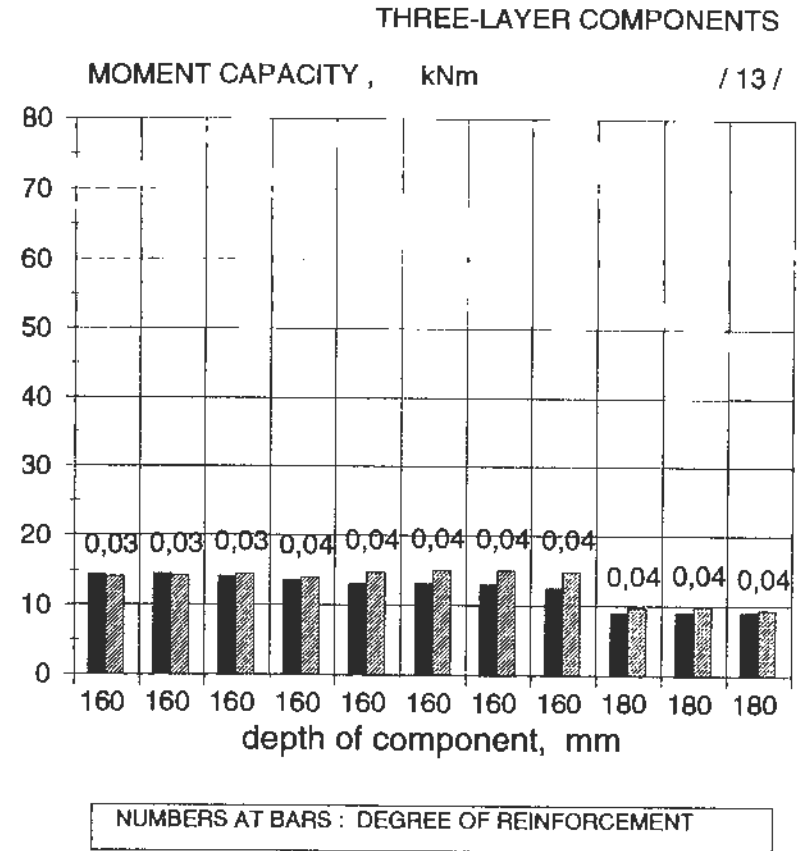
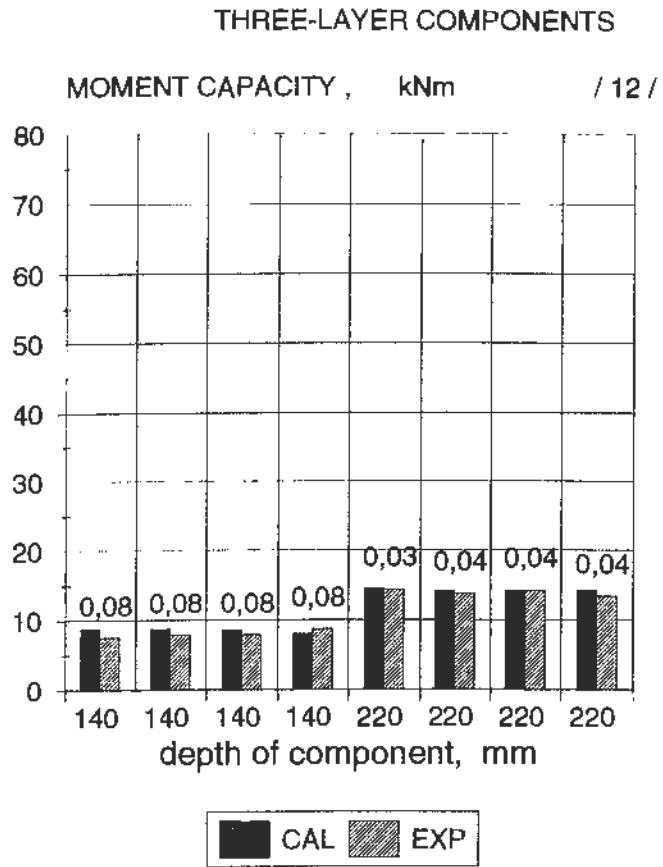
Based on the investigations carried out, it is concluded that the load-bearing capacity of reinforced floor components, made of lightweight aggregate concrete with open structure, can be calculated by means of the expressions in chapter 2.

## **6. ACKNOWLEDGEMENTS**

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Experimental works were carried out at the laboratories of the three manufacturers.

Figure 6. Moment capacities. /12/, /13/.



## 7. NOTATION

$A_s$	area of reinforcement
$b$	width of component
$d$	effective depth of cross-section
$f_{bt}$	bending tensile strength of lightweight concrete
$f_c$	compressive strength of lightweight concrete
$f_{yk}$	tensile yield strength of reinforcement
$f_{0,2}$	0,2% proof-stress of reinforcement
$h$	depth of component
$l$	length of component
$M_{cal}$	calculated bending moment capacity
$M_{exp}$	experimental bending moment capacity
$V_{cal}$	calculated shear force capacity
$V_{exp}$	experimental shear force capacity
$\rho$	density of lightweight concrete
$\tau_u$	basic shear strength of lightweight concrete
$\Phi$	degree of reinforcement
$\varphi$	ratio of reinforcement

## 8. REFERENCES

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