LIGHTWEIGHT CONCRETE WALLS AND COLUMNS: DETERMINING THE LOAD-CARRYING CAPACITY

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ABSTRACT: Precast walls and columns of lightweight aggregate concrete with open structure (LAC) or aerated autoclaved concrete (AAC) are extensively used in houses and structures in Northern Europe. The work on an European CEN-standard /1/ for precast LAC-components identified a number of national differences on requirements and design rules, which lead to a more detailed evaluation of the different design rules and calculation models.

This paper presents and compares the different, relevant models estimations of load-carrying capacities and compares these with the experimental results. The LAC in the test specimens cover a large range of designs with compressive strengths in the range of 2 to 20MPa, densities between 500 and 1800kg/m^3 and thicknesses between 50 and 250mm. The models represent walls and columns with cross-sections where the reinforcement and the tensile strength may be neglected or included in the individual model, since the national design traditions and the actual amount of structural reinforcement in the test specimens differs.

The CEN-standard /1/ for LAC-components and the CEN-draft for AAC /2/ prescribes the Ritter and the Navier-models for the structures without structural reinforcement and presents the design assumptions for the linear model of the structurally reinforced component. This paper document these formulas to be reasonably optimal and conservative.

KEYWORDS: Load-carrying capacity, lightweight, concrete, walls, columns, CEN.

1. INTRODUCTION

Prefabricated walls and columns of lightweight aggregate concrete with open structure (LAC) or aerated autoclaved concrete (AAC) are extensively used in houses and structures in Northern Europe /3/. This have recently resulted in the development of an European standard for precast LAC-components.

A number of national differences were identified during the standardisation work and seems to be due to the differences in requirements and traditions. These differences can best be illustrated by the maximal allowed slenderness, (ratio between the column-length (L_c) and the thickness of the wall or column (h)), as shown on Figure 1.1.

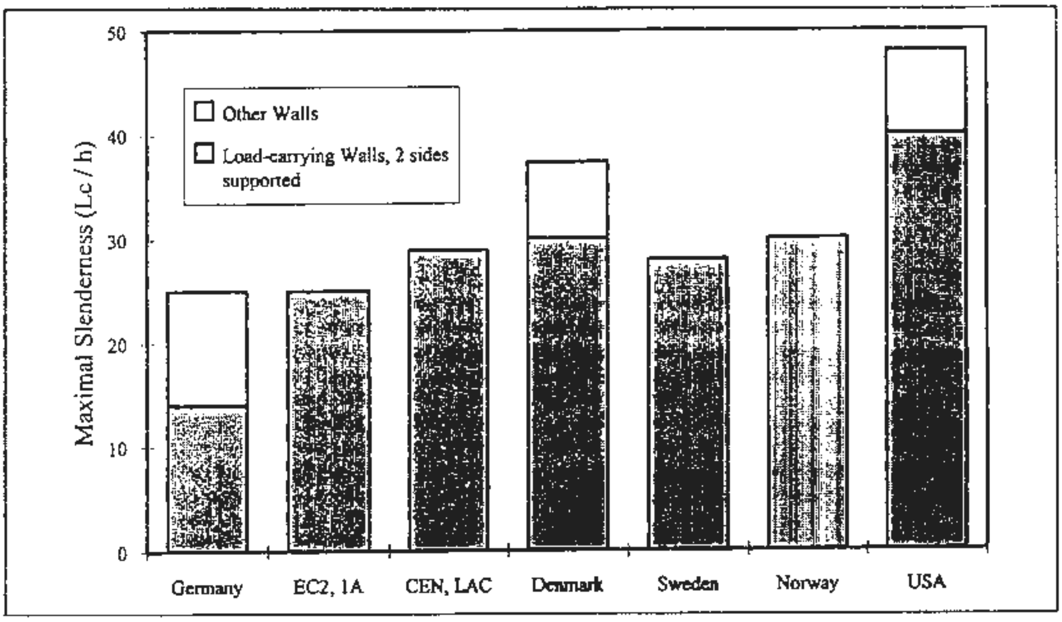

Figure 1.1. Maximal slenderness limits (L_c/h) in different countries.

The limits depends on the intended use of the components, on whether they are used for conventional load-carrying purposes with 2/3/4 supported sides or only used in positions, where they are not essential to the overall stability of the structure (e.g. in-fill walls) /1/,/5/,/6/,/8/,/10/,/11/ and /12/. The definition of the column length (L_c) differs as well from one country to another, which does affect the design. The European standards /4/,/6/ for normalweight and lightweight aggregate concrete with closed structure have no limitations for the slenderness of reinforced walls, but the standard /5/ for plain or lightly reinforced concrete do contain certain restrictions, equal to those imposed on masonry structures /7/.

The differences in the maximal slenderness illustrates the large potential for more slender structures in most European countries, since the USA-limits does exceed all others significantly (and the USA-requirements are for LAC-components with densities below 800kg/m^3).

This paper evaluates the different, possible design rules and formulas and compare their estimations with the available test results for LAC and AAC components in order to present a documentation for the proposed European design rules /1/, /2/.

The walls and columns have usually compressive strengths in the range of 2 to 20MPa, densities between 500 and 1800kg/m^3 and thicknesses in the range of 50 to 250mm. The components do contain some reinforcement, but the amount differs much from one country to another and may be quite insignificant for load-carrying purposes.

2. DISCUSSION OF THEORETICAL APPROACH

Most of the calculation methods are based on elasticity-theory or a more plastic approach, most of them combined more or less empirically with factor's which account for e.g. the non-linearities in the performance of the components, due to either a non-linear stress-strain relationship in the concrete or 2nd order effects.

The loading on the tested components is an eccentric, vertical load and/or a horizontal load. The load-carrying-capacity for eccentric, vertical loads is usually estimated on the basis of compressive strength (f_c) and modulus of elasticity (E_c), whereas the horizontal load requires either bending or flexural tensile strength (f_t) or reinforcement to be taken into account.

The in-situ cast normal weight concrete walls will usually contain reinforcement, which transfers the bending moments, however, this reinforcement does not prevent a brittle failure and neither is it always prescribed. The flexural tensile strength is usually only taken into account in normal weight concrete structures for the transfer of shear stresses, whereas masonry structures often take a flexural tensile strength into account.

The flexural tensile strength has for a number of decades been taken into account in Scandinavia for the transfer of bending moments in LAC-walls and in autoclaved, aerated concrete (AAC)-walls, whereas the reinforcement is mainly intended for the transport loads and the shrinkage. Reinforcement will, however, always give a higher load-carrying capacity and may be necessary in e.g. cases with a large horizontal load (soil pressure).

3. ECCENTRICALLY LOADED COLUMNS OR WALLS ($e_N/h<0.5$)

The eccentric vertical loads have eccentricities below h/2, since the vertical loads (N) must be positioned on top of the wall as shown on Figures 3.1 and 3.2. (This eccentricity is also the maximal limit for obtaining any load-carrying capacity without taking the reinforcement or the flexural tensile strength into account).

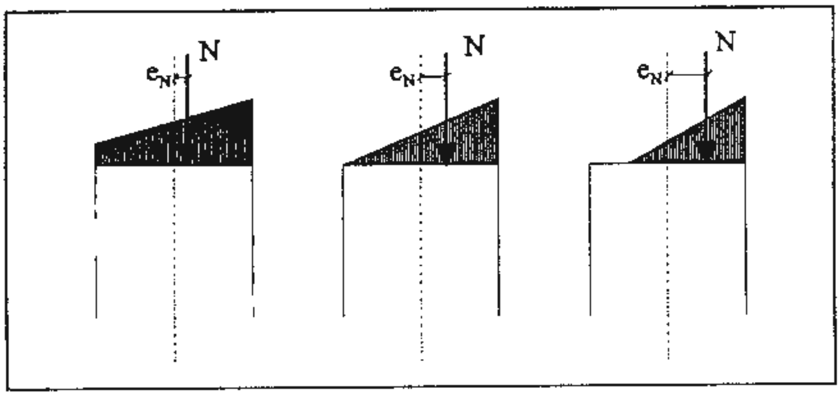

Figure 3.1. Loading on a wall or column. ($e_N/h<1/6$, $e_N/h=1/6$, $e_N/h>1/6$).

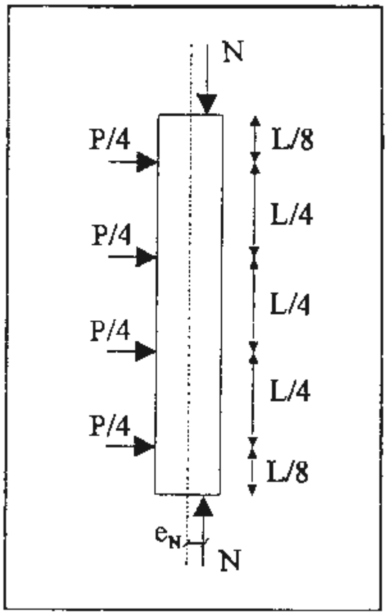

Figure 3.2. Load combination on a wall or a column.

A horizontal load (P) may be applied on the component, either as a uniform load, or (as in some of the tests) as four point-loads as indicated on Figure 3.2. This load-combination corresponds to a higher equivalent eccentricity (e), calculated as:

$$e \quad = \{N \cdot e_N + P \cdot L/8\}/N \qquad (3.1)$$

where

L is the height of the component, which in the test bench is equal to the column length L_c

In the tested components the parameters f_c, f_t, E_c were usually measured on samples from a set of 3 similar components and averaged for each type /13/,/15/,/18/, /19/. In some tests, however, the parameters were measured directly on the actual component /14/,/16/,/17/,/19/.

A number of established set of formulas will be presented in the following along with a short explanation of their backgrounds. Comparisons with experimental results will evaluate the different set of formulas.

3.1. RITTER'S METHOD

The austrian engineer Ritter, developed a method for calculating the load-carrying capacity of an eccentrically loaded, unreinforced column or wall. This method have been used for as well masonry as unreinforced concrete (both normal weight and lightweight and with closed or open structure). The method have also been used for decades in e.g. danish codes /8/ and have been implemented in the CEN-standard for LAC /1/ and the CEN-draft for autoclaved aerated concrete /2/.

The formulas for the load-carrying capacity (N_{cr}) are listed in the following.

$$N_{pl} = b \cdot h \cdot f_c \quad (3.1.1)$$
$$N_{eu} = K_c \cdot N_{pl} \quad (3.1.2)$$
$$K_c = 12 \cdot (K/\pi)^2 \quad (3.1.3)$$
$$K = (f_c/E_c)^{1/2} \cdot (L_c/h) \quad (3.1.4)$$
$$\varepsilon = e/h \quad (3.1.5)$$
$$h_c = h - 2 \cdot e$$
$$= h \cdot (1 - 2 \cdot \varepsilon) \quad (3.1.6)$$
$$N_{cr}/N_{pl} = (1 - 2 \cdot \varepsilon)/(1 + K_c/(1 - 2 \cdot \varepsilon)^2) \quad (3.1.7)$$

where

b is the width of the wall or column.

The method takes the part of the cross-section symmetrical to the loading point into account and neglect the rest of the cross-section, which is assumed to be cracked. The method reduces the capacity of the column/wall with a factor $(1+K_c/(1-2 \cdot \varepsilon)^2)$, which depends on the eccentricity and the slenderness of the wall. The method uses a conservative estimate of the size of the uncracked part of the cross-section and accounts for the 2nd order effects through the use of the K_c-factor.

3.2. ELASTIC MODEL WITHOUT TENSILE STRENGTH OR REINFORCEMENT

A classic elastic model can be based on the concept of a linear-elastic material without any tensile strength and by neglecting the reinforcement. The column or wall will in this model fail in one of the following three types of failures:

I: Failure due to crushing in uncracked cross-section
II: Failure due to crushing in cracked cross-section
III: Failure due to stability failure in cracked cross-section

The model have been established and discussed by /20/,/21/ who showed that the three cases are separated by two limit-values for the separating dimensionless first order eccentricities:

$$\varepsilon_{L1} = (2 - K_c)/12 \quad (3.2.1a)$$
$$\varepsilon_{L2} = (1 - \{K_c/2\}^{1/2})/2 \quad (3.2.1b)$$

The formulas for the load-carrying capacity in those references are listed in the following.

Case I: Failure due to crushing in uncracked cross-section ($0 \leq \varepsilon \leq \varepsilon_{L1}$)

$$N_{cr}/N_{pl} = (1/2)(6\varepsilon+1)/K_c + 1 - \{((6\varepsilon+1)/K_c+1)^2 - 4/K_c\}^{1/2} \quad (3.2.2a)$$

Case II: Failure due to crushing in cracked cross-section ($\varepsilon_{L1} \leq \varepsilon \leq \varepsilon_{L2}$)

$$N_{cr}/N_{pl} = (3/4)(1/2 - \varepsilon)(1 + \{1 - K_c/9/(1/2 - \varepsilon)^2\}^{1/2}) \quad (3.2.2b)$$

Case III: Failure due to stability failure in cracked cross-section ($\varepsilon_{L2} \leq \varepsilon \leq 0.5$)

$N_{cr}/N_{pl} \quad = (1-2\varepsilon)^3/K_c$ (3.2.2c)

The assumption of a ideally linear-elastic stress-strain relationship may be less conservative than required, however, the classic linear model can be used as a theoretical reference for other formulas.

3.3. EVALUATION OF THEORIES

The estimates of the load-carrying capacity according to the elastic model (3.2.2) and the Ritter model (3.1.7) are compared on Figure 3.3.1 and indicates up to 50% higher load-carrying capacity by using the classic, elastic model.

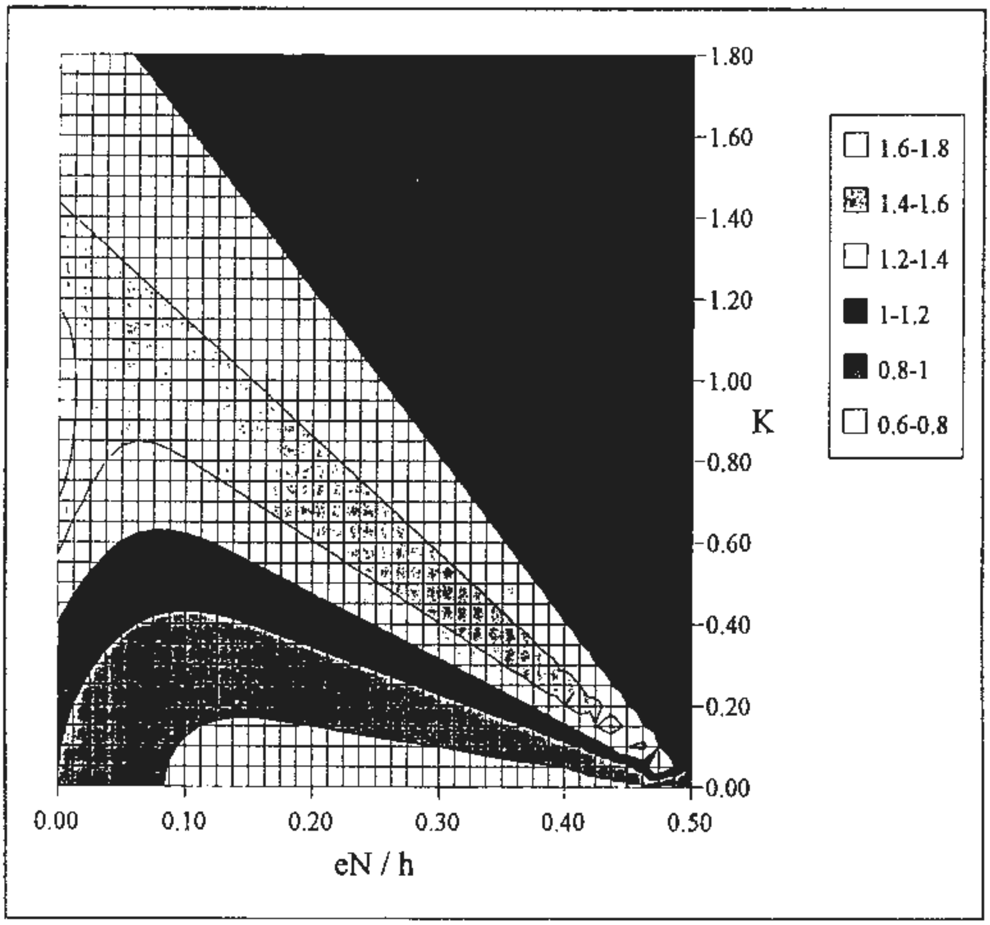

Figure 3.3.1. Diagram for ratio between estimated N_{cr} after linear model and Ritter model as a function of the dimensionless eccentricity (ε) and slenderness factor (K).

The test results have been found in a number of danish test reports from the period 1958-1993 /13/,/14/,/15/,/16/,/17/,/18/,/19/. The parameter range of the test specimens are indicated on Figure 3.3.2. The curves on Figure 3.3.2 indicates the limits between the three types of failure in the elastic model in clause 3.2 and illustrates the different elastic failure modes as well as the slenderness of the test specimens. the maximal L_e/h-value in the test in 35, which is above the limits of the European codes. the two models are compared to test results on Figure 3.3.3, where the Ritter model is shown to be conservative, but where the classic elastic model is nonconservative in a few cases.

The Ritter model shows a good correlation to the test results for small imperfections, but does give a rather conservative estimate for larger eccentricities as seen on Figure 3.3.4. The dimensionless eccentricity (ε) with a properly designed structural joint will, however, usually be in the range of 0.05 to 0.16 and the Ritter-model will thus give an acceptable and conservative design of the wall or column.

The Figure 3.3.6 shows that the linear model will be nonconservative for centrally loaded structures, where ε=0. The model will give more conservative estimates for the larger imperfections and can thus be used for design purposes.

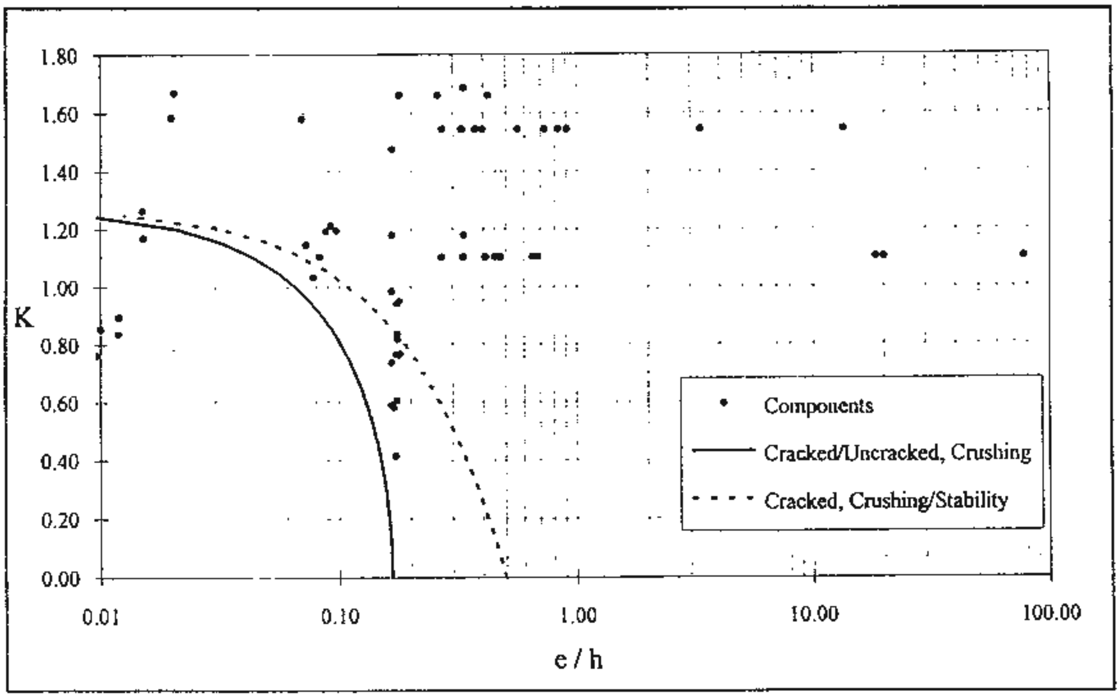

Figure 3.3.2. Dimensionless eccentricity and slenderness of tested components.

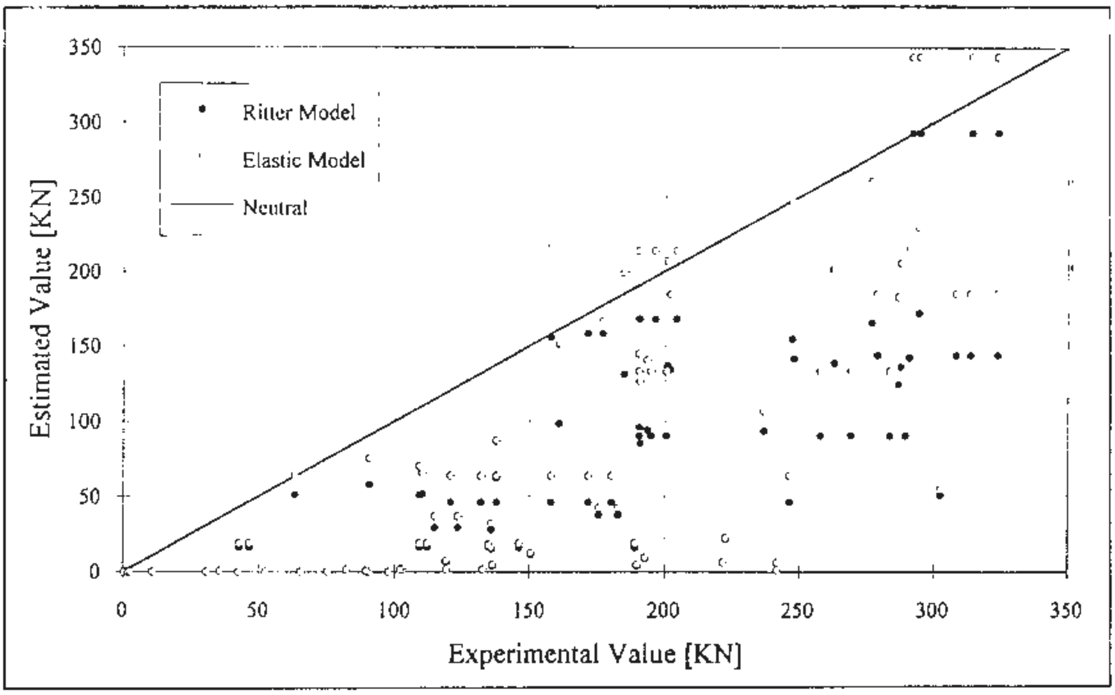

Figure 3.3.3. Estimated load carrying capacity versus experimental capacity.

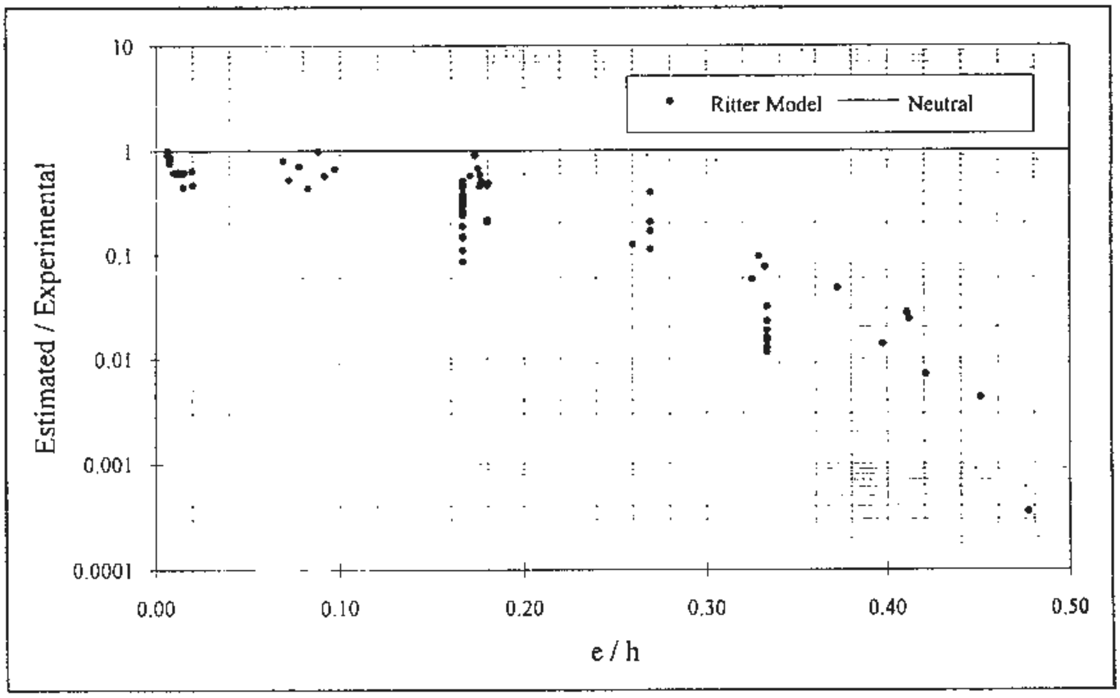

Figure 3.3.4. Ratio between estimated and experimental capacity versus dimensionless ecce..tricity (ε) for the models.

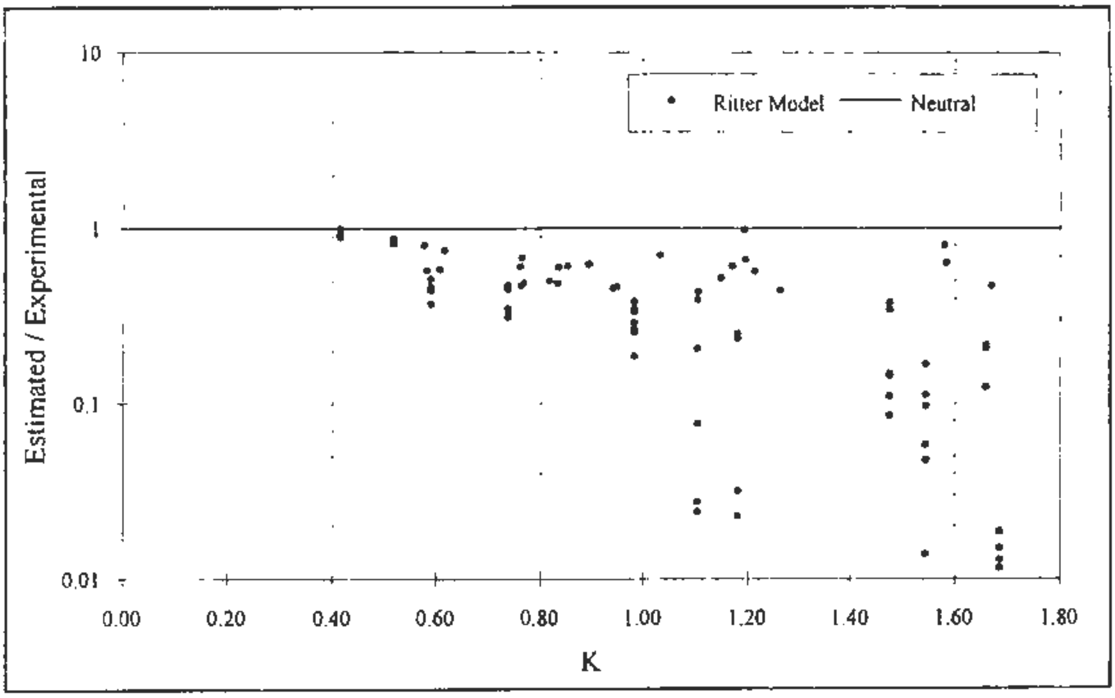

Figure 3.3.5. Ratio between estimated and experimental capacity versus dimensionless slenderness (K) for the Ritter model.

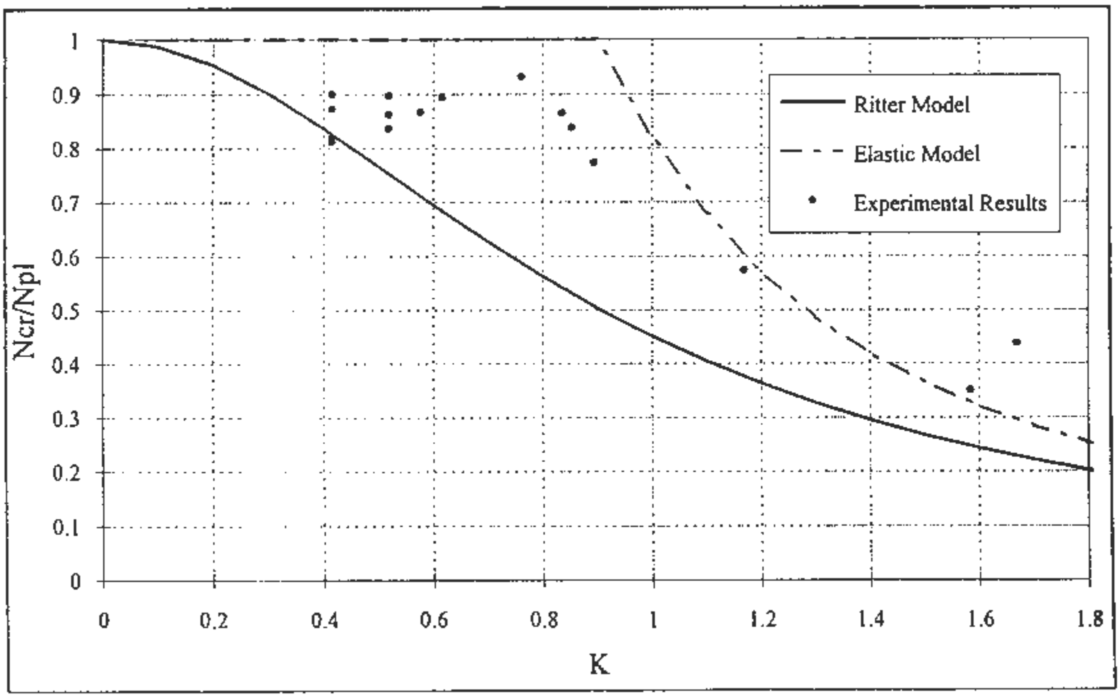

Figure 3.3.6. Experimental and estimated load-carrying capacity of centrally loaded components ($\varepsilon=0$).

4. COMBINED VERTICAL AND HORIZONTAL LOADING

4.1. NAVIER'S METHOD

Navier's model is used in the danish standard /8/ and in the proposed CEN-standards /1/, /2/ and combines the results from Ritter's method with bending from a horizontal load, thus taken the non-linearity and the 2nd order effects into account. The model use the flexural tensile strength for the transfer of the bending moments, but ignores the reinforcement.

N_e is calculated by Ritter's model in clause 3.1 as N_{cr} with the actual eccentricity of the permanent load /1/,/8/. In this clause an eccentricity of $e_1 = L_c/500$ has, however, been used for the calculation of N_e, since all the loads are short-term loads in the tests.

The stresses in the component are calculated as:

$$\sigma_c = N_{cr}/A + N_{cr} \cdot e/W \cdot N_e/(N_e - N_{cr}) \tag{4.1.1}$$
$$\sigma_t = -N_{cr}/A + N_{cr} \cdot e/W \cdot N_e/(N_e - N_{cr}) \tag{4.1.2}$$
$$A = b \cdot h \tag{4.1.3}$$
$$W = b \cdot h^2/6 \tag{4.1.4}$$

The two cases are separated by a limit value of the first order eccentricity (ε_{L1}):

$$\varepsilon_{L1} = (1+\alpha)/(1-\alpha) \cdot (2-(1-\alpha) \cdot K_c)/12 \tag{4.1.5}$$
$$\alpha = f_t/f_c \tag{4.1.6}$$
$$K_c = N_{pl}/N_e \text{ (in this clause)} \tag{4.1.7}$$

The load-carrying capacity is reached at the lowest N_{cr}-value where the material fails in tension or compression as:

Case I: Failure due to crushing in uncracked cross-section ($0 \leq \varepsilon \leq \varepsilon_{L1}$)

$$N_{cr}/N_{pl} = (1/2)[(6\varepsilon+1)/K_c + 1 - \{((6\varepsilon+1)/K_c+1)^2 - 4/K_c\}^{1/2}] \tag{4.1.8a}$$

Case II: Failure due to cracking of cross-section ($\varepsilon_{L1} \leq \varepsilon$)

$$N_{cr}/N_{pl} = (1/2)[(1-6\varepsilon)/K_c - \alpha + \{((1-6\varepsilon)/K_c - \alpha)^2 + 4\alpha/K_c\}^{1/2}] \tag{4.1.8b}$$

4.2. ELASTIC CALCULATION OF REINFORCED WALL OR COLUMN.

The reinforcement is assumed linear elastic and placed symmetrically in the cross-section, but not necessarily in the centre line. The concrete is assumed linear elastic without a tensile strength. The cross-section may be cracked to some extent, determined according to the elastic modelling of the stresses and deformations. The design assumptions for this classic, elastic approach are also listed in the CEN-standard /1/ and must be used in order to account for the influence of the reinforcement.

The ultimate compressive or tensile strain in the reinforcement is larger than in the concrete, which means that a compressive failure in the reinforcement is not possible, since the concrete will be crushed prior to this. The component fail in one of the following four types of failures:

I: Failure due to crushing in uncracked cross-section
II: Failure due to crushing in cracked cross-section
III: Failure due to stability failure in cracked cross-section.
IV: Failure in tension in the reinforcement.

Formulas for this type of model are difficult to establish and voluminous unless one uses iterations for e.g. determining the size of the uncracked zone. The simple elastic model for a reinforced cross-section without tensile strength /22/ have been entered in a spreadsheet program (EXCEL). The iterations have been carried out by the optimization tools in the programtaking the relevant conditions (strength etc.) into account. The test results have in each case been compared to the estimated values for a cross-section with reinforcement and tensile strength either included or neglected.

4.3. EVALUATION OF THEORETICAL FORMULAS

The estimated capacities are compared to test results on the Figure 4.3.1 to 4.3.6. Figure 4.3.1 shows that the classic elastic models predicts up to 50% higher capacity than the Navier-model. The Navier-model gives reasonable and mainly conservative estimates of the load-carrying capacity for all slenderness and eccentricities. The elastic model will lead to quite conservative estimates in the tested specimens. The specimens were, however, quite lightly reinforced and the loads are only slightly eccentric as shown in Figures 4.3.2, 4.3.5 and 4.3.6. This means that the elastic model will only be relevant for more eccentrically loaded components with a higher degree of reinforcement.

The minimum amount of reinforcement in normal weight concrete cross-sections is defined as the amount where a brittle failure in pure bending is avoided, that is where

$$M_c > M_u$$

where

M_u is the bending capacity of the uncracked cross-section
M_c is the bending capacity of the cracked cross-section
f_y is the yield strength in the reinforcement
A_r is the reinforcement area

and

$$M_u = 1/6 \cdot b \cdot h^2 \cdot f_t$$

$$M_c = A_r \cdot f_y \cdot (h_{eff} - (2/3) \cdot A_r \cdot f_y / (f_c \cdot b))$$

The ratio between experimental and estimated capacity is shown as a function of the reinforcement degree and the ratio between the two bending capacities in Figure 4.3.5 and 4.3.6. These figures shows that the Navier-model leads to a too conservative estimate of the load-carrying capacity for more reinforced cross-sections, (which is expected since the model neglects the reinforcement), and than the elastic model will be relevant for those designs..

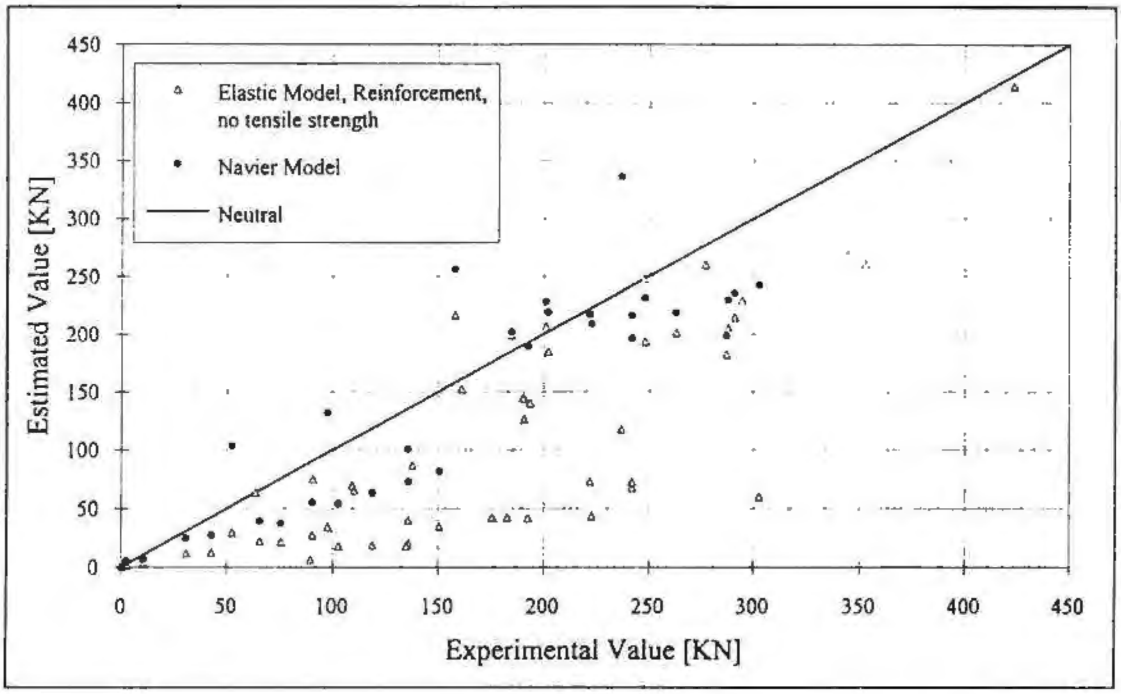

Figure 4.3.1. Estimated load-carrying capacity versus experimental capacity.

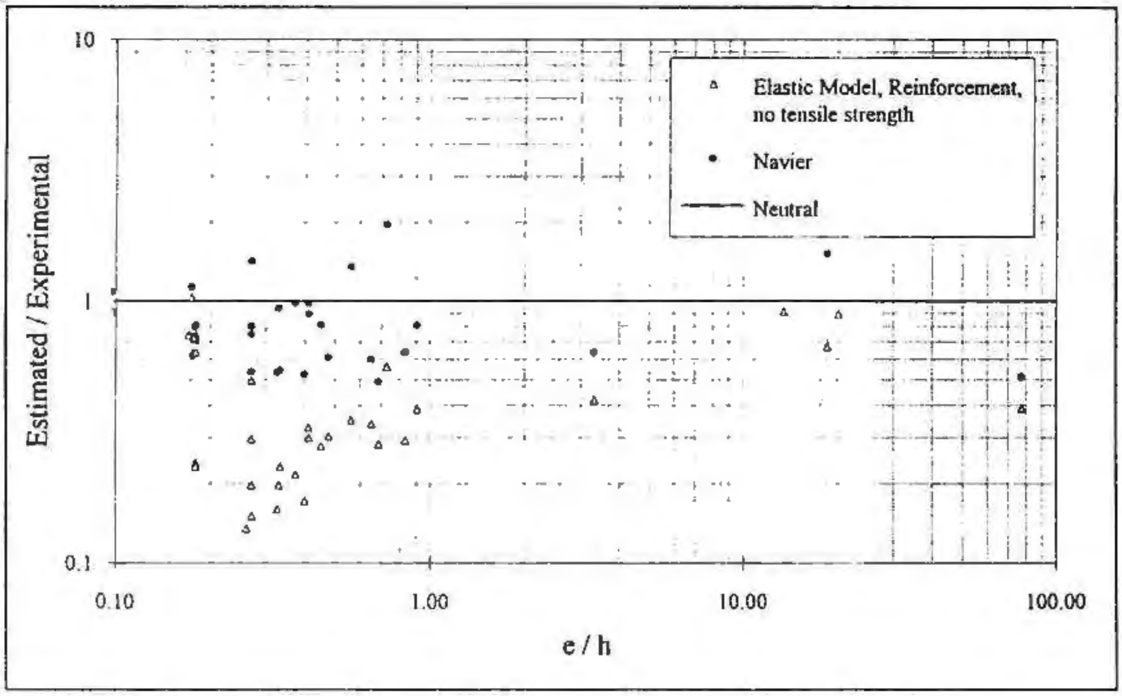

Figure 4.3.2. Ratio between experimental and estimated load-carrying capacity versus dimensionless eccentricity (ε).

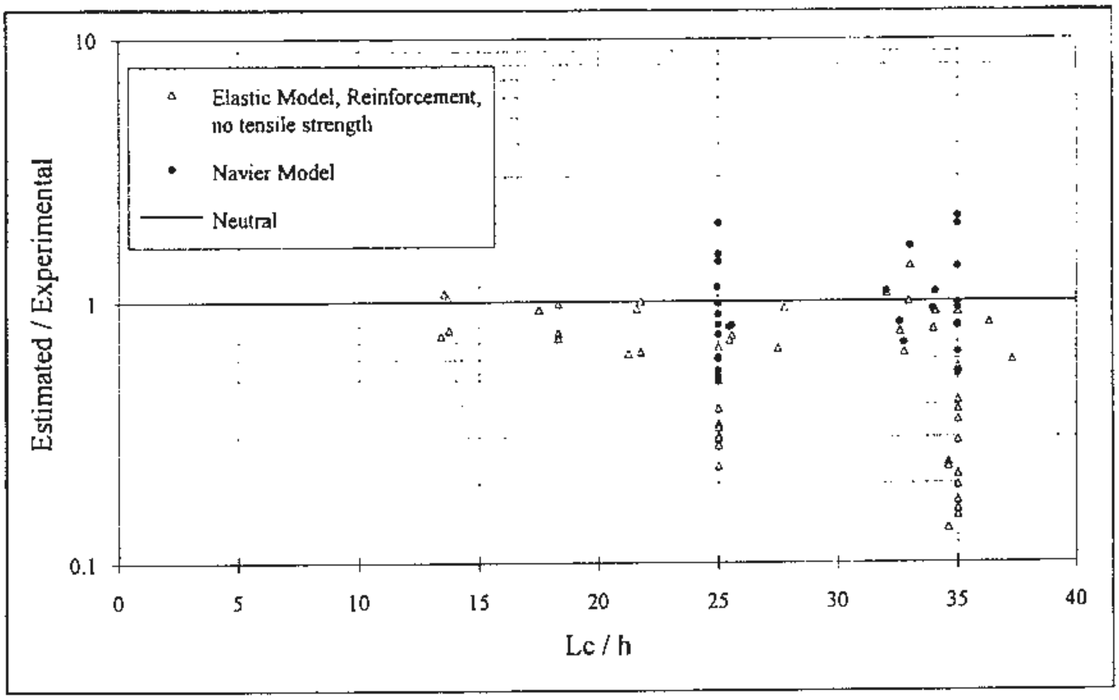

Figure 4.3.3. Ratio between experimental and estimated load-carrying capacity versus slenderness (L_c/h).

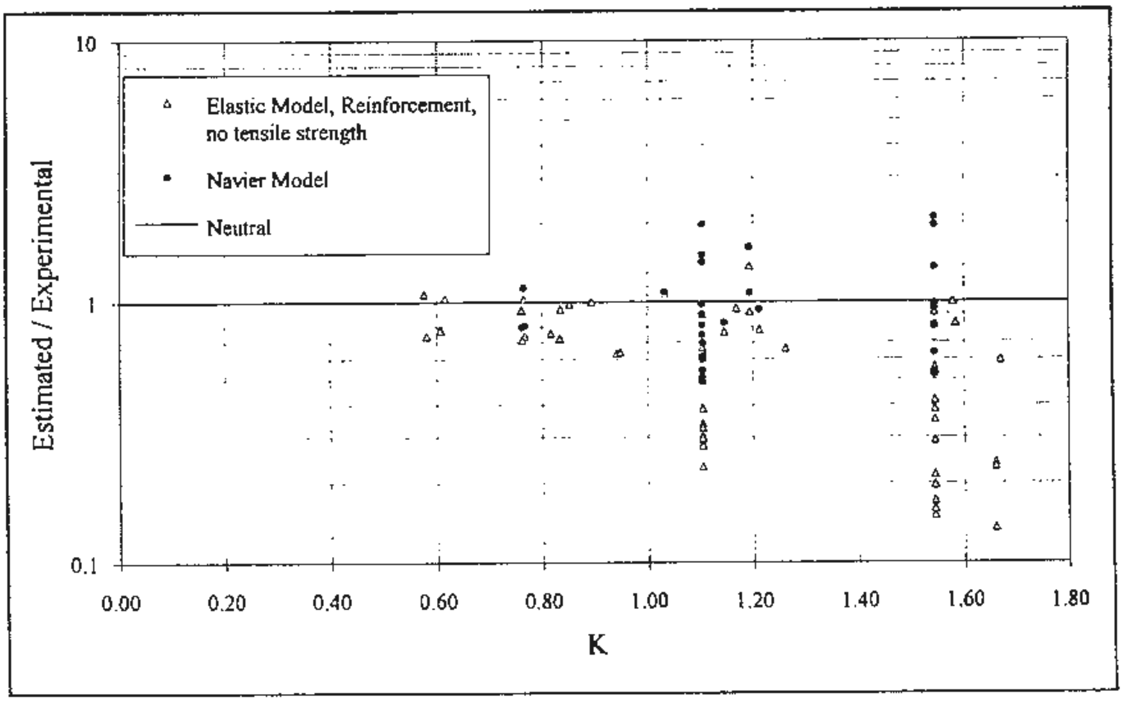

Figure 4.3.4. Ratio between experimental and estimated load-carrying capacity versus slenderness factor K

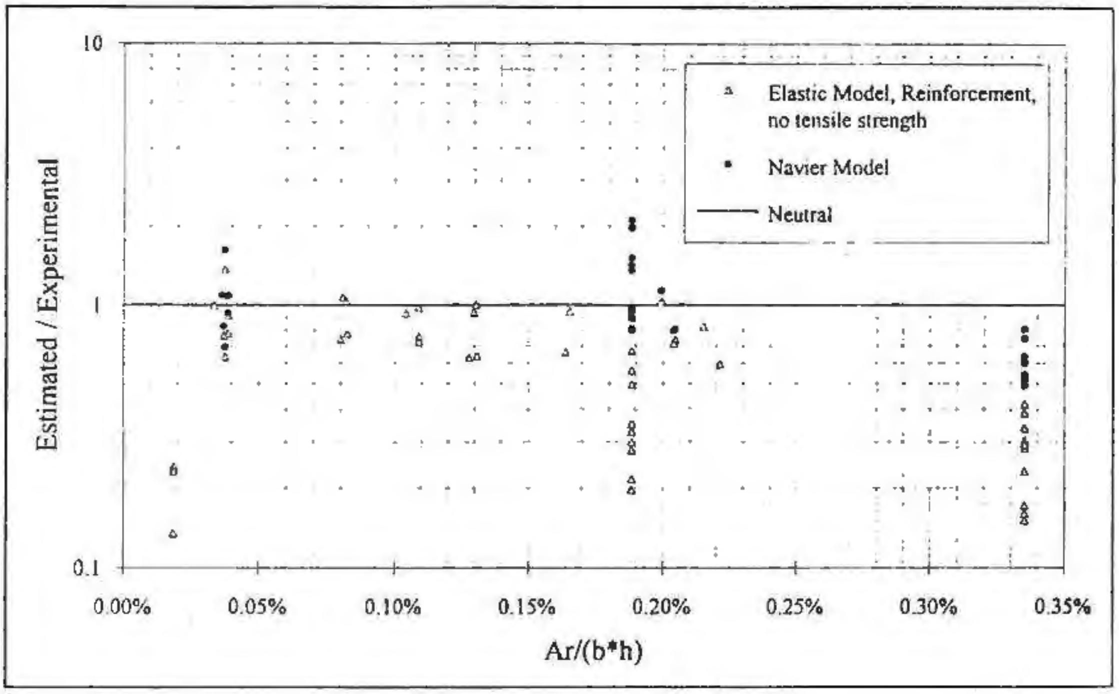

Figure 4.3.5. Ratio between experimental and estimated load-carrying capacity versus reinforcement degree.

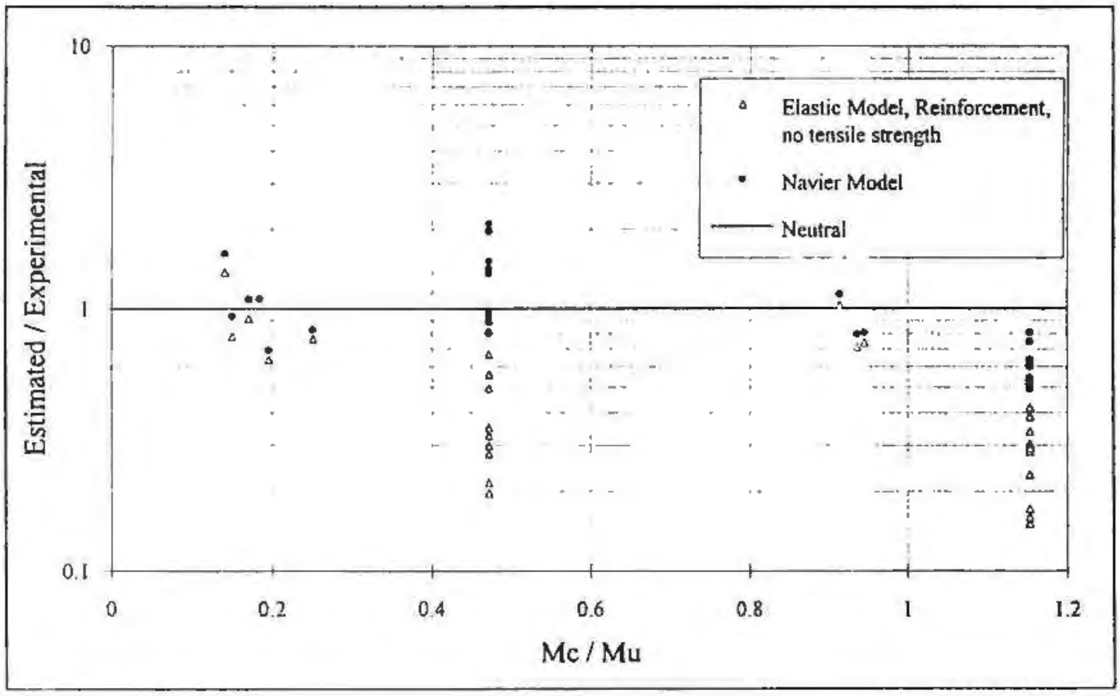

Figure 4.3.6. Ratio between experimental and estimated load-carrying capacity versus ratio of bending capacity of cracked/uncracked cross-section.

4.4. INTERACTION DIAGRAMS

The interaction between the horizontal and the vertical loads are often shown as curves in a diagram, where the curves may have more or less complicated variations. A number of test results /13/ for components of the same LAC can be used to compare the experimental interaction curves to the estimated curves.

	Wall	Column
L /mm/	2500/3500	
b /mm/	1000	300
h /mm/	100	
f_c /MPa/	16.20	
f_t /MPa/	4.5	
E_c /MPa/	8320	
A_r /MPa/	188.50	100.53
h_{eff} /mm/	50	72
f_y /MPa/	500	

Table 4.4.1. Parameters for the tested components.

The vertical load N has an eccentricity of h/4, whereas the horizontal load P is applied at four points, thus creating a moment variation similar to the variation from a uniform load. The load-carrying capacities are estimated with Navier's model using the full cross-section.

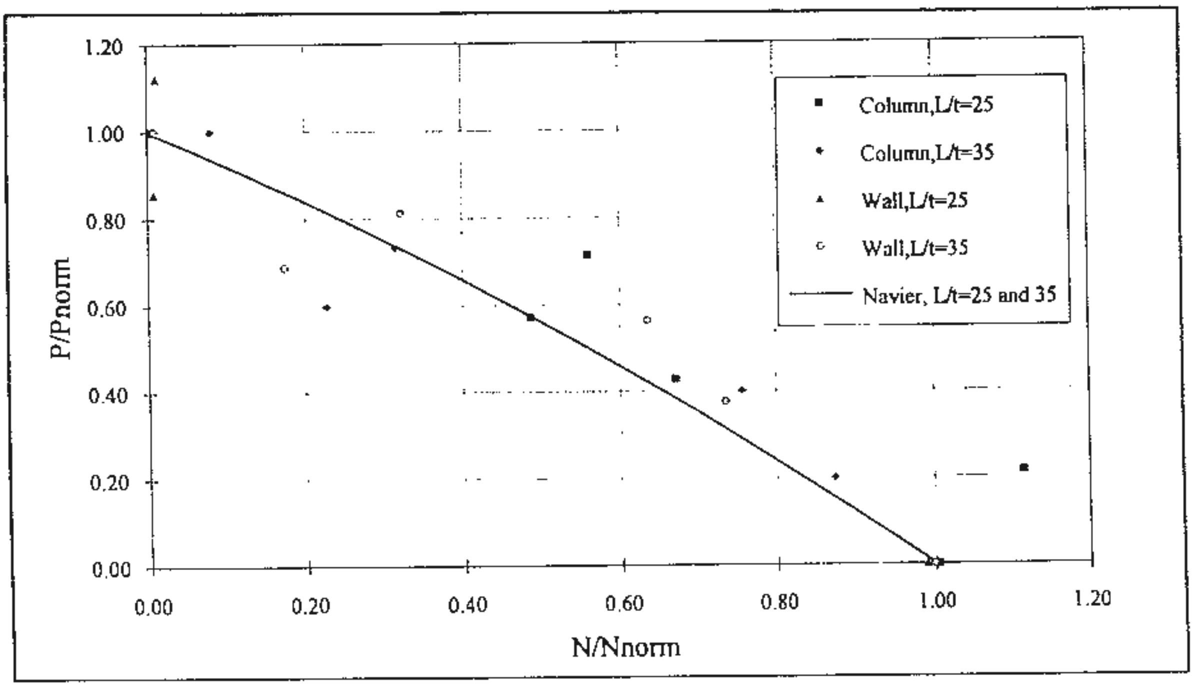

Figure 4.4.1. Interaction curves between N and P.

The interaction curves have been individually normated, which leads to identical interaction curves for the Navier-model for L/h=25 and 35. The curves for the columns and the walls are also identical, since the reinforcement is neglected.

The Figure 4.4.1 shows a good correlation between the estimated interaction curves and the experimental results, although the level of the capacities are different as shown previously.

5. CONCLUSIONS

The paper have verified Ritter's model as a good and conservative model for designing unreinforced walls or columns, subjected to an eccentric vertical load, whereas the pure linear model may lead to less conservative values.

The walls or columns subjected to horizontal loads as well may be designed with Navier's model, which takes the flexural tensile strength into account, but neglect the reinforcement. This model will lead to conservative and reasonable estimations of the load-carrying capacity. The design in practise is based on the eccentricity of the permanent load and will thus reduce the uncracked part of the cross-section further and lead to lower and more conservative values of the load-carrying capacity.

The larger horizontal loads can only be transferred by the linear model, which accounts for the reinforcement, but neglect the flexural tensile strength. The elastic, reinforced model is also conservative, especially for the lightly reinforced or plain components.

The CEN-standard /1/ for LAC-components prescribes the Ritter and the Navier-models for the structures without structural reinforcement and presents the design assumptions for the linear model of the structurally reinforced component. The proposed formulas and assumptions enable thus a reasonably optimal but conservative structural design.

6. NOTATION

A_r	Reinforcement in cross-section.
b	Width of the cross-section
E_c	Modulus of elasticity of the concrete
e_1	First order eccentricity of component
f_c	Compressive strength of concrete
f_t	Flexural tensile strength
f_y	Yield strength.
h	Height/thickness of the cross-section
h_c	Height/thickness of the compression zone
h_{eff}	Effective height of reinforcement
L_c	Column length of component
M_c	Bending capacity of cracked cross-section
M_u	Bending capacity of uncracked cross-section
N	Vertical compressive load on component
N_{cr}	Load-carrying capacity of component
N_{eu}	Critical Euler-load on component
N_{pl}	Plastic load-carrying capacity of cross-section
ε	Dimensionless first-order eccentricity

σ_c Compressive stress
σ_t Tensile stress

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