



## STRENGTH AND STIFFNESS RELATIONS IN LIGHTWEIGHT AGGREGATE CONCRETE WITH OPEN STRUCTURE

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**ABSTRACT:** The paper establish and documents formulas for estimation of bending tensile strength and modulus of elasticity in lightweight aggregate concrete with an open structure (LAC). The extensive test material covers LAC with dry density ( $\rho$ ) in the range of 500-1800kg/m<sup>3</sup> and compressive strength ( $f_c$ ) in the range of 2-30MPa. The paper presents a representative documentation for the LAC-types used in roof or floor components and in wall components and will provide the danish producers and designers with an alternative to the currently obligatory testing.

### 1. INTRODUCTION

Precast components of Lightweight Aggregate Concrete (LAC) with open structure produced as described in /1/ are used as roof or floor slabs or as walls in buildings.

The current danish code /2/ request the producer to document the roof or floor components capacity through full-scale testing as described in /3/, whereas the wall components can be documented through calculations based on material parameters.

The full-scale testing is probably the most correct approach to document the produced components and allows producers to declare fairly high load-carrying capacities. The approach is unfortunately also very time consuming and expensive and the interpretation of the test results quite difficult.

The major danish producers of roof and floor components of LAC (Dansk Leca, Fibo, H+H) therefore initiated a project in 1992 with financial support from the Danish Board of Technology in order to establish and document proper design rules for roof and floor components.

A large test program with a total of several thousand test specimens have been carried out and establish an extensive documentation for e.g. the relationship between modulus of elasticity (E) and the compressive strength ( $f_c$ ) and the dry density ( $\rho$ ) as well as the relationship between bending tensile strength ( $f_{bt}$ ) and  $f_c$  and  $\rho$ .

These relationships have until now not been documented for LAC, nor covered by any standard, code or handbook, except the new product and design standard for precast LAC-components /4/.

This paper will discuss and document formulas for the relationships.

## 2. SAMPLING AND TESTING OF PARAMETERS

### 2.1. ROOF AND FLOOR COMPONENTS

A total of 106 pairs of roof or floor components were produced by Dansk Leca A/S, A/S FIBO and H+H Industry A/S over a period of 1½ year. The components had either monolithic or sandwich (multilayer) cross-sections /1/.

The two components in such a pair were produced immediately after each other, subjected to the same curing and have the same cross-section and width, but not necessarily the same length. The concrete in the two components should thus be as similar as possible.

The primary component in such a pair were subjected to a full-scale functional testing according to /3/, whereas the secondary component were used for determination of the relevant material properties which may be necessary for the evaluation of the full-scale tests. These parameters are dry density ( $\rho$ ), compressive strength ( $f_c$ ), bending tensile strength ( $f_{bt}$ ) and modulus of elasticity ( $E$ ) and will be used to document the formulas for  $f_{bt}$  and  $E$ . The information and the results of the full-scale testing will be presented in other papers.

The test specimens were cut as sets of specimens, where each set contains the specimens which are necessary for the determination of  $f_c$ ,  $f_{bt}$  and  $E$ . The specimens in a set are cut at the same position in the secondary component, in order to avoid too large variations of e.g. dry density between the specimens in a set. Three sets were cut at different positions from each layer in each components and will represent an individual set of data in this paper. The size of the test specimens are indicated in Table 1.

Cross-section	Compressive strength	Bending tensile strength	Modulus of elasticity
Sandwich, Top layer	2 Prisms 20*100*100mm	None	2 Prism 20*100*200mm
Sandwich, Center layer	Cylinder Ø100*100mm	Prism 100*100*400mm	Cylinder Ø100mm
Sandwich, Bottom layer	2 Prisms 20*100*100mm	None	2 Prism 20*100*200mm
Monolithic	Cylinder Ø100*100mm	Prism 100*100*400mm	Cylinder Ø100mm

Table 1. Dimensions of test specimens.

The actual testing were carried out according to the danish test standards DS434.3/5/, DS434.4 /6/ and DS434.5 /7/. These methods are similar to the test standards /8/, /9/, /10/ from CEN/TC177, which are going to replace the current national standards in a couple of years.

The standard cylinder used in tests is a Ø100 \* 200mm-cylinder. The test specimens from the

top and bottom layer in the sandwich cross-section are thin prisms, since the layers are too thin to cut standard cylinders from. The prisms will be tested for  $f_c$  and  $E$  as a pair as indicated in Figure 1. The cylinders from the center layer and the monolithic cross-section will be  $\text{Ø}100\text{mm}$  but usually only 100mm high.

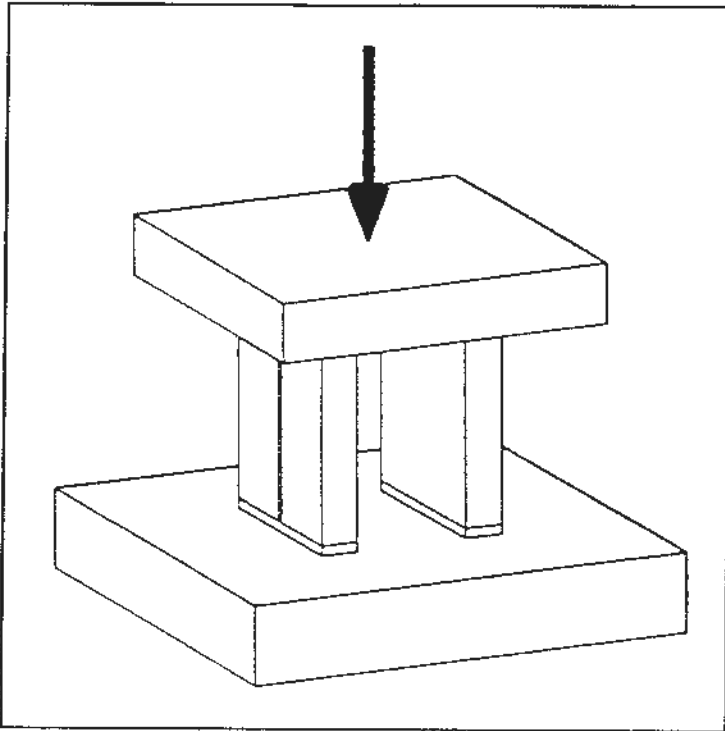


Figure 1. Testing arrangement of a pair of prisms.

Earlier investigations /11/, /12/ have shown that the compressive strengths of the short cylinder and the prisms are the same as in the standard cylinder. This means that the size-effect can be neglected in this paper.

## 2.2. WALL COMPONENTS

A number of test results on  $f_c$ ,  $f_{bt}$ ,  $E$  and  $\rho$  measured on standard test specimens have been obtained from producers of wall components /13/.

The specimens for compressive strength, modulus of elasticity and bending tensile strength are prisms (100\*100\*400mm). Three specimens for the determination of each parameter are cut from different positions in the component. The average values for a wall component are used as one set of data since the position of the samples may differ from one component to another.

## 3. MODULUS OF ELASTICITY

### 3.1. CALCULATION RULES

The modulus of elasticity were measured according to DS434.5 /7/ and corresponds to a chord value at a loading from 0.1 to 0.5 times the expected compressive strength. A similar modulus of elasticity is used in CEB /14/, in the CEN-standards for lightweight aggregate concrete with

closed structure (LACC) /15/, the CEN-standard for precast components of LAC /4/ and in ACI's concrete manual for LACC /16/, whose formulas will be considered in the following.

The formulas considered take  $f_c$  and  $\rho$  into account as:

$$E = K \cdot f_c^\alpha \cdot \rho^\beta \quad (1)$$

where  $f_c$  is in MPa and  $\rho$  is in  $\text{kg/m}^3$ . ACI's formula /16/ is:

$$E = 0.05 \cdot f_c^{0.5} \cdot \rho^{1.5} \text{MPa} \quad (2)$$

where the danish recommendations for LACC-structures, NP146R /17/ prescribe:

$$E = 0.046 \cdot f_c^{0.5} \cdot \rho^{1.5} \text{MPa} \quad (3)$$

which is 92% of the ACI's formula (2). The CEB /14/ uses a formula with other factors. The CEB-formula is also adopted in the CEN-standard for LACC-structures /14/ and in the CEN/TC177 draft standard for precast LAC-components /4/:

$$E = 9.5 \cdot f_c^{0.33} \cdot (\rho/2200)^2 \text{GPa} \quad (4)$$

### 3.2. VERIFICATION OF FORMULA

The experimental values of  $f_c$  and  $E$  varies within each roof or floor component due to variations in  $\rho$ . Taking this variation into account, it can be shown that  $f_c$  in the  $f_c$ -specimens corresponds well to the strength in the  $E$ -specimens, although the loading procedures is slightly different. The  $f_c$  and  $\rho$  values measured on the  $E$ -specimens will be used in the comparisons in this chapter, so inaccuracies can be kept as low as possible. The experimental results from the roof and floor components /1/,/19/ will therefore be used as individual values, whereas the results from the wall components /13/ represents average values from a component.

The symbols on Figure 2 and following figures represents:

- Square: Values from monolithic cross-sections
- Diamond: Values from top or bottom layer in sandwich cross-section.
- Triangle: Values from center layer in sandwich cross-section
- Circle: Values from wall components.

Each producer is represented with a specific shading of the marker, in order to indicate any systematic difference, which may be found.

The curves on Figure 2 shows at which  $(f_c, \rho)$ -combinations the ratio between CEN's estimate (4) and ACI's estimate (2) of  $E$  has a constant value. The Figure 2 shows that the two formulas leads to estimates, which deviates less than 10% in the range where test results have been obtained, except for dry densities below  $1000 \text{kg/m}^3$ .

The CEN-formula will in the following be compared to the experimental values. A comparison between the other formulas (2),(3) and the test results would lead to approximately the same estimate and will therefore not be presented in the following.

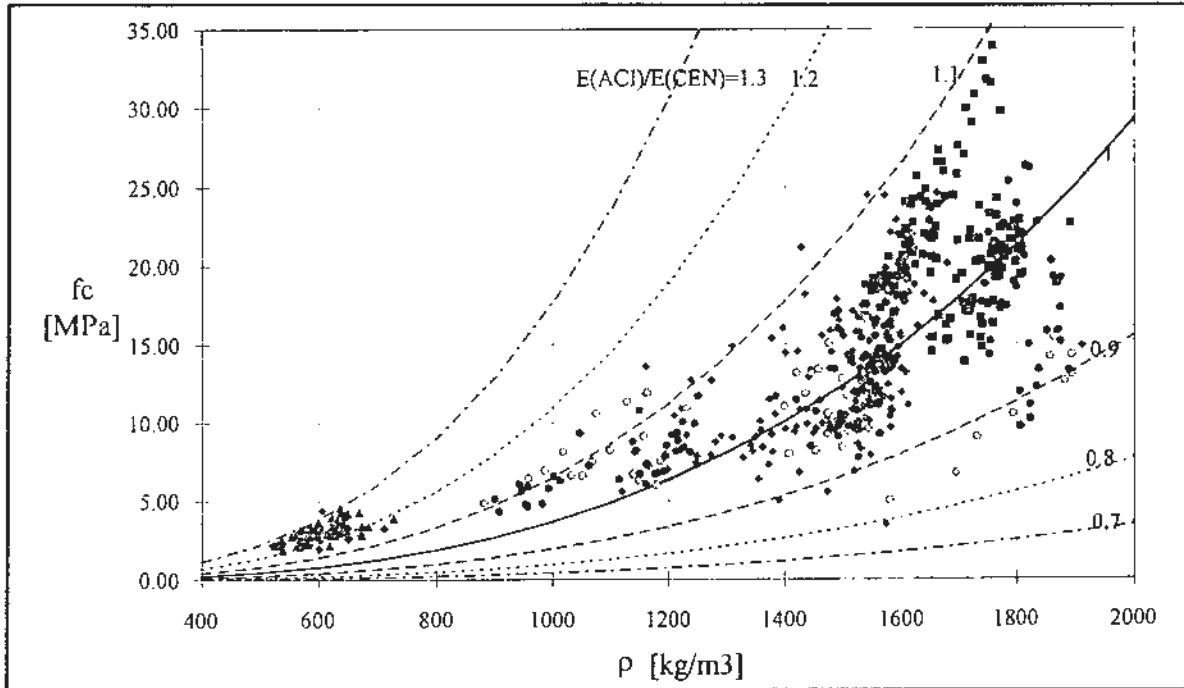


Figure 2. Compressive strength ( $f_c$ ) versus dry density in the test specimens. Ratios CEN/ACI-estimates are shown.

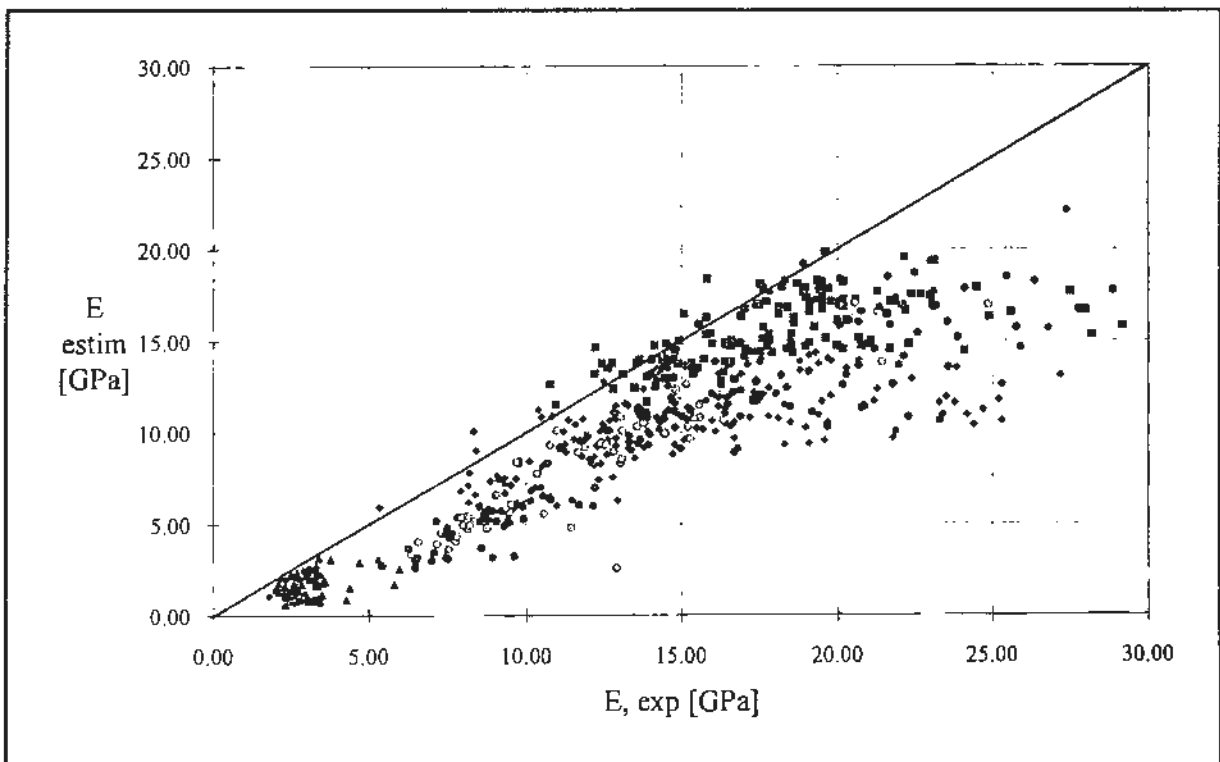


Figure 3. Modulus of elasticity  $E$ . CEN-estimations versus experimental values.

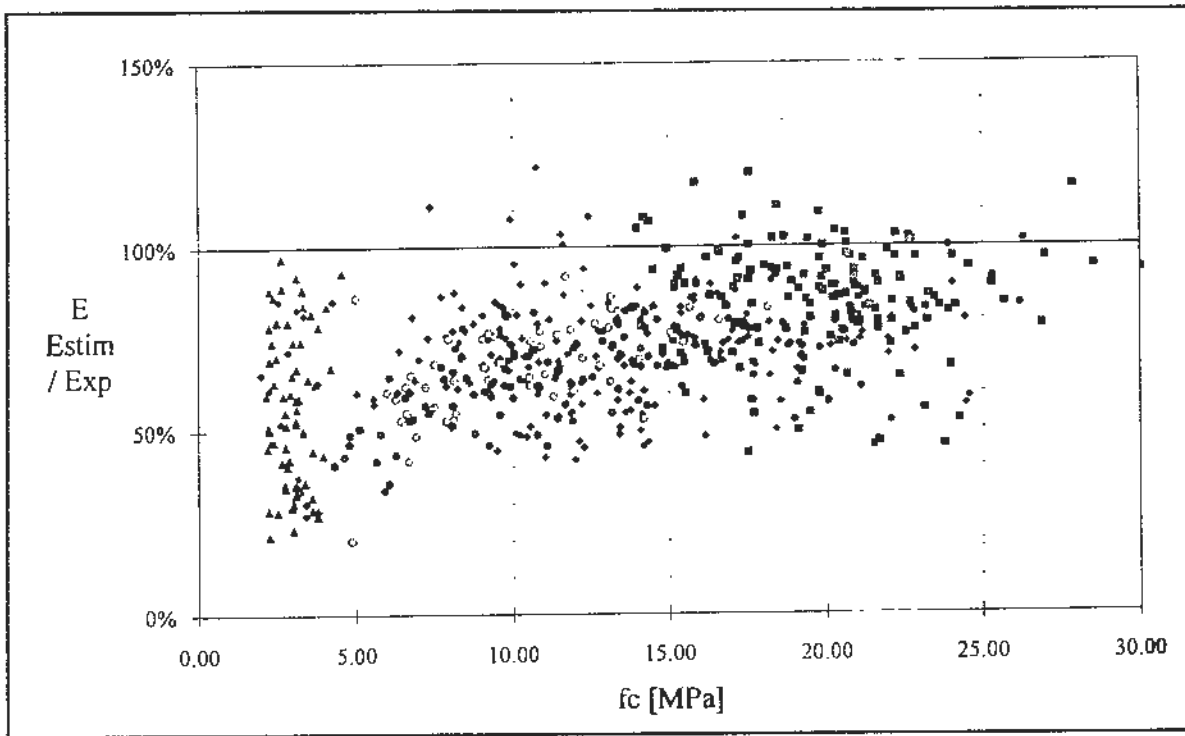


Figure 4. Modulus of elasticity  $E$ . Ratio CEN-estimate / experiments versus compressive strength  $f_c$ .

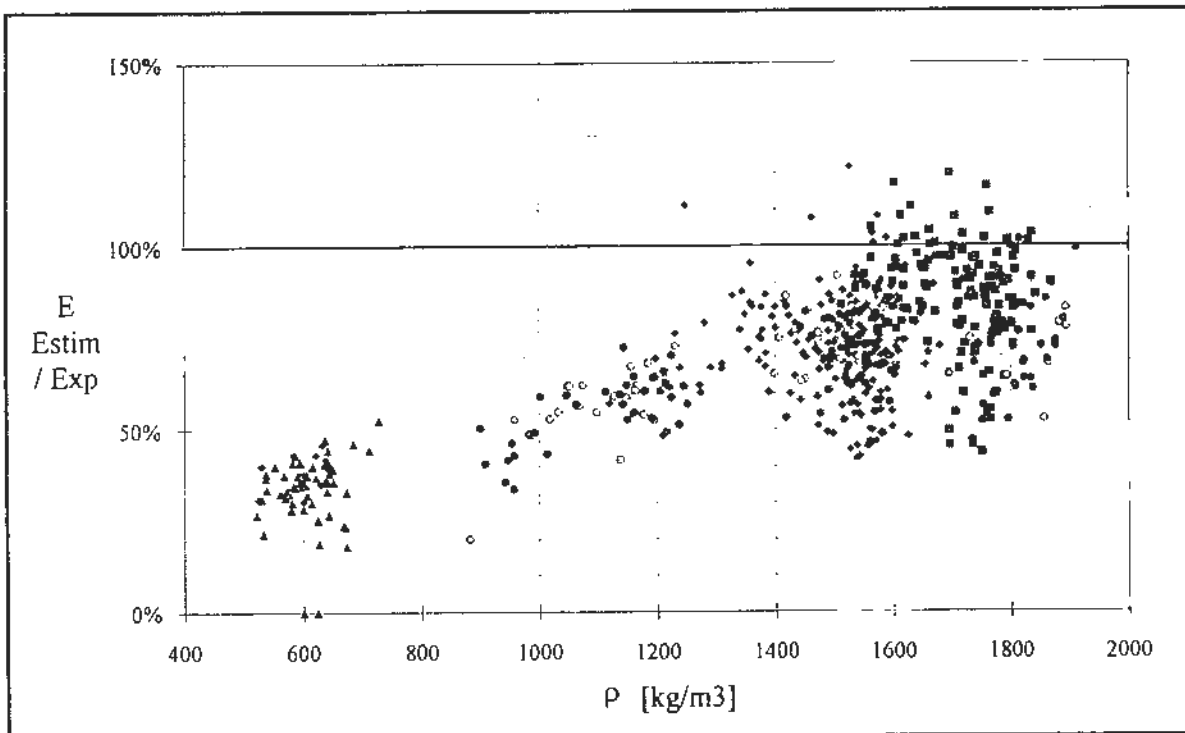


Figure 5. Modulus of elasticity  $E$ . Ratio CEN-estimate / experiments versus dry density  $\rho$ .

Figure 3 shows that the formula leads to an acceptable, but somewhat conservative estimate of  $E$ . The conservatism is better illustrated by the ratios between estimate and experimental value shown in Figure 4 and 5 as functions of  $f_c$  and  $\rho$ .

The Figures 4 and 5 shows that the formula is equally conservative at all  $f_c$ -values and at all  $\rho$ -levels above  $\text{kg/m}^3$ . The formula is quite conservative at lower densities, where it may underestimate  $E$  with up to 50%. The Figures indicate that the formulas from NP146R (3) and ACI (2) as well would be quite conservative for the lowest densities.

## 4. BENDING TENSILE STRENGTH

### 4.1. CALCULATION RULES

Design standards usually defines the tensile strength as the uniaxial tensile strength  $f_t$  and presents formulas for  $f_t$  based on  $f_c$  and  $\rho$ , although the most commonly tested tensile strength is the flexural tensile strength  $f_{bt}$ . The  $f_{bt}$ -test results are then multiplied with a conversion factor 0.5 in order to obtain the uniaxial tensile strength.

The CEN/TC177-standard for precast LAC-components uses  $f_{bt}$  in the formulas for load-carrying capacities and presents a formula based on /15/ and /20/:

$$f_{bt} = 0.42 \cdot f_c^{2/3} \cdot (0.4 + 0.6 \cdot \rho / 2200) \text{ MPa} \quad (5)$$

where  $f_c$  is in MPa and  $\rho$  in  $\text{kg/m}^3$ .

### 4.2. VERIFICATION OF FORMULA

The set of test specimens from the roof and floor components contains also specimens for the bending tensile strength  $f_{bt}$ , except the sets from the thin top and bottom layers. A number of averaged test results from wall components /13/ have also been obtained.

The dry densities in the  $f_{bt}$ -specimen ( $\rho_{bt}$ ) and in the  $f_c$ -specimen ( $\rho_{fc}$ ) in a set taken from the monolithic cross-sections differs with 50 to  $100 \text{ kg/m}^3$  /1/. The  $f_c$ -value used in the estimations for those sets are therefore corrected as:

$$f_c^* = f_c + K_c \cdot (\rho_{bt} - \rho_{fc}) \quad (6)$$

where the data on Figure 2 shows that

$$K_c = 0.080 \text{ MPa} / (\text{kg/m}^3) \quad (7)$$

The formula from CEN (5) is used for the estimations in the comparison with the experimental values as indicated on Figure 7.

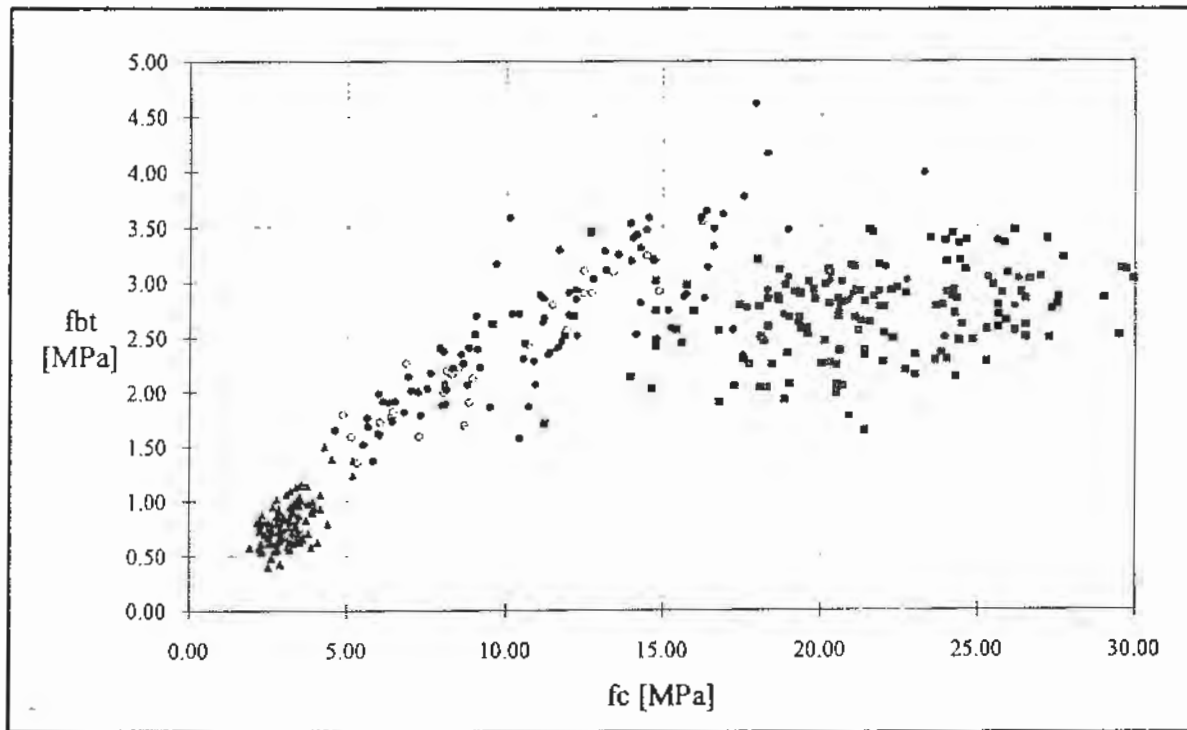


Figure 6. Bending tensile strength  $f_{bt}$  versus compressive strength  $f_c$  in a set of specimens.

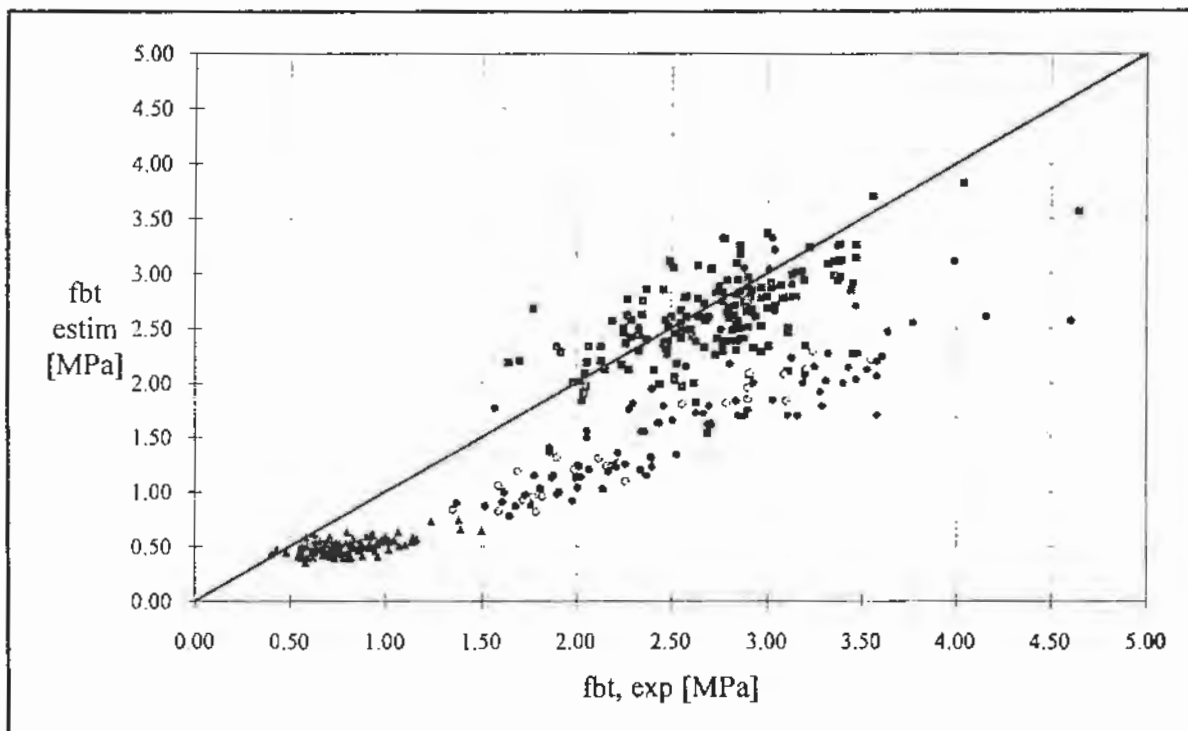


Figure 7. Bending tensile strength  $f_{bt}$ , CEN-estimates versus experimental values.



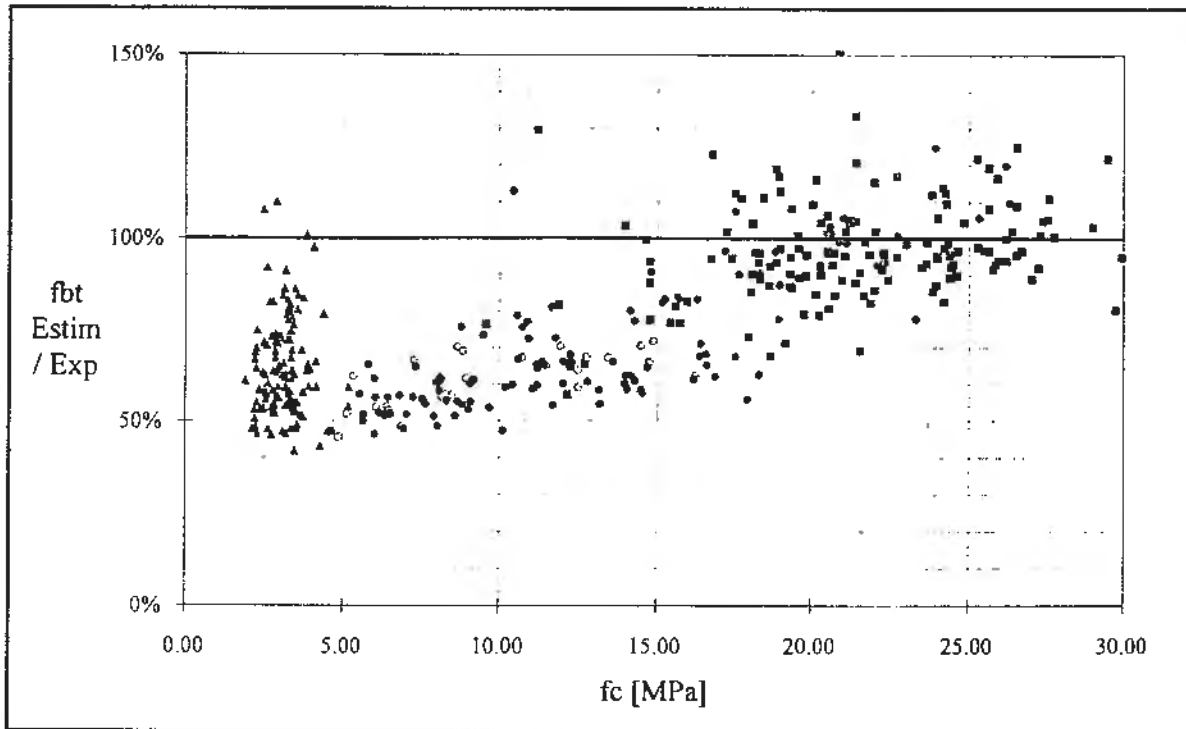


Figure 8. Bending tensile strength  $f_{bt}$ . Ratio CEN-estimation / experiments versus compressive strength  $f_c$ .

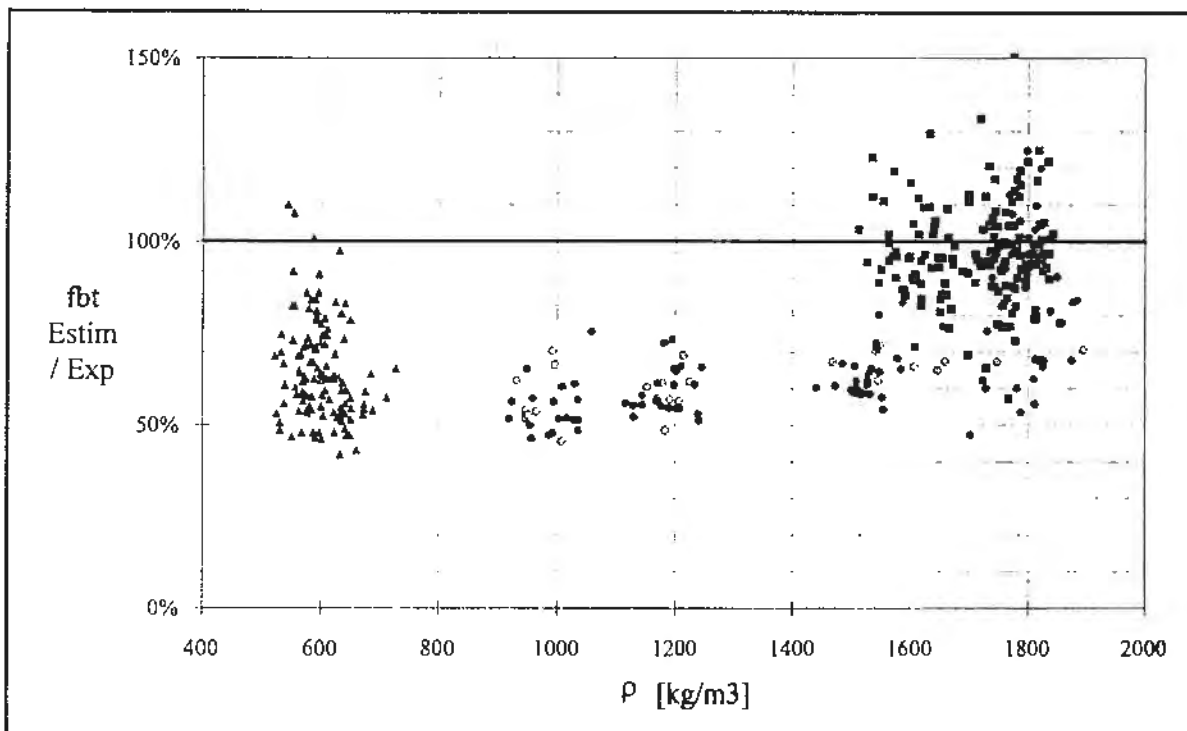


Figure 9. Bending tensile strength  $f_{bt}$ . Ratio CEN-estimation / experiments versus dry density  $\rho$ .

Figure 7 shows that the formula (5) leads to an acceptable estimate of  $f_{bt}$ . The estimation is illustrated by the ratios between estimate and experimental value shown in Figures 8 and 9 as functions of  $f_c$  and  $\rho$ .

Figure 7 to 9 shows that the formula (5) gives an acceptable estimate of  $f_{bt}$ , which is quite conservative for some LAC-types.

## 5. CONCLUSIONS

The estimation of modulus of elasticity  $E$  with the formula from CEN (4) is conservative and does for most LAC-types give similar values as the formula from ACI (2). The estimation is very conservative for the densities below  $800\text{kg/m}^3$ .

The estimation of bending tensile strength  $f_{bt}$  with the formula from CEN (5) is conservative, especially for the lower densities.

The conservativeness at the low densities is acceptable since these LAC-types can have a rather open structure and thus enable quite different productions of LAC with a resulting large possible spectre of the  $f_c$ ,  $f_{bt}$ ,  $E$  and  $\rho$ -parameters.

## 6. ACKNOWLEDGEMENT

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## 7. NOTATION

$E$	Modulus of elasticity
$f_{bt}$	Bending tensile strength
$f_c$	Compressive strength
$f_t$	Tensile strength
$\rho$	Dry density

### Indices

$E$	Value i E-specimen
est	Estimated value
exp	Experimental value
fc	Value in $f_c$ -specimen
fbt	Value in $f_{bt}$ -specimen
k	characteristic value
m	Mean value

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