AIR PERMEABILITY OF CONCRETE CONTAINING FLY ASH

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ABSTRACT

Air permeability of concrete samples (28 day compressive strength 25 MPa to 55 MPa) containing different amount of a fly ash (Class F) was tested. The results are compared with the moisture diffusivity estimated according to data obtained in the drying process. The concrete samples studied were sealed from moisture loss for 28 days after casting, then stored in 50% RH, 20°C. A series of samples were further dried at 60°C to constant weight, and the air permeability was studied.

It is demonstrated here that the concrete under a higher pressure difference (over the range of 0.01 to 0.30 MPa) exhibits a lower air permeability. This is ascribed to the influence of concrete pore size on gas flow.

Coefficient of air permeability (*K*) of 50% RH conditioned concretes varied from 1×10^{-17} m^2 to 2×10^{-16} m^2 (tested under gradient 0.10 MPa/5 cm), decreased with the decrease in the water to cement ratio of concrete, and related to the strength. It is shown that the low to medium replacement of cement by the fly ash, i.e. 20%, leads to a decrease in the air permeability. The presence of excess alkali in fly ash concrete leads to a lower permeability mainly due to its effect on the enhancement of pozzolanic reaction of fly ash. Drying at 60°C resulted in two to eight-fold increase in *K*.

Key words: Air permeability, Alkali, Fly ash, Moisture, Strength, Vapour diffusivity.

1. INTRODUCTION

It has been well recognized that the permeability of concrete is one of the major characteristics of concern with the concrete durability, i.e. the resistance to deterioration due to hazard gasses and liquid migration. The permeability or diffusivity of concrete is a multi-factor governed property, in term of the material properties, it depends on the amount, openness and tortuosity of capillaries and other microstructural and chemical characters of concrete. Different methods, such as water absorption and vapour permeability tests were traditionally carried out to evaluate these characters.

In recent years, gas permeability measurement has increasingly been used for evaluation of concrete properties /1, 2, 3, 4/, because its result directly indicate the permeance of concrete and it is relatively quick and easy to carry on. However, it has been well known that the presence of moisture influences significantly the results /5/, and effort was made to eliminate this effect by drying at high temperature, which gives only the results which may seldom be occur in natural conditions.

The air permeability of fly ash concrete which generally has a higher rate of carbonation, has been intensively however not without controversy reported. Nagataki and Ujike /1/ showed concrete with fly ash cured in water for 28 days, addition of fly ash (at the fly ash to binder ratio F/(C+F) upto 20%) reduced the air permeability and the effect increased with longer water curing time, however the effectiveness significantly reduced with the specimen drying time. Dhir and Byars /6/ used a vacuum-decay method to test an air permeability index which showed to be increasing for the concrete with fly ash (F/(C+F)=0.15 and 0.30) compared with normal concrete on the equal water to binder basis. Recently, Costa *et al.* /2/ reported the air permeability of concretes dried at 60°C, the results showed fly ash cement concrete had a lower permeability compared with normal portland cement concrete. Apparently, the degree of hydration of cementitious materials and moisture content in the tested specimens are the main influencing factors for the discrepancies in the reported air permeability.

In the present study, gas permeability is used to evaluate the effect fly ash on concrete. The results of air permeability of concrete conditioned at 50% RH at room temperature and after drying at 60°C are reported. It is a part of an investigation on the influence of alkali on the carbonation of concrete with fly ash. Therefore the influence of water to cement ratio, fly ash content and alkali content on the air permeability are also studied.

In addition, the concrete weight loss during drying is used for evaluating the vapour diffusivity to give supplementary information on the effect of fly ash.

2. EXPERIMENTAL DETAILS

Twelve cubes (100 × 100 × 100 mm) and one cylinder (150 mm in diameter and ca. 300 mm in height) for each concrete were cast. The concrete proportioning varied as the water to binder (cement and fly ash) ratio = 0.35, 0.45, 0.55 and 0.65; the fly ash content = 0, 20%, 40% and 60% by weight of binder; and cement : sand : stone = 1 : 1.23 ~ 1.64 (adjust to keep the volume fraction of water = 185 kg/m^3 constant) : 2.10 (8-16 mm crushed granite, 1110 kg/m^3) by weight; potassium hydroxide (KOH) content = 0, 1%, 2% and 3% by weight of cement. The details of the mix proportions is shown in Table 1, Appendix I.

The concrete cubes were cured in water at 20°C until 28 days, and the compressive strength at 1, and 28 days were determined. Test results reported as the average of three specimens for each concrete are shown in Table 1. The other cubes were then stored in 50% RH at 20°C for other tests, and the weight loss during drying was recorded.

Because the moisture in concrete partly or totally blocks the air flow path, and the drying of concrete is usually very slow at a normal condition, the concrete used for air permeability measurement were not cured in water. Instead, they were sealed by plastic film after demoulding, and by epoxy at about 3-week-old to ensure the proper hydration and a not-too-high moisture content. This also eliminated the might-be uneven moisture distribution in the test specimens' cross section.

The cylinders of 4-week-old were then sawn to discs of 5 cm thick. Water was used for the cooling of the saw blade, therefore a small amount of water was absorbed by the concrete during sawing. However, it was seen by breaking up some specimens after sawing, the wetted layer was very thin. This was due to the water absorption of the concrete was already very low at the age of 28 days, and the time of sawing was short (about 10 min). The sawn specimens were blown by pressed air to get rid of surface water, and then stored in 50% RH at 20°C for drying.

During the period of drying, the air permeability of the concretes was determined twice or three times. The total air conditioning time was about 9 months, when the weight change indicated the total relative moisture loss was more than 90% and the air permeability tested in the interval of 2 months changed very little. To evaluate the influence of moisture content on the air permeability, one specimen from each concrete was further dried at 60 ± 3 °C for about one to two months, until the weight loss at an interval of one week was less than 0.05%; and the air permeability of was determined again. The drying at this temperature was reported not to induce extra drying cracks observable by microscope /2/.

The concrete air permeability was tested on a permeability cell (Fig. 1). The inlet air was regulated to give a constant pressure; and the exit air flow measured by a bubble flow-meter. The air permeability coefficient is calculated by /1, 3/

$$K = \frac{2\,\eta\, q_V L\, P_O}{P_1^2 - P_2^2} \quad (m^2) \qquad \textbf{(1)}$$

where η is the air viscosity ($\eta_{20°C} = 1.808 \times 10^{-5}$ Pa·s), q_V is the measured flow rate ($m^3 \cdot s^{-1} \cdot m^{-2}$), L is the specimen thickness (m), and P_1 and P_2 are the absolute pressures (Pa) at the inlet and outlet surface of the specimen, and here $P_O = P_2$.

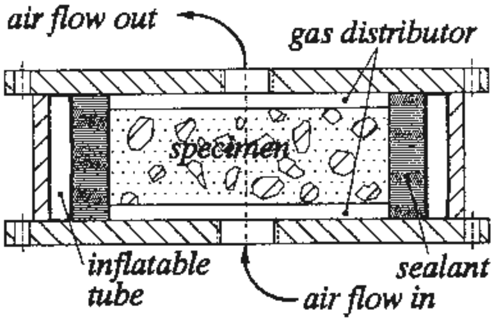

Figure 1 Air permeability cell.

3. RESULTS AND DISCUSSION

3.1 Air permeability of concrete

It was observed that the air flow tested at lower pressure gradient is relatively higher which is a general pattern for all the concrete specimens tested. A typical example is shown in Figure 2, the air permeability (*K*) determined at 0.2 MPa/m is about 50% higher than that at 2.0 MPa/m (1 bar / 0.05 m). As is also shown in Figure 2, the difference between the values is less at higher pressures, and slightly reduces with drying time.

It may be pointed out that Eq.1 which is used for calculating *K* is the same as Poiseuille's formula for gas flow /7/. Kärger and Ruthven /8/ stated that the gas flow in a solid whose pore size varies from micro to macro level is governed by different mechanisms, e.g. Knudsen flow, molecular diffusion, Poiseuille flow, etc. The Poiseuille's flow dominates mainly in larger pores ($r > 1\ \mu m$) or at a higher pressure. In the present results, the deviation from Poiseuille's mechanism, expressed by the variation of *K* at different pressure gradient is obvious. It can be

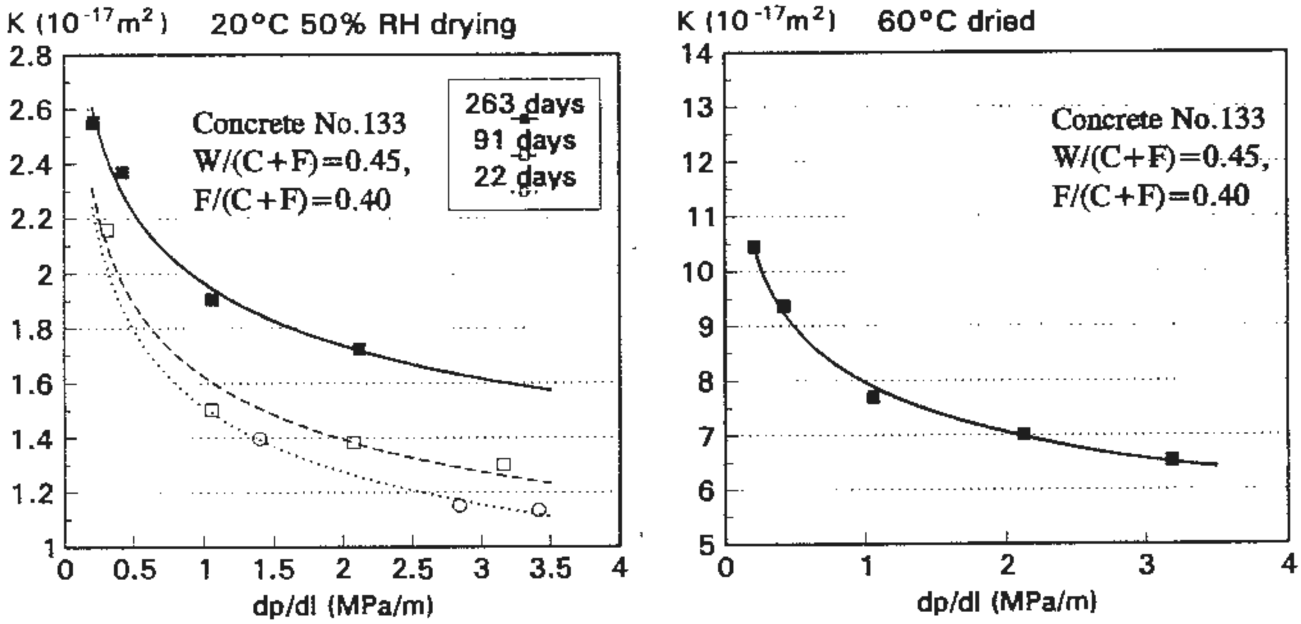

Figure 2 Air permeability varies with pressure gradient.

explained by the collision between air molecules and the wall of pores whose size is comparable to or smaller than the mean free path of air molecules (670 Å at 20°C). Åhlén /9/ showed sandstones tested at higher ΔP (P_{avg}=3-4 bar) had a lower *K*, which was explained by that the Reynolds number changed under high pressure. The results on concrete air permeability reported by Häkkinen /10/ also indicate the higher the pressure (ΔP= 1-10 bar) the lower the *K*. However, such a trend was not noted in some studies /1/, more research work is needed to clarify this. To avoid confusion, the air permeability determined (or interpolated by smoothing the *K* vs dp/dl curve) at dp/dl=2.0 MPa/m are used for further analysis.

Because the concrete was not saturated with water even at the very beginning, *K* determined after 3 weeks drying at 50% RH was already about 10^{-17} to 10^{-16} m^2. After drying the concretes to about 9 months when the weight change rate became considerably smaller, the determined *K* increased to some 20% higher (Figure 3). This increase was not significant and

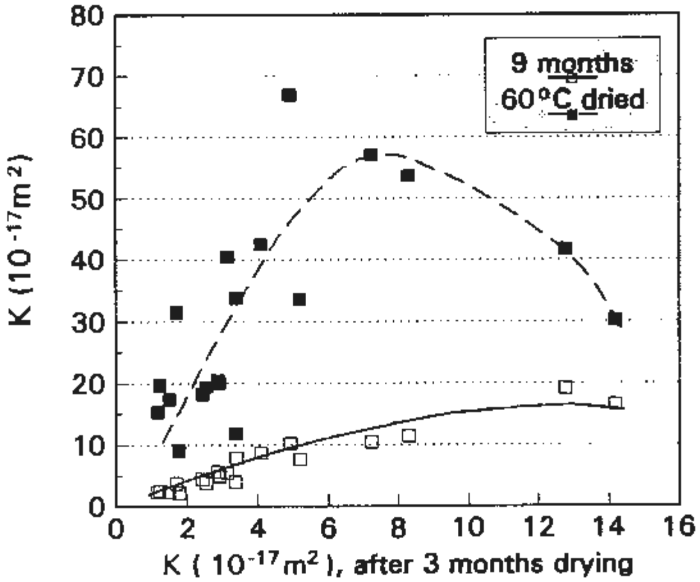

Figure 3 Air permeability of concrete increases with drying.

further increase in *K* at this drying condition would take much longer time, thus the results at 9 months were considered to be representative to the air permeability of the concretes dried at 50% RH, 20°C. Further, the *K* values determined after drying the concretes at 60°C to constant weight increased about 10-fold to 1 - 6 × 10^{-16} m^2/s (Figure 3) which is somewhat higher but in the same order of magnitude reported by Costa *et al* for 180-day water cured concrete of the same designed strength /2/.

The influence of concrete composition on *K* is shown in Figure 4 and 4 based on the analysis of variance. The trend of *K* (of both 50% RH and 60°C dried concrete) increase with water to binder ratio is in full agreement with the results obtained by Costa *et al* /2/, and Dhir and Byars /6/. The effect of fly ash, however, depends both on the content of fly ash, and on the content of alkali in the binder, the latter in fact may accounts for its acceleration effect on the dissociation of fly ash glass phase so as to enhance the pozzolanic reaction fly ash /11/.

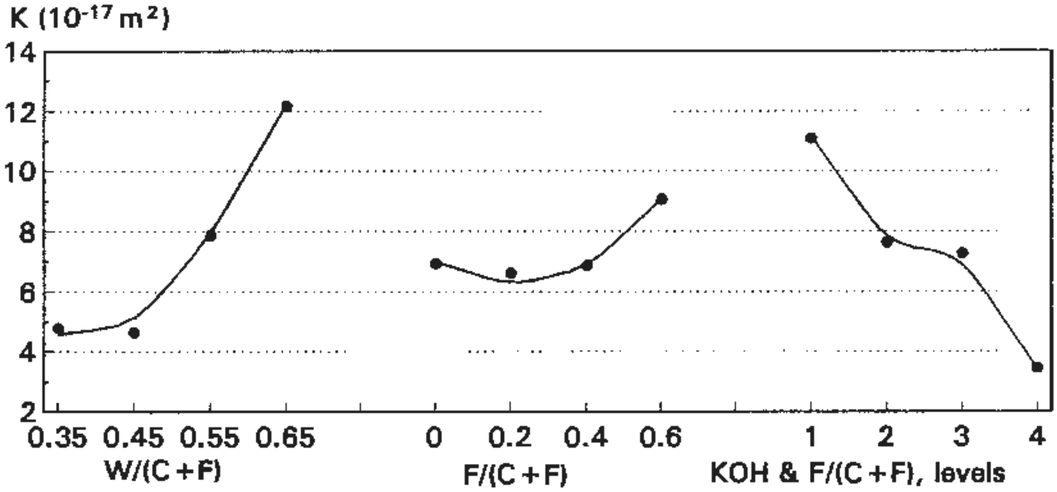

Figure 4 Factor analysis of *K* (9 months drying at 50% RH 20°C). The levels of KOH & F/(C+F) are 0&0, 0.05&0.20, 0.10&0.40 and 0.15&0.60 for 1, 2, 3 and 4 respectively.

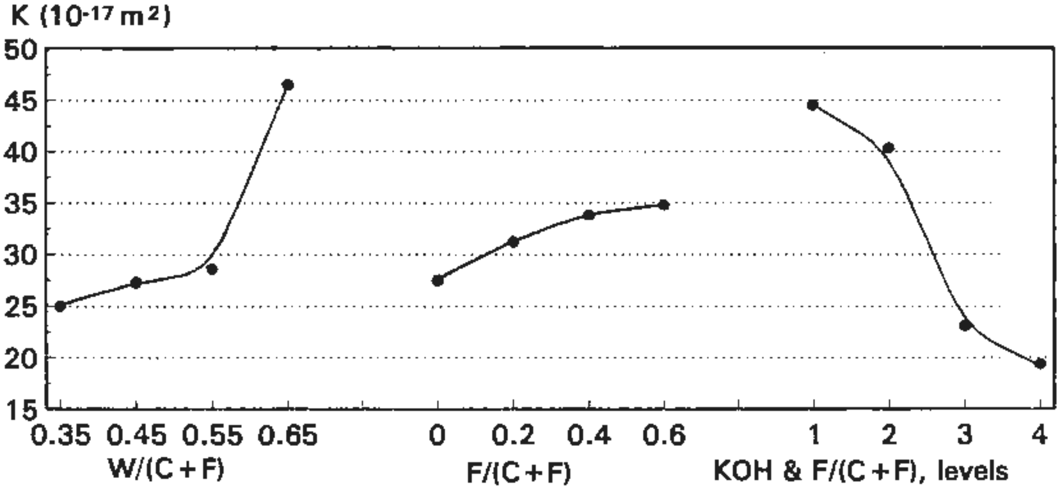

Figure 5 Factor analysis of *K* (60°C dried specimens); levels of KOH & F/(C+F) are the same as in Figure 4.

It appears that the partial replacement cement with fly ash, when F/(C+F) is low (between 0.20-0.40), results in some reduction in *K* of the concrete conditioned at 50% RH, which was also seen in the results of Nagataki *et al* /1/ and Dhir and Byars /6/. However, the 60°C dried concretes all showed an increase in *K* with the F/(C+F) ratio. On the other hand, once again a strong "two-factor-interact-ion" occurred between the F/(C+F) and alkali content, as is shown in both Figure 4 and Figure 5, i.e. the high fly ash content accompanied with high alkali content leads to a linear decrease in *K*. This is to be compared with the results of Costa *et al* which indicate the fly ash-cement concrete had a lower air permeability. In these cases when fly ash is better dispersed (fly ash cement) and fly ash is more hydrated (due to the presence of alkali), the microstructure of the concrete is better developed. Thus it may be postulated that the air permeability can be even lower if the concrete, especially those with fly ash which tends to lose moisture easily at early stage /12/, is cured for longer time.

3.2 Relation between air permeability and open porosity

It is well known that the presence of moisture blocks or reduces the air flow path through concrete. With the presence of moisture gradient from drying surface to the center of the concrete specimen, this blocking effect becomes more complicated. In such a system, the dependence of *K* on porosity can be explained by considering the flow in a porous solid composed of a series of slices with different open porosity.

On the reasoning that the mass flow is uniform in the concrete, the effective permeability can be derived as /13/

$$K_{eff} = \frac{k\,\epsilon}{\frac{x_1}{\varepsilon_2} + \frac{x_2}{\varepsilon_2} + \ldots + \frac{x_n}{\varepsilon_n}} \tag{2}$$

where k and ϵ are respectively the permeability and porosity when all the pores are uniformly open, x_i and ϵ_i are the relative length and relative porosity respectively. Compared with /8/

$$K_{eff} = \frac{k\,\epsilon}{\tau} \tag{3}$$

where τ is tortuosity, Eq2 indicates the presence of moisture which causes the reduction in local porosity effectively increases the tortuosity. Drying, on the other hand, makes the ϵ_i increase with time. Obviously, the increase in porosity makes an increase in *K* more than linearly because of a reduction in effective tortuosity.

This is appreciated by an almost linear increase in *K* of a concrete dried at 50% RH and 20°C between 3 weeks to 9 months (Figure 6). This result is in agreement with that of Parrott and Chen /14/ who showed a near-linear increase in the logarithmic *K* versus weight loss. The effect of porosity on permeability is also expressed in the Kozeny-Carman equation /9/, in which *K* is proportional to $\epsilon^3/(1-\epsilon)^2$. It should be pointed out that, the air-flow in concrete is in fact a combination of Knudsen's flow and Poiseuille's flow. The latter, is proportional to r^2 (r = pore radius), which results in a greater increase in *K* compared to the enlargement of pores.

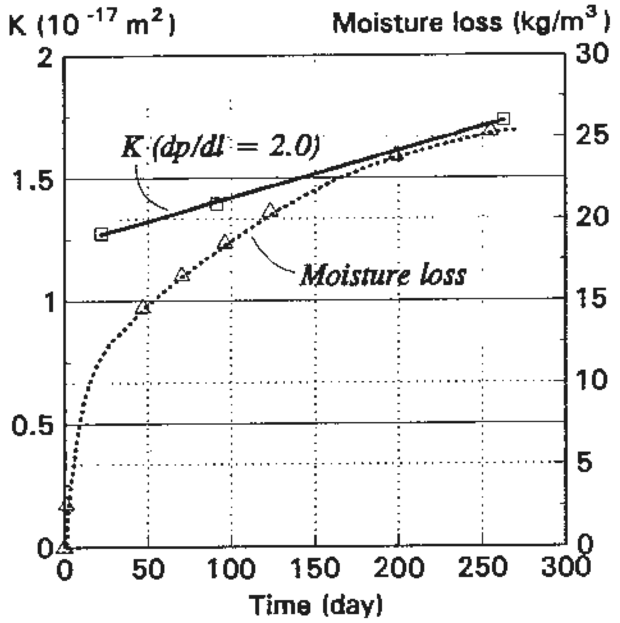

Figure 6 The air permeability determined under dp/dl=2.0 MPa/m and weight loss with drying time, the same concrete as in Figure 2.

3.3 Initial and near-equilibrium moisture diffusivity

The results of drying rate, as will be discussed below, provide a supplementary information about the water vapor transport properties, i.e. the initial diffusivity (D_0, diffusivity at the starting of drying) and near-equilibrium diffusivity (D_e). Figure 7 shows a typical drying process of concrete at 20°C 50%RH. As anticipated, the total moisture loss and the initial drying rate of the seal-cured concrete were less than those of water cured concrete. This was due to the self-desiccation effect /15/ which led to a lower moisture content in the concrete, and the concentration dependence of moisture diffusivity of concrete. Figure 7 also shows the weight

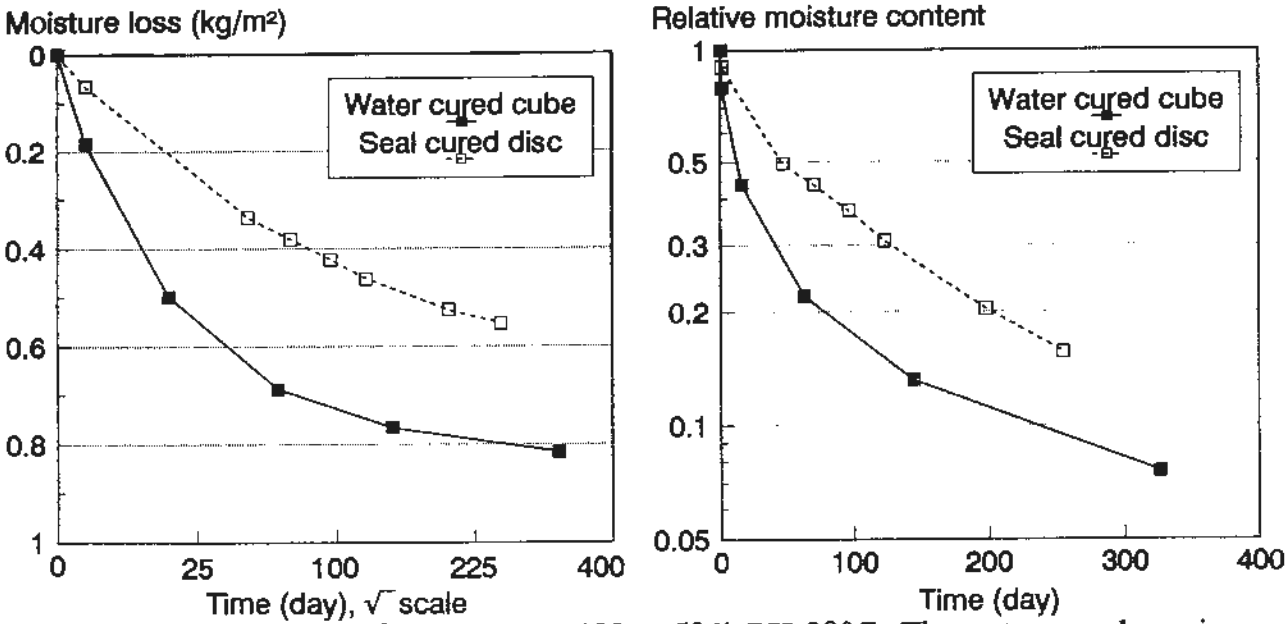

Figure 7. Drying process of concrete No.133 at 50% RH 20°C. The water cured specimens were saturated with water at the starting.

loss is approximately proportional to the square root of time at the starting of drying, and logarithmic of relative moisture content is linearly related with time at later drying stage.

These phenomena can be used for estimating the moisture diffusivity on the assumption that drying process follows Fick's second law:

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial x}\left[D\frac{\partial C}{\partial x}\right] \tag{4}$$

$C(x,t) = C_0$, for $0 \leq x \leq L$, at $t=0$; and $C(x,t) = C_e$ for $x = 0$ and L, at $t > 0$; where $C(x,t)$ is the concentration, and the subscripts "0" and "e" denote for the initial and equilibrium state. It is known that the moisture diffusion coefficient of concrete is higher at high RH /16/ and decreases rapidly at lower RH (over the range RH > 50%). This fact makes it impossible to analytically solve the Fick's second law in general, however D_0 and D_e can be estimated by

$$D_0 = (S_0 \frac{L}{4})^2 \pi \tag{5}$$

where S_0 is the slope of $(W_0-W(t))/(W_0-W_e)$ (W denotes for water content) versus $\sqrt{t}$ curve at $t=0$, (Crank /13/); and

$$D_e = S_e \left[\frac{L}{\pi}\right]^2 \tag{6}$$

where S_e is the slope of $\ln[(W(t)-W_e)/(W_0-W_e)]$ versus t curve at long time ($(W(t)-W_e)/(W_0-W_e) < 0.6$ or lower). A brief clarification of the latter is given in Appendix II. The results of D_0 and D_e are shown in Table 1 and 2 in Appendix I. The process of drying at 60°C in an oven appeared to be more rapidly at later stage than at the beginning. The estimated D_0 at 60°C is only slightly higher than D_e at 20°C, whereas D_e at 60°C is about 1 order of magnitude higher than D_e at 20°C, Table 2. Though in general a high *K* corresponds with a high D, no linear relation between them can be drawn out on the base of the data obtained. Figure 8 shows the concrete with fly ash has a greater *K* than normal concrete compared on the basis of same D.

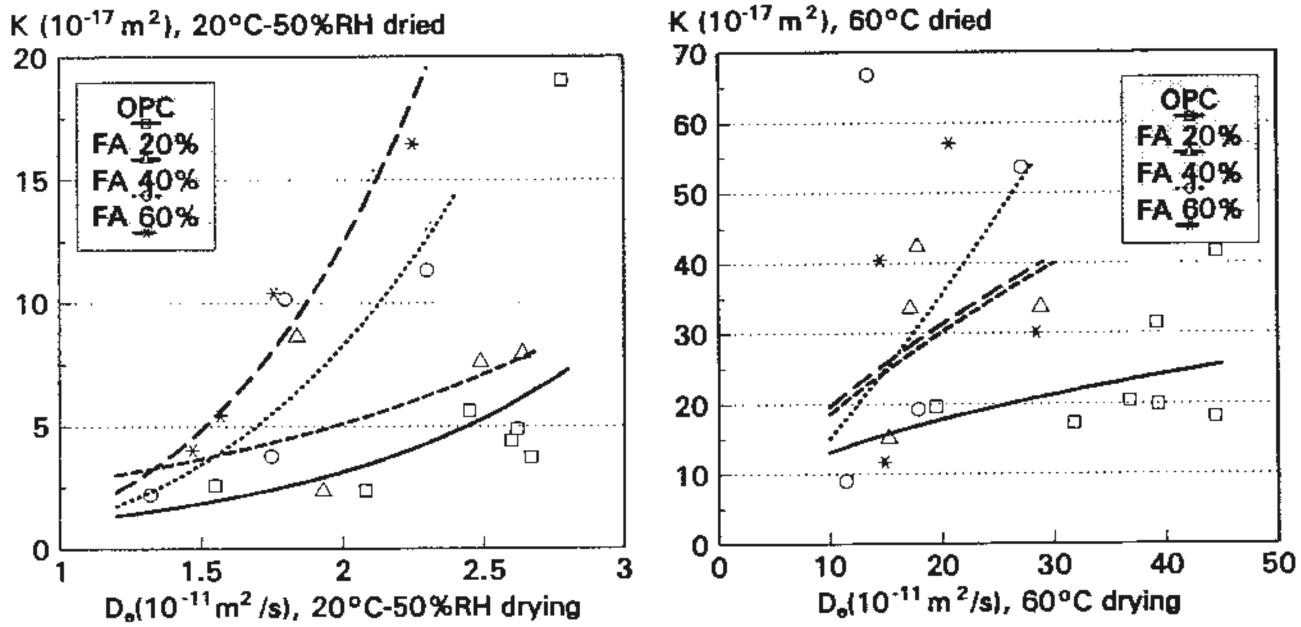

Figure 8 Relation between D and *K* depends on concrete type. The data are fitted by the least-square method to show the tendency.

The deviation in D - *K* relationship is mainly due to the fact that different mechanisms govern the moisture diffusion and air flow in concrete. For example, when moisture condensation occures in a capillary, it blocks air flow while cuts short the moisture diffusion path /8/; and air permeability is greater in larger cappilary because of the resistance is lower /7/ whereas the moisture diffusivity in the sence of molecular diffusion changes little. A commen feature shared by D and *K* is that both increase with porosity (Figure 9).

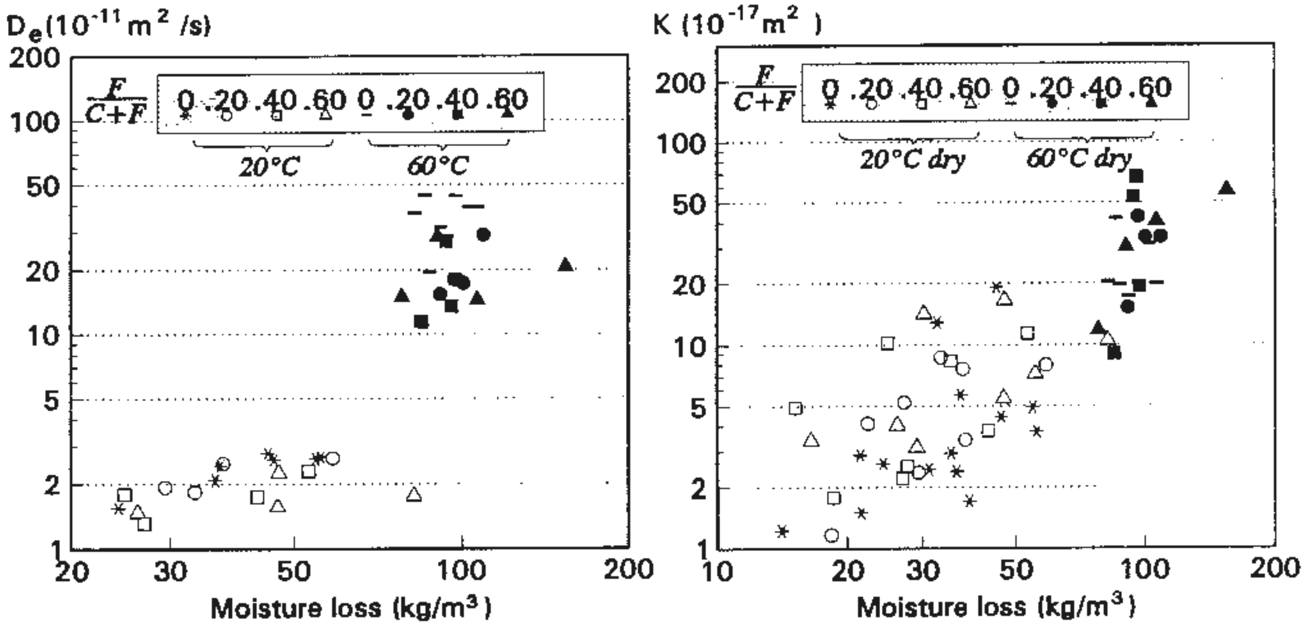

Figure 9 D and *K* increase with porosity.

3.4 Relation between concrete strength and air permeability and moisture diffusivity

The plot of the concrete initial moisture diffusivity vs. strength, and the air permeability vs. strength can be fitted by a power function (Figure 10), similar to that reported by Costa *et al.*

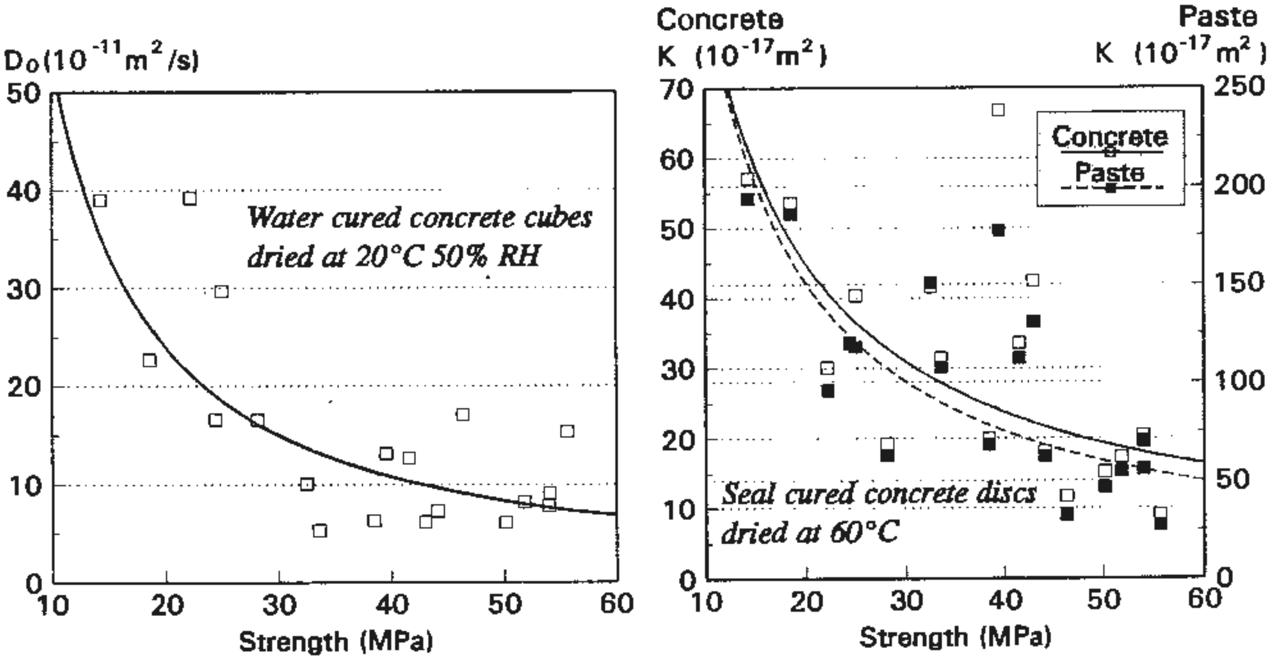

Figure 10 D_0 and *K* decrease with 28-day compressive strength.

/2/, however the deviation is rather large. Considering aggregate is impermeable to air flow, the paste permeability (60°C dried) can be approximated by $K_{\mathrm{concrete}} / (1-V_{\mathrm{aggregate}})$ where V means the volume fraction, which is more consistent to the permeability-strength dependence, Figure 10. Here, the effect of aggregate-paste transition zone (which is usually more porous) is considered as part of the paste quality for the sake of simplicity. In Figure 10, the obvious deviation indicates different effects of pores on permeability and on the strength, e.g. small pores which have little influence on strength may still be permeable to gas flow.

4 CONCLUSIONS

Air permeability of the tested concrete dried at 50% RH in room temperature varies over the range of 0.2 to 2×10^{-16} m^2, depending on the concrete composition. After 60°C drying, the value rises to 0.9 to 7×10^{-16} m^2. It appears the K value obtained in this study is somewhat high, due to the fact the specimens were only seal-cured for 28 days, which influences more on the concrete with larger volume fraction of fly ash.

In the presence of some extra alkali, the concrete with high volume fly ash showed a strong tendency of decrease in K. This can be explained by the enhanced pozzolanic reaction of fly ash due to alkali, and shows the potential of fly ash in the improvement of permeability.

Although closely related with the strength, the air permeability cannot be well predicted simply according to the strength value, and same is true for the water vapour diffusivity.

This study shows that, for the seal-cured concrete specimen of the thickness 5 cm, the K determined at 9 months drying (20°C, 50% RH) is linearly related to that at 3 weeks, and the difference is only 20% to 40%. Thus for an approximate estimation of K, the results obtained after the short drying time may also be acceptable.

In addition, it is found that the K determined in this study is dependent on the pressure gradient, i.e. the value obtained at lower pressure gradient appears to be significantly higher. This may be due to the influence of capillary walls on the air flow, and more work is needed for the confirmation.

5 ACKNOWLEDGEMENTS

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APPENDIX I TABLES

Table 1 Mix-Proportions and Properties of Concrete (water cured)

No.	$\frac{KOH}{C}$	$\frac{W}{C+F}$	$\frac{F}{C+F}$	Strength (MPa)		D ($10^{-11} m^2/s$)
				1 day	28 days	Initial
111	0	0.65	0	11.6	32.5	10.00
122	0	0.55	0.20	5.8	41.6	12.66
133	0	0.45	0.40	4.3	55.7	15.36
144	0	0.35	0.60	1.3	46.3	17.01
213	0.005	0.65	0.40	1.5	18.5	22.65
224	0.005	0.55	0.60	0.9	22.1	39.16
231	0.005	0.45	0	28.9	51.8	8.12
242	0.005	0.35	0.20	20.1	50.1	6.09
314	0.010	0.65	0.60	0.6	14.3	38.97
323	0.010	0.55	0.46	4.1	28.1	16.60
332	0.010	0.45	0.20	18.7	43.0	6.12
341	0.010	0.35	0	35.3	54.0	9.08
412	0.015	0.65	0.20	9.6	24.4	16.56
421	0.015	0.55	0	19.6	33.6	5.28
434	0.015	0.45	0.60	3.6	25.0	29.69
443	0.015	0.35	0.40	11.7	39.5	13.12
121	0	0.55	0	17.1	54.0	7.72
221	0.005	0.55	0	19.9	44.1	7.23
321	0.010	0.55	0	22.8	38.5	6.24

Table 2 Concrete (seal-cured) Diffusivity and Air Permeability

Specimen No.	Vapour Diffusivity (10^{-11} m^2/s)			Air Permeability (10^{-17} m^2)	
	D_0, 20°C	D_e, 20°C	D_e, 60°C	K, 20°C dry	K, 60°C dry
111	10.37	2.78	44.52	19.05	41.64
122	10.60	3.24	17.17	7.60	33.61
133	5.71	1.32	11.42	2.21	9.02
144	6.27	1.47	14.89	4.02	11.75
213	3.56	2.30	27.14	11.34	53.61
224	8.96	2.25	28.44	16.47	30.08
231	3.71	2.08	31.76	2.38	17.29
242	4.24	1.93	15.27	2.35	15.17
314	2.80	1.76	20.68	10.40	57.05
323	3.31	1.75	17.89	3.78	19.27
332	4.29	1.84	17.82	8.62	42.45
341	4.56	1.55	19.45	2.60	19.66
412	4.56	2.64	28.83	7.93	33.72
421	4.83	2.67	39.20	3.73	31.41
434	1.92	1.57	14.50	5.43	40.44
443	3.04	1.80	13.44	10.15	66.80
121	3.01	2.45	36.74	5.65	20.41
221	4.92	2.60	44.37	4.40	18.14
231	6.06	2.62	39.31	4.90	19.92

II ESTIMATION OF D_e BASED ON THE DRYING PROCESS

Denote the concentration which is a function of position and time, as C(x, t). When only diffusion is involved in the transport, the process is expressed by the Fick's second law (Eq.4)

If the diffusivity is a constant, the relative moisture content in a column of length L obeys

$$\frac{W(t)-W_e}{W_O-W_e} = \frac{8}{\pi^2}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}e^{-\left(\frac{(2n+1)\pi}{L}\right)^2 Dt} \tag{7}$$

where W_0, W(t) and W_e are the water content at starting, at time t, and at equilibrium state respectively. At later stage, i.e. when t is large, the terms with large n value will vanish, so

$$\ln\frac{W(t)-W_e}{W_0-W_e} \approx \ln\frac{8}{\pi^2}-\frac{\pi^2}{L^2}Dt \tag{8}$$

It follows that the slope of ln(W(t)) vs. t line is equal to $D(\pi/L)^2$. When D = D(C) is a rapid descending function of concentration, as is the case of moisture diffusivity of concrete at RH higher than 50%, the $\ln\frac{W(t)-W_e}{W_0-W_e}$ vs. t plot still reveals a straight line at low W(t). This is because of the D(C) is practically equal to D_e when W(t) is low enough, and the problem becomes the diffusion with a constant D and an uneven initial concentration distribution. It can be theoretically proven that the exponential time dependence of the diffusion process will not change in this situation, however, it is sufficient to show this graphically based on results of numerical calculation, Figure 11.

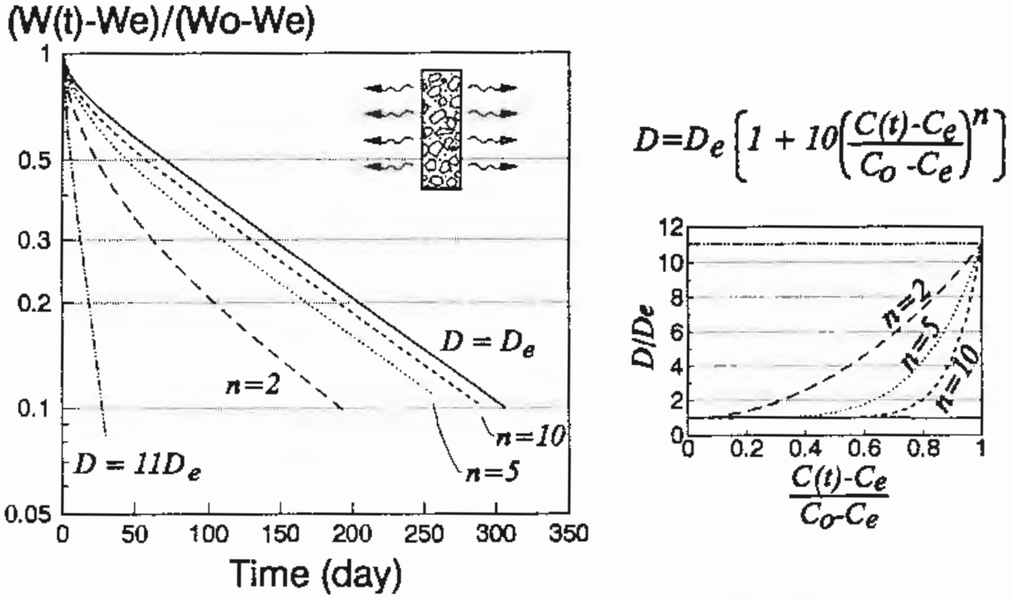

Figure 11 Numerical calculation result of drying process; two surfaces drying, L=5 cm, $D_e=2 \times 10^{-11}$ m^2/s. Note that the slopes of drying curves (of D=D(C)) are almost parallel at the late stage.