A Fatigue Failure Criterion for Concrete Based on Deformation

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ABSTRACT

A hypothesis is presented for fatigue failure of concrete structures. It is based on a deformation formulation, and utilizes the monotonic $F-\delta$ curve and the fatigue creep curve. The hypothesis is applied to flexural fatigue tests on notched beams of a plain high performance concrete. The experimental part comprises monotonic loading in deformation control and constant amplitude loading at three different load levels in flexural tension. The hypothesis is in all essentials consistent with the experimental findings. Furthermore, it provides a deformation formulation for accumulated damage estimation and remaining service life prediction, which takes account for the nonlinear nature of damage development in contrast to the linear Palmgren-Miner hypothesis.

Key words: *plain concrete, fatigue, fatigue failure hypothesis.*

1. INTRODUCTION

Fatigue of concrete is a process of progressive changes in the material during which the load bearing ability of a structure subsequently deteriorates. An accepted view is that microcracks initiate and propagate within the most tensed region and by time coalesce into a governing macrocrack, which becomes responsible for the ultimate failure. The degradation process can be monitored by measurements of e.g. deformation, ultrasonic pulse velocity or acoustic emission.

The traditional way to describe fatigue is to relate the fatigue strength to the static strength. The fatigue strength is defined as the number of repeated load cycles a structure can endure to failure of a stress, or load, of a certain magnitude. Usually the fatigue strength is represented by stress-fatigue life curves, referred to as *S-N curves* or *Wöhler curves*.

A number of serious drawbacks are associated with characterizing fatigue by stress limits: (a) The experimental test results show a large scatter in the number of cycles to failure at each stress level. The scatter in fatigue life can in return be traced back to the scatter in the static strength.

This is a key issue and a big dilemma because there is no method presently available that can be used to determine the strength of a structure (in a nondestructive way) in advance of a fatigue test. (b) Each S-N curve is only applicable for constant amplitude loadings with one of the load limits (the upper or the lower) held fixed, or with a constant ratio of these. A large number of tests are hence needed to establish the fatigue life chart for the most usual combinations of average stress levels and stress amplitudes. (c) Given a set of S-N curves, it is still not possible to predict the fatigue life in case of variable amplitude loading, which is the most common type encountered in reality. For example, a number of load cycles at a given stress level followed by a number of load cycles at a lower stress level tends to reduce the fatigue life more compared with the reversed order of load application. Thus, fatigue of concrete displays sequence effects which cannot be accounted for by S-N curves. (d) There are rate effects where a reduction in the speed of loading seems to decrease the fatigue life. This also complicates the relation between fatigue and static strength, as the latter normally is determined at a much slower rate of loading. In addition to all this, environmental influences, experimental noise, material property variations etc, also blurr the experimental findings and complicate deductions of general validity.

Therefore, strong motives indeed exist for finding better methods to describe fatigue. This paper investigates the potential of a fatigue failure hypothesis suggested by G. Balázs, see for example /1/, /2/, which is based on deformation instead of stress. The hypothesis makes use of the fatigue creep curve in combination with a deformation failure criterion that is related to the monotonic $F-\delta$ curve. In this study the hypothesis is applied on flexural fatigue tests on a plain high performance concrete performed by /9/.

2. DESCRIPTION OF THE FATIGUE FAILURE HYPOTHESIS

The hypothesis of Balázs assumes that fatigue failure occur when the largest deformation at each load cycle attains the algebraic value of the deformation corresponding to the maximum load in a comparable monotonic (static) test. It is anticipated that this happens at the end of the linear section of the fatigue creep curve, point C in Figure 1. Balázs suggested this relationship from observations on the stress-slip behaviour of a steel bar embedded in concrete subjected to repeated loads, /1/.

Hence, by monitoring the development of deformation in fatigue tests, and relating it to the deformation capacity of identical structures in static loading, it is in principle possible to predict the fatigue failure. The general applicability of the hypothesis for different kinds of fatigue problem however, still needs to be verified. Before entering into details of the hypothesis, a discussion of the deformability of materials from a fracture mechanics point of view might be appropriate.

Many quasi-brittle materials (rock, concrete, mortar etc) appear to have a limit for their ability to deform. When strained beyond this limit the material cracks. Consider a specimen of a homogeneous and isotropic material which is loaded in an ideal mode I situation, where a uniform stress distribution develops and remains during loading until failure. Under such circumstances a linear relation exists between load and deformation $(F-\delta)$, or equivalently between stress and strain $(\sigma-\varepsilon)$, up to the point where the material strength is reached and the body cannot sustain further loading. Hence, the theory of linear elasticity can be applied to calculate the load bearing capacity. It is trivial if the failure criterion is formulated in stress or strain. Upon reaching the maximum stress/strain the specimen will expose a brittle failure, due to that, as a consequence of the nonvarying material properties, every region of the cross-section simultaneously attains its strength. At this stage there are no possibilities to maintain equilibrium (and stress redistribution is not an option).

However, failure without a synchronous stress redistribution is unlikely to occur in real concrete structures. Variations in material properties (strength, stiffness) and boundary conditions promote stress redistribution as soon as cracking is initiated, which is manifested in a nonlinear $F-\delta$ relation. The driving force is the strive of nature towards minimizing the overall energy of the system. This kind of structural behaviour is even recognized in fracture mechanics testing on the simplest concrete structure: a bar. Deformation controlled uniaxial tensile tests on notched specimens reveal that the specimen flex to the side during cracking, which result in a nonuniform crack opening, see for example /8/, /11/, /5/. In /5/ it was observed that the nonuniform crack opening initiates on the ascending branch of the $F-\delta$ curve, which implies that stress redistribution is already active when the maximum load is reached. And it plays an even more important role when the post-peak regime is entered. In fact, softening of concrete is not possible without the participation of stress redistribution. Alternatively, it can be expressed as *concrete softening requires a strain gradient.*

Due to the structural behaviour, the maximum load does not coincide with macro-crack initiation. For a structure the concept of strain must therefore be abandoned and replaced by some deformation that is able to reflect the failure mode. This is also justified from the point that fracture is a localized phenomenon, see for example /6/. Recently, it has also been experimentally verified that tensile fatigue fracturing localizes to a narrow zone, /8/.

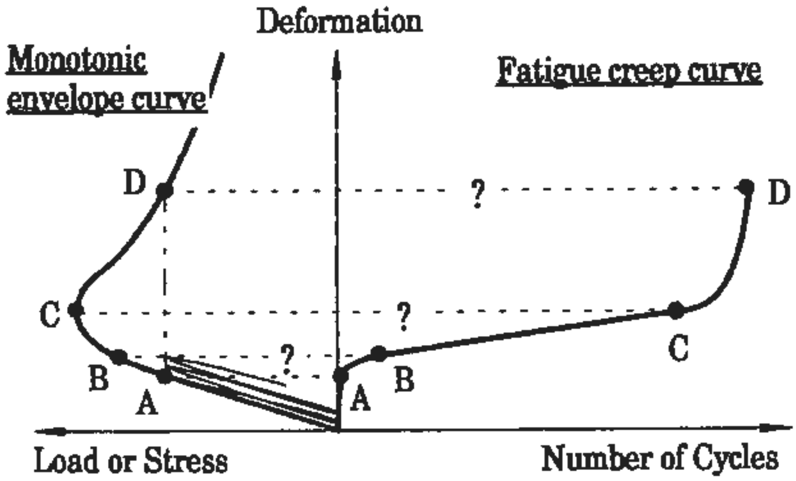

Figure 1: Schematic description of the fatigue failure hypothesis.

To turn back to the failure hypothesis, there are question marks in Figure 1, to accentuate that the lines intersecting the points on the monotonic and the creep curve respectively are suggested relationships. They have not yet been experimentally verified, at least not to the authors knowledge. Nevertheless, some possible interpretations regarding their physical meaning may be discussed. Point A obviously corresponds to the instant deformation at load application. Point B is thought to represent the onset of stable crack propagation, which prolongs to point C. The slope of the linear section B-C is thus a measure of the rate of crack propagation (mm/cycle or mm/s). When the linear section is entered (at B) and the stable crack propagation begins, the governing macrocrack is assumed to be born. Therefore, the nonlinear part A-B might be due to the formation of a microcracked zone (process zone) from which the macrocrack subsequently emanates. The nonlinear part C-D manifests the accelerated deterioration of the material structure that reduces the cross-section and inevitably leads to the ultimate failure.

From controlled uniaxial tensile tests it is concluded that localization starts on the ascending branch, before the peak load is reached, /5/. The starting point of nonuniform crack-opening often coincided with the proportionality limit, that is, where the ascending branch begins to become nonlinear. It is possible that point B on the creep curve corresponds to the proportionality limit on the monotonic $F-\delta$ curve.

3. TEST PROGRAMME

Pettersson & Pöntinen /9/ conducted both monotonic and fatigue tests in three-point bending on a plain high performance concrete. In total 7 monotonic tests and 13 fatigue tests were executed. The test procedure basically followed the suggested method for determining the fracture energy proposed by the RILEM technical committee TC-50, /7/. Due to problems with the data-acquisition system, the results from three of the fatigue tests had to be excluded in this analysis because of incomplete deformation measurements.

3.1 Test setup, control and measuring system

The test setup is schematically presented in Figure 2. The load was supplied by a closed loop servohydraulic press with a capacity of 50 kN in both compression and tension. The deflection was measured at midspan on the top surface of the beam by two LVDT gauges. The notch mouth openingen was measured by a COD gauge. The two LVDTs were fixed to a steel frame located on the top of the beam, and with its supports straight above the supports of the beam to ensure that no bending moment was induced, which could affect the deformation measurements.

The monotonic tests were performed in deformation-control with the COD signal as feedback to the controller. The rate of the notched mouth opening was set to 0.002 mm/s. Each test lasted approximately 5-7 minutes, and the maximum load was attained after 50-60 seconds.

The fatigue tests were performed in load control with a sine waveform at 1 Hz. The lower load limit was kept constant at 10% of the average static flexural strength, while the upper load limit was changed between three different levels; 70%, 80% and 90% of the average flexural strength.

3.2 Concrete composition

The high performance concrete mix included ordinary low heat Portland cement, sand and coarse granitic gravel, microsilica equal to 10% of the cement by weight, and a melamine superplasticizer. The ratio $w/(c+s)=0.27$.

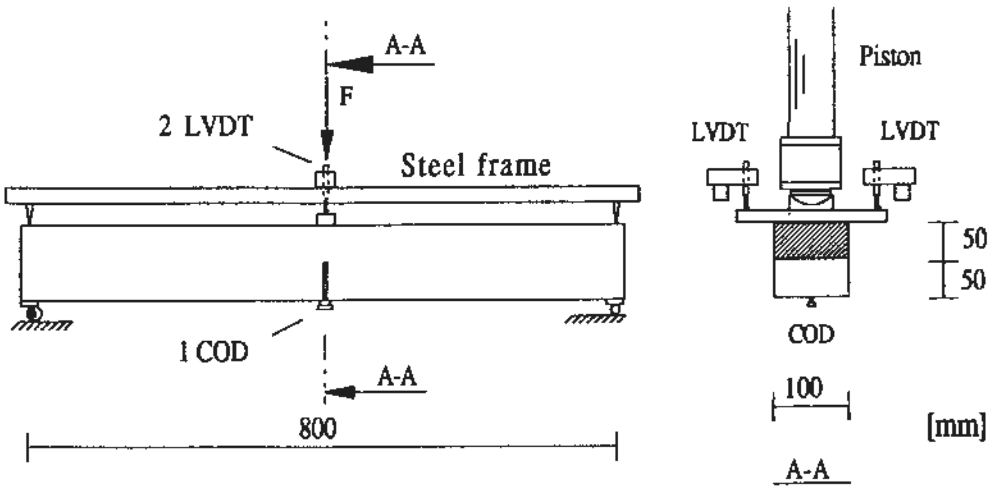

Figure 2: Test setup for the monotonic and the fatigue tests.

Splitting tests and compressive tests were done 28 and 79 days after casting on 100 mm cubes which had been stored in the laboratory together with the beams. These tests were performed according to the Swedish Code for Concrete Construction (BBK 79). The mix composition and the mechanical properties are shown in Table 1.

Table 1: Mix composition and mechanical properties

Mix proportions [kg/m³]		*Mechanical properties*	
Cement	539	f_{cc} (28d)	124 MPa
Water	161	f_{cc} (79d)	124 MPa
Sand 0-8 mm	777	$f_{c,spl}$ (79d)	7.5 MPa
Gravel 12-16 mm	1087	G_F	208 Nm/m^2
Microsilica	53.9		
Superplasticizer	1.8 % of Cement		

4. RESULTS

4.1 General

The results from the monotonic tests are given in Table 2, and the results from the fatigue tests in Table 3. The S-N relation from /9/ is shown in Figure 3. The best obtained regression line, according to the least square method and a probability of failure of 50%, is

$$\log N = 15.5 - 15.5 \cdot S_{max} \qquad (S_{min} = 0.10) \qquad (1)$$

where $S_{max} = F_{max}/F_{flex}$, $S_{min} = F_{min}/F_{flex}$ and F_{flex} is the flexural loadbearing capacity. The S-N curve shows a quite large scatter for the number of cycles to failure at the different load levels, a consequence of the fact that the real strength of the specimens is unknown.

Table 2: Results from monotonic tests

Beam *No*	F_{flex} [kN]	$\delta_{C,m}$ [mm]	δ_{tot} [mm]	G_F [Nm/m^2]
1*	(1.53)	(0.11)	-	-
2	1.82	0.22	1.0	174
3	1.79	0.17	1.3	193
4	1.92	0.18	1.2	190
5	1.96	0.15	1.2	203
6	1.86	0.15	1.7	235
7	2.01	0.17	1.5	251
Average	1.893	0.173	1.32	208
Stand. dev.	0.085	0.026	0.25	29.4

* Broke on the ascending branch

Table 3: Results from fatigue tests

Load Limits $F_{min} - F_{max}$	*Beam* *No*	$\delta_{C,f}$ [mm]	N_C Cycles to point C	$\delta_{D,f}$ [mm]	N_D Cycles to failure	$\frac{N_C}{N_D}$
10%-70%	18	0.11	25 848	0.25	39 496	0.65
	20	0.13	22 400	0.26	30 336	0.74
	21	0.14	258 560	0.22	344 960	0.75
10%-80%	10	0.14	2 781	0.29	3 990	0.70
	14	0.16	14	0.24	21	0.67
	16	0.13	98	0.29	174	0.56
	19	0.14	6 481	0.26	8 646	0.75
10%-90%	11	0.16	10	0.19	12	0.83
	13	0.19	29	0.32	56	0.52
	17	0.13	43	0.26	65	0.66
Average		0.143		0.258		
Stand. dev.		0.022		0.037		

The subindices C and D refers to Figure 4:

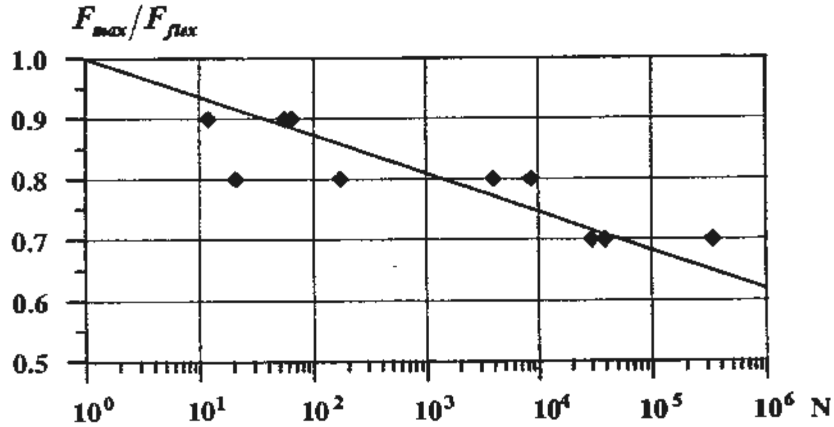

Figure 3: S-N curve from Pettersson and Pöntinen /9/.

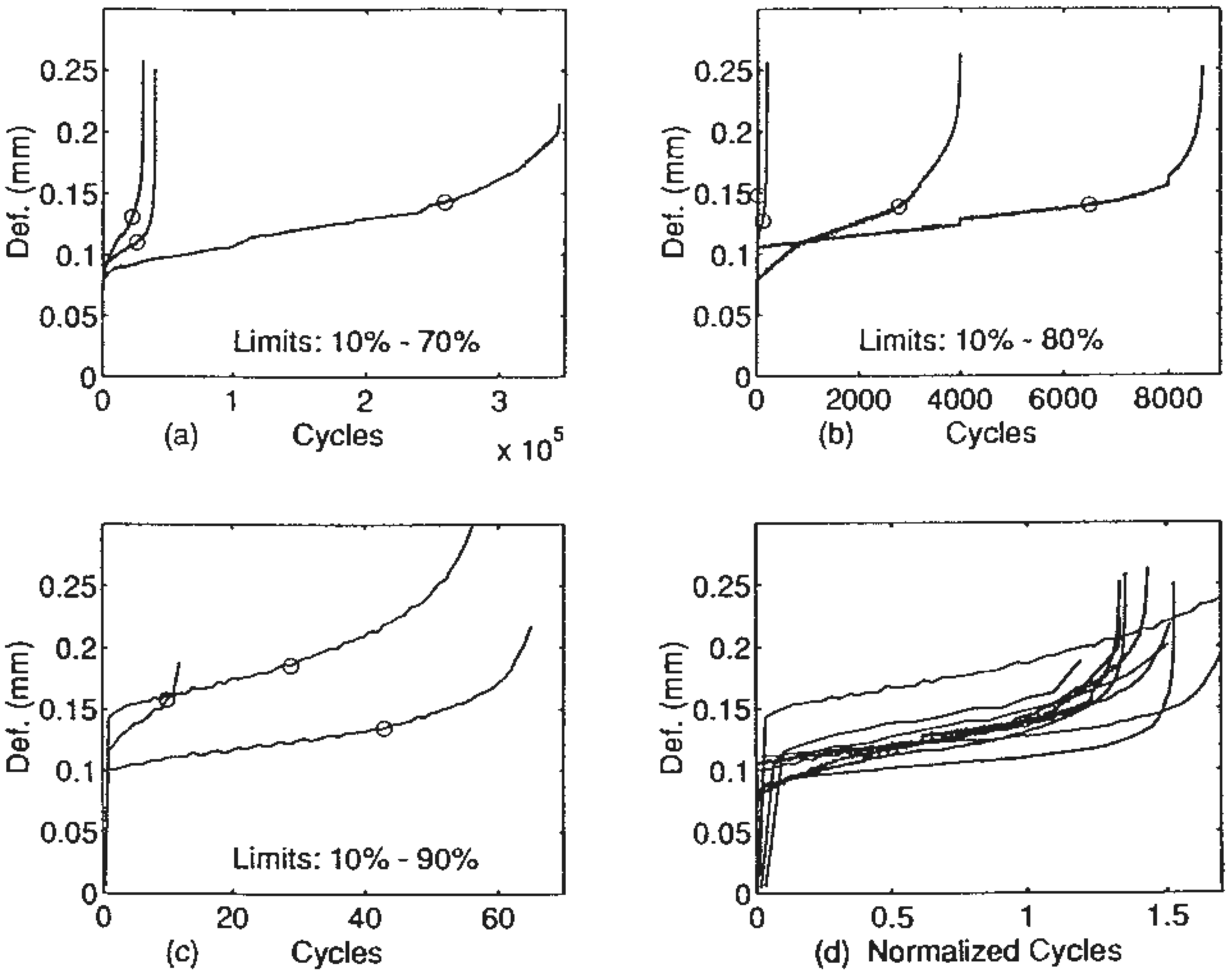

Figure 4: (a)-(c) Fatigue creep curves for the different load levels, (d) Normalized fatigue creep curves, with respect to point C, for all tests.

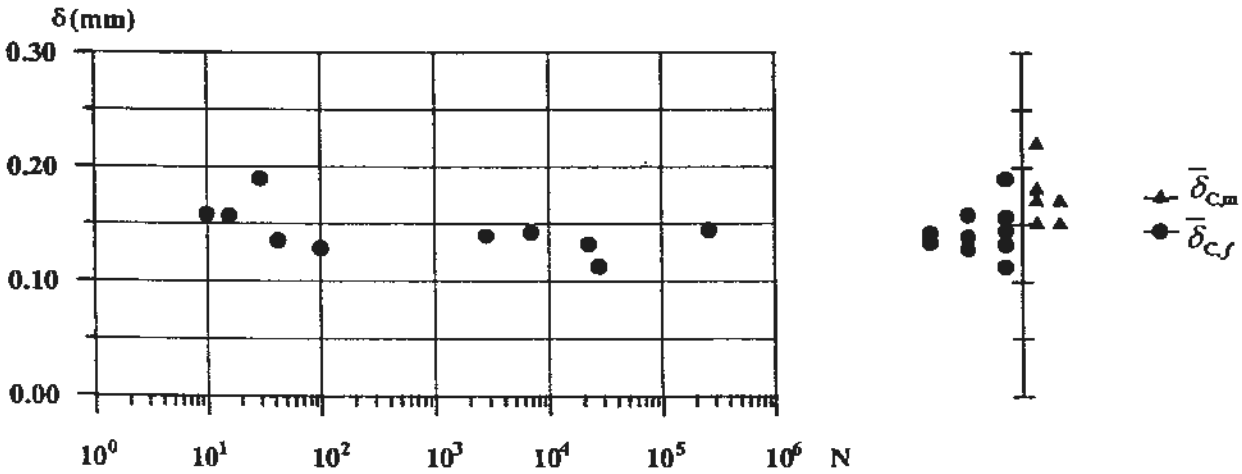

Figure 5: (left) Points representing the end of the linear section of the creep curves. (right) Distribution of deformation of the fatigue and the monotonic tests.

Figure 4 displays the fatigue creep curves. The points corresponding to the end of the linear section (point C in Figure 1) are marked and compiled as a function of cycle number in Figure 5 for all tests.

The values of $\delta_{C,f}$ seems to be rather independent of the number of cycles, although the number of tests are low, and the scatter is relatively lesser compared to the S-N curve in Figure 3. For a comparison $\delta_{C,m}$ from the monotonic tests are included. There appears to be a correspondence of deformation averages. To check if the observed correspondence is significant, the difference between the average values are subjected to a statistical treatment.

4.2 Aspects of Evaluation

The evaluation procedure needs some comment. The point corresponding to the end of the linear section of the fatigue creep curve may in reality be difficult to establish. Due to experimental noise (and presumable also to other unknown factors) the linear section was not real straight for some of the tests, which of course obstructs the attempt to define the point where the curve starts to deviate. Therefore, a regression analysis on the "linear" section was performed for all curves. The point of deviation was then defined visually by studying the difference between the creep curve and the regression line.

The definition of the deformation corresponding to the maximum load in a monotonic test is more straightforward. However, beam No 5 did expose a plateau at the peak. A plateau is a manifestation of considerable stress-redistribution. One might then ask what kind of rate of deformation would result, if such a redistribution occurs in a fatigue test. Would it yield secondary creep or tertiary creep? Here a rather conservative approach was adopted in order not to overestimate the deformation capacity. Since secondary creep is of interest, the deformation $\delta_{C,m}$ was set to the beginning of the plateau, see Figure 6. Due to the circumstances mentioned the deformation values are rounded off and presented with only two decimals.

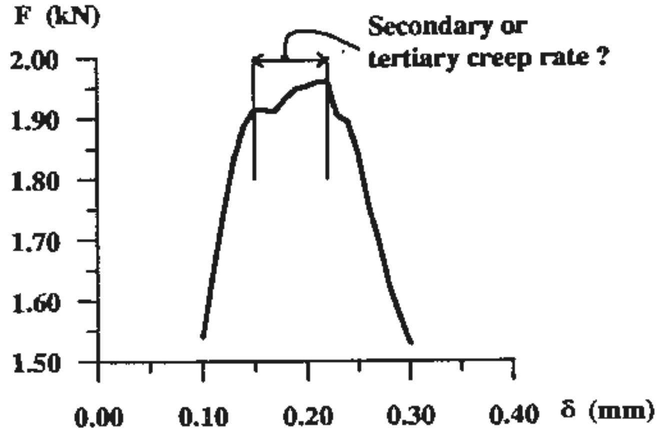

Figure 6: $F-\delta$ curve of beam No. 5 with a plateau at the peak.

4.3 Statistical Analysis

To investigate if the observed difference between the average deformations $(\bar{\delta}_{C,m}-\bar{\delta}_{C,f})$ is statistically significant, a two sided significance test is performed for the difference in the population means. The procedure for an unpaired design (fully randomized test) is adopted, see for example /3/.

A null hypothesis H_0 is formulated and tested against an alternative hypothesis H_1. H_0 is that the difference in means is zero. Hence the deformation at maximum load in a monotonic test equals the deformation corresponding to the end of the linear section of the fatigue creep curve. H_1, on the other hand, is that the difference is greater or less than zero. With $\Delta\eta=\eta_m-\eta_f$, the difference in the population means, this can be stated

$$H_0:\ \Delta\eta=\Delta\eta_0=0,\ \text{and}\ H_1:\ \Delta\eta=\Delta\eta_1\neq 0$$

The distributions of $\delta_{C,m}$ and $\delta_{C,f}$ are unknown, and so are the population standard deviations $\sigma_{C,m}$ and $\sigma_{C,f}$. On the assumption that $\bar{\delta}_{C,m}$ and $\bar{\delta}_{C,f}$ are random samples from two normal distributions with the same variances and differing, if at all, only in their means, a t-distribution can be used as a reference distribution. The appropriate t-statistic is then

$$t=\frac{\Delta\delta-\Delta\eta}{s\sqrt{\frac{1}{n_m}+\frac{1}{n_f}}} \tag{2}$$

where $\Delta\delta=\bar{\delta}_{C,m}-\bar{\delta}_{C,f}$ is the difference in average deformations and $s\sqrt{1/n_m+1/n_f}$ is the standard deviation of the difference in averages, with n_m and n_f the numbers of monotonic and fatigue tests. The estimates of the standard deviation are combined into a pooled estimate of σ as

$$s^2=\frac{(n_m-1)s_m^2+(n_f-1)s_f^2}{n_m+n_f-2} \tag{3}$$

which is allowable if the population variances for the reference distributions are approximately equal. Numerically, $\Delta\delta=\bar{\delta}_{C,m}-\bar{\delta}_{C,f}=0.030\ mm$ and $s\sqrt{1/n_m+1/n_f}=0.012\,mm$. The hypothesis that the difference in the population means $\Delta\eta$ has some value $\Delta\eta_0$ can be tested by refering

$$t_0=\frac{0.030-\Delta\eta_0}{0.012} \tag{4}$$

to a t-table with $(\eta_m-1)+(\eta_f-1)=14$ degrees of freedom. In particular for $\Delta\eta_0=0$ (the null hypothesis), $t_0=0.030/0.012=2.50$. The question is how often t would exceed 2.50 or fall short of -2.50. The required probability is

$$\Pr(|t|>|t_0|)=2\cdot\Pr(t>2.50)\approx 0.028 \tag{5}$$

Hence, if the true difference were zero, a deviation in either direction as large as that experienced, or larger, would occur by chance about 28 times in 1000.

Additional information may be obtained by a confidence interval. A two sided 1-α confidence interval for the true difference $\Delta\eta$ is defined by the probability $\Pr(|t|>t_{\alpha/2})$ where α is the chosen level of probability and $t_{\alpha/2}$ corresponds to a single tail area $\alpha/2$. For a 95% confidence interval with 14 degrees of freedom, $t_{\alpha/2}=2.145$. Thus, all values of $\Delta\eta_0$ for which

$$\left|\frac{0.030-\Delta\eta_0}{0.012}\right|<2.145 \tag{6}$$

would not be discredited at the 0.05 confidence level. In particular for the null hypothesis, $\Delta\eta_0=0$, the confidence limits become

$$[0.004,\ 0.056]\ (mm). \tag{7}$$

The limits for a 98% confidence interval, with $t_{\alpha/2}=2.624$, is analogous [-0.001, 0.061] (mm). Since zero is not within the limits of (7), H_0 is discredited with 95% probability. Hence, it does exist, in statistical terms, a difference between η_m and η_f on the 5% confidence level. On the other hand, H_0 cannot be discredited with 98% probability.

As a comparison, the Mann-Whitney test, also known as the Wilcoxon test, is employed. Such nonparametric tests (also refered to as "distribution free" tests) do not make any assumption about the reference distribution, but they still rely on the random sampling hypothesis. The test statistic for the Mann-Whitney test is

$$T=S-\frac{n(n+1)}{2} \tag{8}$$

where S is the sum of ranks assigned to one of the two populations and n the corresponding number of samples. Refering T to an appropriate table gives that H_0 is discredited at the 0.05 significance level, but not at the 0.02 level. The agreement between the t-test and the Mann-Whitney test, supports the assumption that $\bar{\delta}_{C,m}$ and $\bar{\delta}_{C,f}$ have a normal distribution.

The importance of the statistical inference should not be overemphasized. The sample averages $\bar{\delta}_{C,m}$ and $\bar{\delta}_{C,f}$ are based upon a small number of tests, 6 and 10 respectively, which implies that outliers have a large impact on sample averages and standard deviations. Moreover, the observed difference in deformations is on the *safe* side. Contemplation of the 98% interval [-0.001, 0.061] in relation to the difference, $\Delta\delta=\bar{\delta}_{C,m}-\bar{\delta}_{C,f}$, shows that the probability that $\bar{\delta}_{C,f}$ should exceed $\bar{\delta}_{C,m}$ is quite small, which is important if $\bar{\delta}_{C,m}$ is to be used as a failure criterion.

5. FATIGUE DAMAGE ACCUMULATION

Damage caused by fatigue is generally described with the Palmgren-Miner hypothesis. The supposition is that damage accumulates linearly with the number of cycles applied at a particular load level. The Palmgren-Miner failure criterion is

$$\sum_{i=1}^{k} \frac{n_i}{N_i} = 1.0 \qquad (9)$$

where n_i is the number of cycles at stress level i, N_i is the number of cycles to failure at stress level i and k is the number of stress levels. The hypothesis does not accurately reflect the degradation process of concrete. For instance, sequence effects are not considered. Nevertheless, it is used in design practice, mainly due to its simplicity and because there is no better method available.

A hypothesis for damage accumulation may also be expressed in deformation. A plausible formulation for the criterion of failure is

$$\frac{1}{\delta_{C,m}} \cdot \sum_{i=1}^{n} \Delta\delta_i = 1.0 \qquad (10)$$

where $\delta_{C,m}$ is the deformation at the maximum (static) load in a monotonic test, and $\Delta\delta_i$ the increase in deformation due to load cycle i. $\Delta\delta_i$ is a measure of the partial damage obtained during cycle i. The similarity with the Palmgren-Miner hypothesis is obvious. However, this formulation accounts for the nonlinear characteristics of damage accumulation. Moreover, it can be applied without having detailed insights in the relationship between the incremental damage $\Delta\delta_i$ and the number of cycles to failure N, by monitoring the deformation of a structure during service and comparing it with the static deformation capacity.

The rate of deformation increase in the linear section of the cyclic creep curve, $\dot{\delta}_{sec}$, can also be utilized to estimate the time to failure. Several investigators have demonstrated the strong correlation between the rate of deformation and the nnmber of cycles to failure, although most of them have preferred to express it as strain rate, $\dot{\varepsilon}_{sec}$, see for example /4/, /10/. Caution should though be practiced when expressing deformation as strain for a partially cracked material.

The relation for the experiment in study is visualized in Figure 7, where both N_C and N_D are plotted versus $\dot{\delta}_{sec}$ in a logarithmic scale. The regression curves are parallel, due to that the ratio N_C/N_D is rather independent of the cycles to failure, see Table 3. Hence, both relations can be utilized. Combining the ultimate failure criterion, $\delta_{C,m}$, with the rate of deformation increase $\dot{\delta}_{sec}$, the remaining fatigue life can be estimated. Under the presumption that the linear part of the creep curve is reached and the rate of deformation does not change due to any altered loading scheme, the relation is

$$rfl = \frac{\delta_{C,m} - \delta_i}{\dot{\delta}_{sec}} \qquad (11)$$

where *rfl* is the *remaining fatigue life* and δ_i is the current deformation (at time i or cycle i). *rfl* is either expressed in cycles (N) or time (s), depending on the chosen unit for $\dot{\delta}_{sec}$.

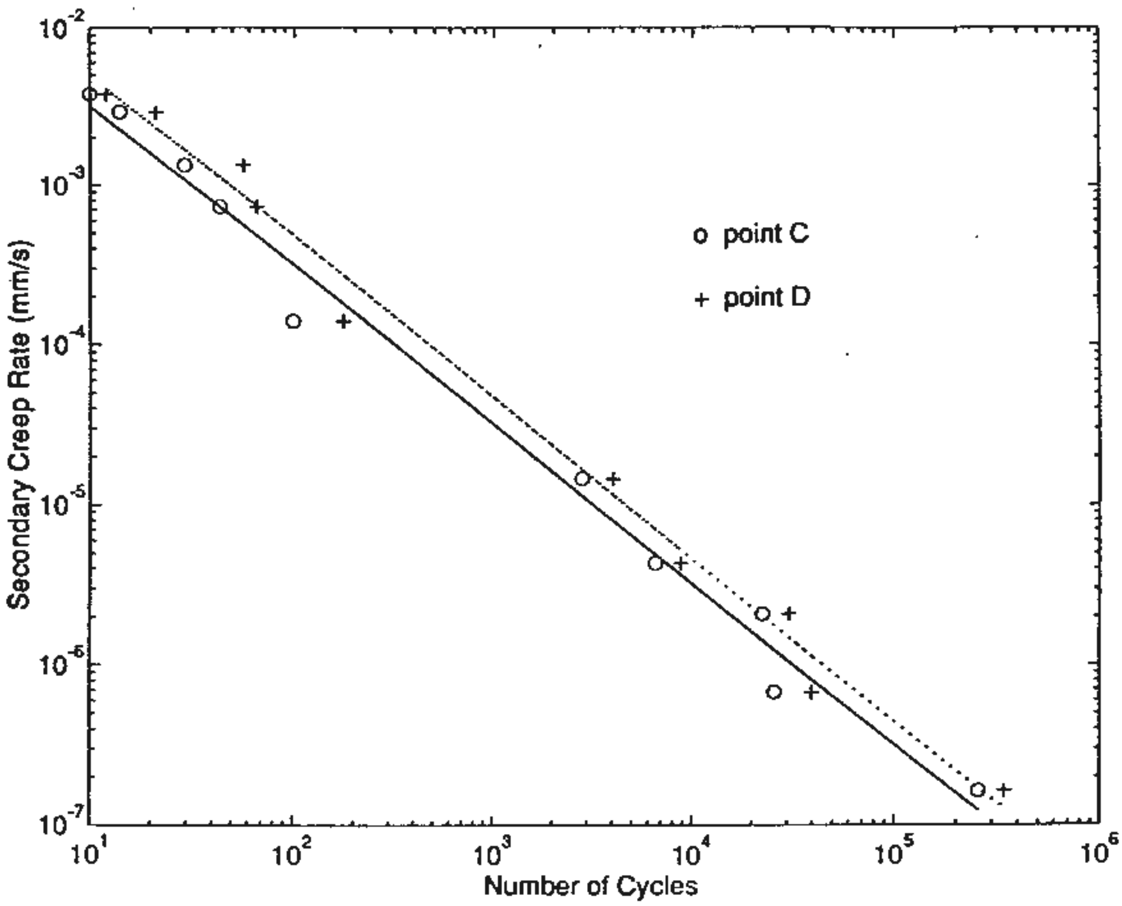

Figure 7 : Relation between fatigue creep rate $\dot{\delta}_{sec}$ and fatigue life, defined as N_C and N_D.

6. DISCUSSION

The results presented in the previous sections indicate that the deformation corresponding to the monotonic maximum load, $\delta_{C,m}$, can be applied as a fatigue failure criterion. However, the experiments which this study is based on encompasses only repeated loading in flexural tension. It is possible that reversed loading, in tension-compression, might enlarge $\delta_{C,f}$ compared to repetitive tensile loading. Such an observation is reported in /8/ in uniaxial tension-compression fatigue. This behaviour can be explained by the fact that compressive loading causes micro-cracking distributed over the bulk volume. The deformation from the part of the volume which are situated within the measuring length of the displacement gauge thus adds to the deformation caused by localization, resulting in an apparently higher deformation.

The suggestion that the deformation at ultimate failure, $\delta_{D,f}$, should correspond to the monotonic post-peak deformation, $\delta_{D,m}$, does not get any support from the creep curves. Considering the very large difference in loading rate between monotonic and fatigue tests in general, it is doubtful that such a relation, if it exist, is possible to verify experimentally.

6.1 How to apply the failure criterion in practice

If the failure criterion, Eq. 10, is to be used for damage accumulation estimations, the static deformation capacity, $\delta_{C,m}$ has to be known. Thus, full-scale tests has to be performed as $\delta_{C,m}$ is a structural quantity. That of course limits the applicability to small-size and relative cheap structures, for example different types of precast structural elements. If $\delta_{C,m}$ is known damage estimations can be accomplished during service. Of course, it is important that the deformation is measured in the same way when the structure is subjected to fatigue loading.

However, the failure criterion can be employed without knowing the static deformation capacity, $\delta_{C,m}$. It is in principle sufficient to monitor the development of deformation during service to get information of the status of the structure. A requirement is that the structure is instrumented such that the deformation measurements can capture the mode(s) of failure. The creep curve can then be produced in "real time". The tendency of the deformation development indicates the condition of the structure. Further, the deformation measurements need not to be performed at periods of high load levels. It can be done at periods of no loading or low loads, because the creep curves for the two latter parallels the first. And as tendencies are of importance, they will appear as well. Appropriate measures can then be considered if the rate of deformation starts to increase.

7. CONCLUSIONS

Based on this study the following conclusions are drawn.

- The deformation at the end of the linear section of the fatigue creep curve, $\delta_{C,f}$, was in average found to be lower than the monotonic (static) deformation capacity, $\delta_{C,m}$. The statistical analysis contradict a relationship between $\delta_{C,f}$ and $\delta_{C,m}$, at the 0.05 level of confidence, but not at the 0.02 level of confidence. However, more important than the statistical inference is the practical consequence this observation will have on the applicability of the hypothesis. In that respect it can be advantagous that $\delta_{C,f}$ are somewhat smaller than $\delta_{C,m}$, if it is a general trend, as it increases the safety margin to ultimate failure. Consequently, there seems to be no objections of principal nature for applying the hypothesis as a fatigue failure criterion.

- For damage accumulation estimations and for remaining service life predictions it is required that the static deformation capacity, $\delta_{C,m}$, is known . By recording the behaviour of a structure during service, the remaining service fatigue life can be predicted in "real time" in terms of damage accumulation. A prediction of the number of cycles to failure, or the time to failure, based on deformation is more accurate than estimates based on stresses, because deformation represents the actual damage in the material. It is important that the deformation measurements start at the time of activation.

- If $\delta_{C,m}$ is unknown, the hypothesis may still be used. Necessary information can be obtained by studying the slope of the fatigue creep curve. The rate of deformation increase tells the status of the structure.

- An inherent feature of the hypothesis is that it does not overestimate the fatigue life; it is a safe criterion in this respect. The safety margin increases, in terms of number of cycles to ultimate failure, the lower the relative load level is. Moreover, it is in principle valid for any load spectrum, and it is independent of the order of load application, so called sequence effects.

TABLE OF NOTATION

δ, ε	deformation, strain	F, σ	force, stress
F_{min}, F_{max}	lower and upper load limits in a fatigue test	F_{flex}	flexural load bearing capacity
δ_i	deformation at time *i* or cyle *i*	$\dot{\delta}_{sec}$	rate of deformation of the linear section of the fatigue creep curve
$\delta_{C,f}, N_C$	deformation and number of cycles corresponding to the end of the linear part of the fatigue creep curve, see Figure 1	$\delta_{C,m}$	deformation corresponding to the maximum load in a monotonic (static) test
$\delta_{D,f}, N_D$	deformation and number of cycles corresponding to ultimate failure, see Figure 1	rfl	remaining fatigue service life
η	population mean	σ, s	population standard deviation and sample standard deviation

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