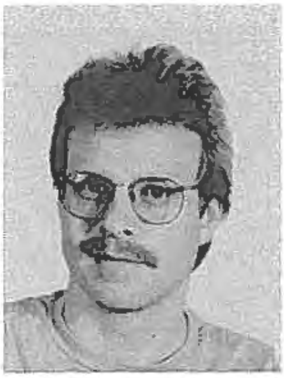

Cast-in-Place Bonded Anchors

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Abstract

By use of the theory of plasticity a theoretical model has been developed for the anchorage strength of cast-in-place bonded anchors subjected to direct tension. The model is general, meaning that it covers i.a. both cast–in and drilled–in thread and reinforcing bars.
Tests with cast–in thread bars are reported, and the theoretical expresssions for the load carrying capacity are compared to the test results. The correspondence between the test results and the predictions is satisfactory. It is shown by the use of the test results and the theoretical results that the load carrying capacity of cast–in bonded anchors can be presented in a very simple expression. In a specific case the expression is similar to an emperical expression for drilled–in bonded anchors given in the literature.

Key words:
Bonded anchors, tension, tests, theoretical model, thread- and reinforcing bars.

1 Introduction

Transfer of forces between a concrete structure and another structure is often carried out by means of cast–in or drilled–in thread bars (bonded or adhesive anchors). Especially in the case of restoration and rebuilding of existing structures drilled–in bonded or expansion anchors are widely used. However, also in the case of new structures under construction the drilling–in technique is used because it increases the flexibility considerably, and those who should use the connection can place the anchors themselves. Hence, the responsibility for the correct position is in this case very clear.

Unfortunately the designers must in almost all cases rely on empirical rules for determination of the load carrying capacity of cast–in or drilled–in thread and reinforcing bars. The empirical rules and expressions are in some cases developed by the suppliers of fastening systems, and the different methods often result in different values for the load carrying capacity. In many cases the suppliers have not even given any expressions or methods, but the load carrying capacity for a particular fastening system is given in a table. This is quite unsatisfactory. Therefore, amoungst other reasons, it would be valuable if it was possihle to develop a model based on a rational theory to treat these types of anchorage problems. The expansion anchor is in principle simple to treat, hecause the models for punching shear in slabs can be applied in these cases. The bonded anchor is more difficult to treat theoretically, because the failure mechanism cannot directly he related to a similar problem. However, by combining already availahle knowledge it is in a simple way possible to develop a theoretical model for bonded anchors.

The theory of plasticity is used here as theoretical basis for the calculations. Due to the fact that concrete is not a rigid plastic material the effective plastic strengths $f_{cp} = \nu f_c$ and $f_{tp} = \rho f_c$ for compression and tension, respectively, are introduced, wbere the factors ν and ρ are named the *effectiveness factors*. f_c is the uniaxial concrete compression strength measured on a cylinder. The calculations are carried out according to the upper bound theorem in the theory of plasticity. As failure criterion for the concrete, the modified Coulomb failure condition is used. The bars are assumed to consist of a rigid plastic material. See for instance Nielsen /1/ for a more detailed description of the theory of plasticity used to treat load carrying capacity problems in concrete structures.

The failure along the embedment length is separated in two parts, see figure 1; one part close to the concrete surface perpendicular to the longitudinal direction of the bar, and the other far from the surface. The failure close to the concrete surface is assumed to he similar to a punching shear failure, while the remaining failure part is assumed to be like an anchorage failure for ribbed reinforcing bars placed far from concrete edges.

The results presented here cover the cases where the bar is located far from concrete edges parallel to the axis of the bar, and the bar is subjected to tensile loads only. The angle between the longitudinal direction of the bar and the concrete surface is 90°.

2 Theory

As mentioned, the load carrying capacity of embedded thread bars is determined by use of the theory of plasticity, applying the upper hound theorem. The internal work is divided into two contributions; *part I* is the part close to the concrete surface, and *part II* is the remaining part. The failure mechanism is illnstrated in figure 1. The failure in the part I forms a cone and can be treated as a punching shear failure in a slab with thinkness l_c. This failure will be named as *cone failure* in the following. The failure in the part II area forms a cylinder around and along the bar and can be considered as an anchorage failure. In the following the designation *anchorage failure* will be used for this part of the total failure. In the following the cone and anchorage failure will be treated, but not all details will be given. However, the full theory for the different problems can be found in given references.

The cone failure is treated first. As mentioned above, the cone failure can be considered as a punching shear failure in a plain concrete slab, see figure 2.

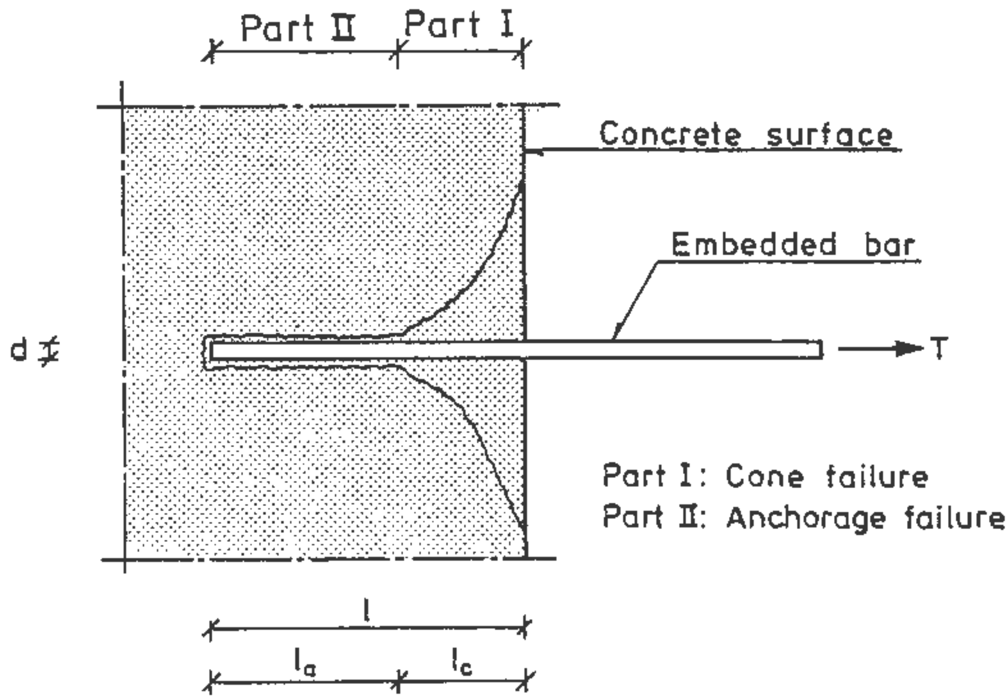

Figure 1: *Failure mechanism for an embedded or bonded bar.*

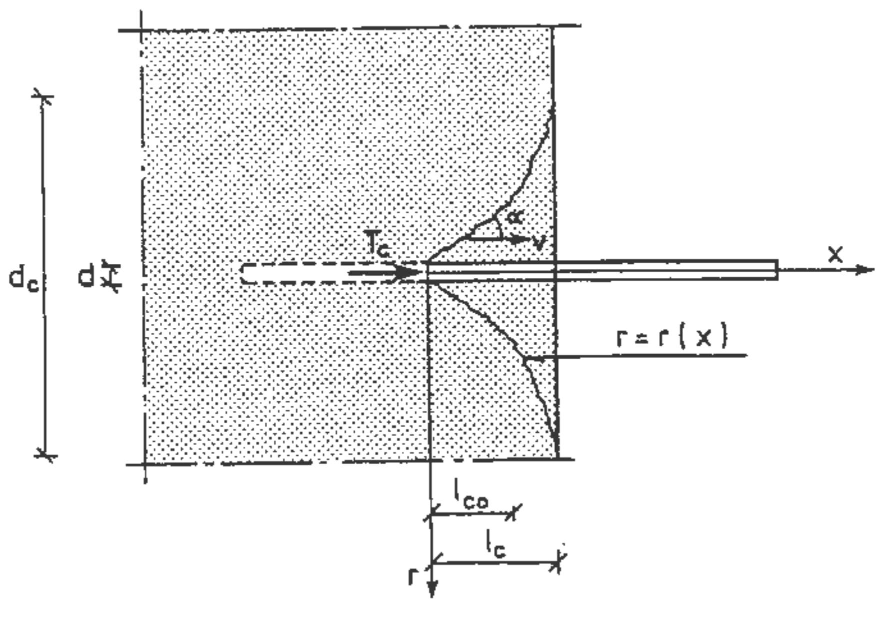

Figure 2: *Failure mechanism for the cone failure (Part I in figure 1).*

Using calculus of variations and the Euler equation it can be found that the optimum solution for the total cone failure (Part I) can be expressed as the sum of two contributions T_{c1} and T_{c2} given by

$$T_c = T_{c1} + T_{c2} \tag{1}$$

$$T_{c1} = \frac{\pi \nu f_c l_{co}(d\cos\varphi + l_{co}\sin\varphi)(1-\sin\varphi)}{2\cos^2\varphi} \tag{2}$$

$$T_{c2} = \frac{\pi}{2} f_c \left(\lambda c(l_c - l_{co}) + \lambda \left(\frac{d_c}{2}\sqrt{\left(\frac{d_c}{2}\right)^2 - c^2} - ab \right) - \mu \left(\left(\frac{d_c}{2}\right)^2 - a^2 \right) \right) \tag{3}$$

where

$$a = \frac{d}{2} + l_{co}\tan\varphi \tag{4}$$

$$\frac{b}{c} = \tan\varphi = \frac{k-1}{2\sqrt{k}} \tag{5}$$

$$c = \sqrt{a^2 - b^2} \tag{6}$$

$$\frac{d_c}{2} = a\cosh\left(\frac{l_c - l_{co}}{c}\right) + b\sinh\left(\frac{l_c - l_{co}}{c}\right) \tag{7}$$

$$\mu = \nu - \rho(k+1) \tag{8}$$

$$\lambda = \nu - \rho(k-1) \tag{9}$$

$$k = 4 \qquad \text{(for concrete)} \tag{10}$$

The plastic solution for the punching shear failure was first given by Bræstrup et al /1/, but a detailed description can also be found in for instance Nielsen /2/.

The anchorage failure along the last part II can he analysed using the failure mechanism illustrated in figure 3.

It can be shown that the angle α in figure 3 is equal to the angle of friction for the concrete φ for thread bars, when the surroundings are strong. The expression for the load carrying capacity for the anchorage failure can then be given as

$$T_a = \pi d l_a \nu f_c \left(\frac{1}{2\sqrt{k}} + \frac{k-1}{2\sqrt{k}} \frac{C}{\nu} \right) \tag{11}$$

where $C = \frac{W_{is}}{\pi d l_a f_c}$ is the dimensionless internal work done in the surroundings, for more explanation see Andreasen /3/.

This solution can also be found by a lower bound calculation valid locally around the bar. It can be shown that the solution (11) requires that $C \geq \frac{\nu}{k+1}(1 - 2k\frac{\rho}{\nu})$. In the case considered here where the bar is located far from concrete edges parallel to the axis of the bar, this

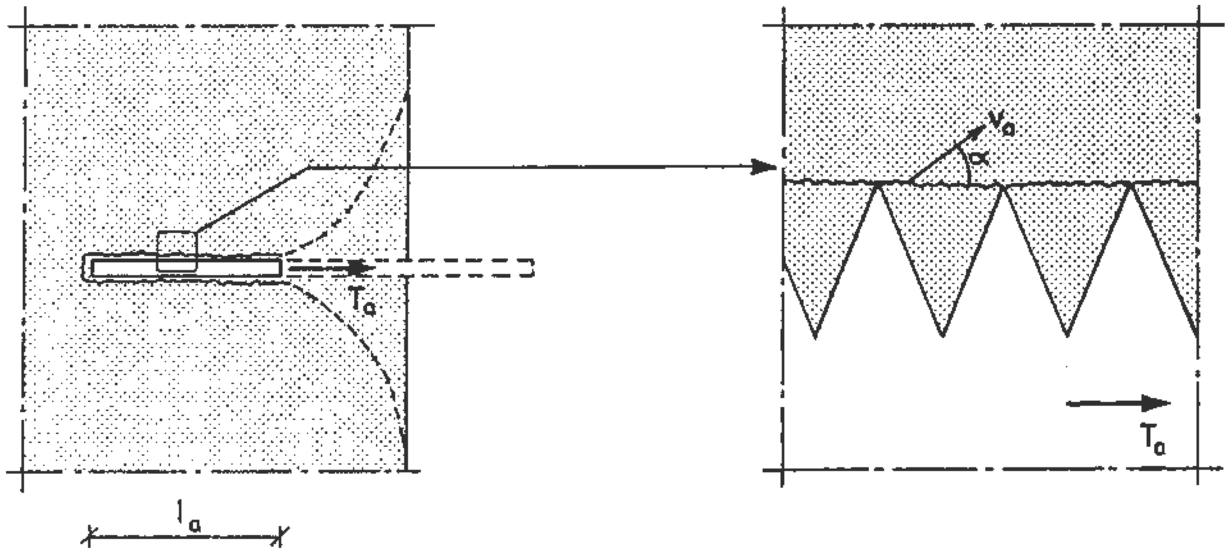

Figure 3: *Failure mechanism for the anchorage failure (Part II in figure 1).*

requirement is always fulfilled. Expression (11) is also valid in the case of anchorage of ribbed reinforcement as for instance Swedish or Danish Kam steel.

Unfortunately, it has not been possible theoretically to determinate the value of the dimensionless internal work from the surroundings. Hence, the value of C has to be determined by comparing the theoretical model with test results.

The expression for the total load carrying capacity of the embedded bar can be given by

$$T = T_a + T_c \tag{12}$$

where T_a is given in (11) and $T_c = T_{c1} + T_{c2}$ from (2) and (3). l_a and l_{co} are free parameters and must be determined so that the minimum value for T in (12) is obtained.

3 Theory compared to test results

Brimnes & Harild /4/ has carried out tests with cast-in-place bonded thread bars placed far from concrete edges parallel to the longitudinal direction of the bar and subjected to direct tension only. A total of 40 tests have been carried out; 10 large concrete blocks with 4 embedded bars in each. The concrete strength (measured on 150x300 mm cylinders) varied between 18 and 47 MPa, the diameter, d, of the bars was constant and equal to 12 mm, and the relative anchorage length $\frac{l}{d}$ ($l = l_a + l_c$) varied between 3 and 14.

In general the bars were of quality 8.8 ($f_{0.2} = 800 * 0.8 = 640$ MPa), but in a few cases where a high load carrying capacity was expected, the quality 12.9 ($f_{0.2} = 1200 * 0.9 = 1080$ MPa) was used. The load and different displacements were measured during each test. For instance the displacement of the bar and the concrete cone relative to the concrete block was measured.

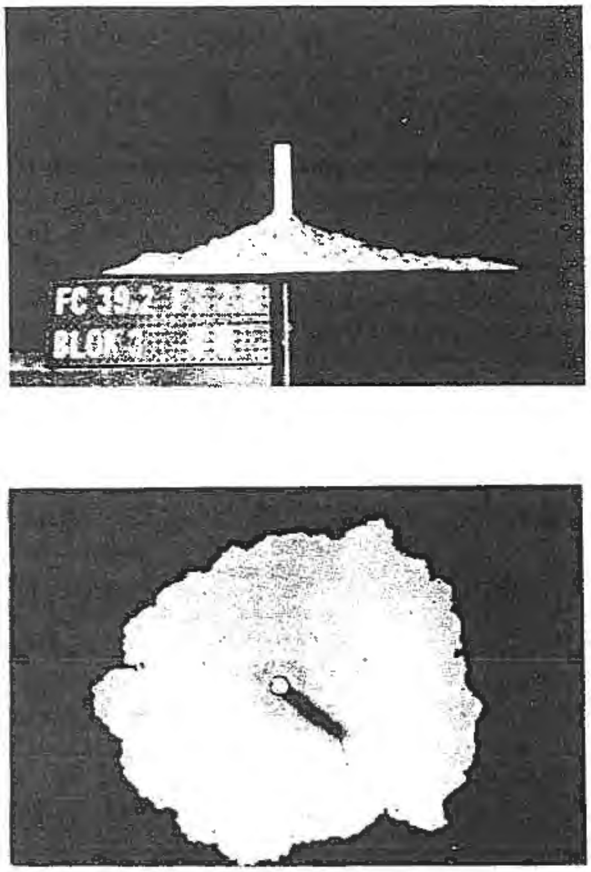

Figure 4: *Thread bar with concrete fixed to it after failure.*

In figure 4 a photo of the thread bar with concrete after failure is shown. As can be seen in figure 4, the failure appearing in the tests is similar to the theoretical failure mechanism illustrated in figure 1.

An example of the force in the emhedded har as a function of the displacement of the bar is shown in figure 5. As noticed the load–deformation curve is similar to the curves obtained from tests with uniaxial compression (or tension) in concrete. After the maximum load is reached the force decreases, hut a considerable force can be carried even though large displacements have occured.

In the comparison between the test results and the theoretical expressions for the load carrying capacity different methods have been used. In the theoretical expressions there are three unknown quantities, the effectiveness factors ν and ρ and the value of the dimensionless internal work C. An estimate for ν, ρ, and C can be obtained by comparing the theoretical expressions with the test results.

Assuming $\nu = \frac{k_\nu}{\sqrt{f_c}}$, $\frac{\rho}{\nu}$ =constant, and $\frac{C}{\nu}$ =constant satisfactory agreement between tests and theory is obtained, and with these assumptions it is possible to simplify the expressions for the load carrying capacity. For

$$\nu = \frac{2.0}{\sqrt{f_c}} \tag{13}$$

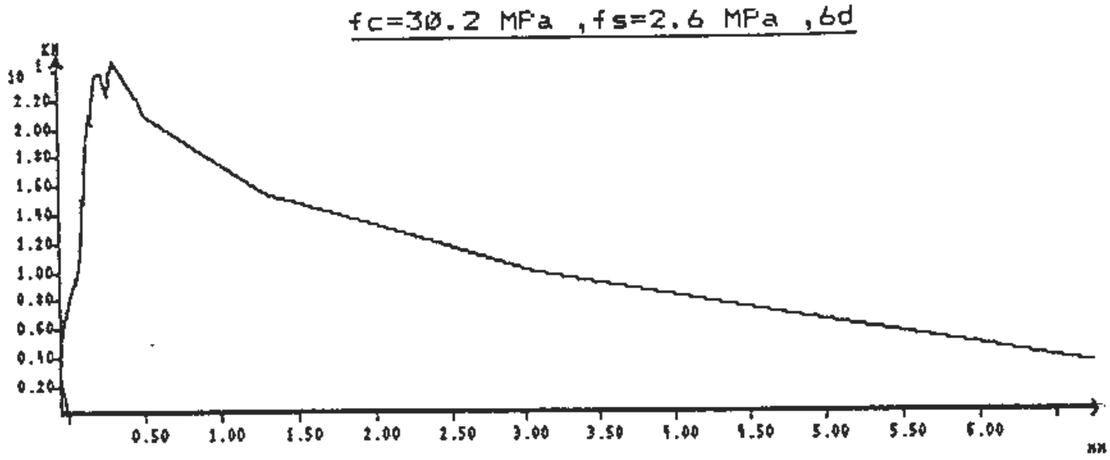

Figure 5: *The force in the bar as a function of the measured displacement of the bar.*

$$\frac{\rho}{\nu} = 0.01 \tag{14}$$

$$\frac{C}{\nu} = 1.05 \tag{15}$$

the mean value and the coefficient of variation on the ratio between T_{test} and T_{theory} are 1.00 and 0.21, respectively. The results from the analysis are given in figure 6.

The theoretical load carrying capacity in figure 6 is obtained using (12) and (13)–(15). Inserting (13)–(15) in the expressions for (12) it is found that the expression for the load carrying capacity can be simplified to

$$\sigma = \frac{T}{\frac{\pi}{4}d^2} = \frac{1}{2}\nu f_c \left(8.3\frac{l}{d} - 9.6\right) \tag{16}$$

where f_c is in MPa. The tests covered the following range of the parameters:

$f_c \in [18.5; 46.9]$ MPa,
$\frac{l}{d} \in [3; 14]$,
and $d = 12$ mm.

Use of the parameters given in (13)–(15) results in a failure mechanism where the geometry for the cone mechanism is independent of the total dimensionless anchorage length l/d and the concrete strength f_c. Therefore, the expressions (1)–(12) can he given in the simple expression (16).

For very small embedment lengths ($l/d < 2.8$) expression (16) is not valid, due to the fact that the failure will only consist of a cone failure. However, (16) is conservative for small values of l/d. A more correct estimate for the load carrying capacity in this case can be found setting $T_a = 0$ in (12) and $l_c = l$ in (1)–(10), valid for the cone mechanism.

In the literature it has been pointed out that the failure mode changes, when the embedment length is larger than 9 to 10 times the diameter of the bar. However, for $f_c \simeq 30$ MPa and $l/d \simeq 12$ the stress in the bar σ is approximately 500 MPa according to (16). This indicates

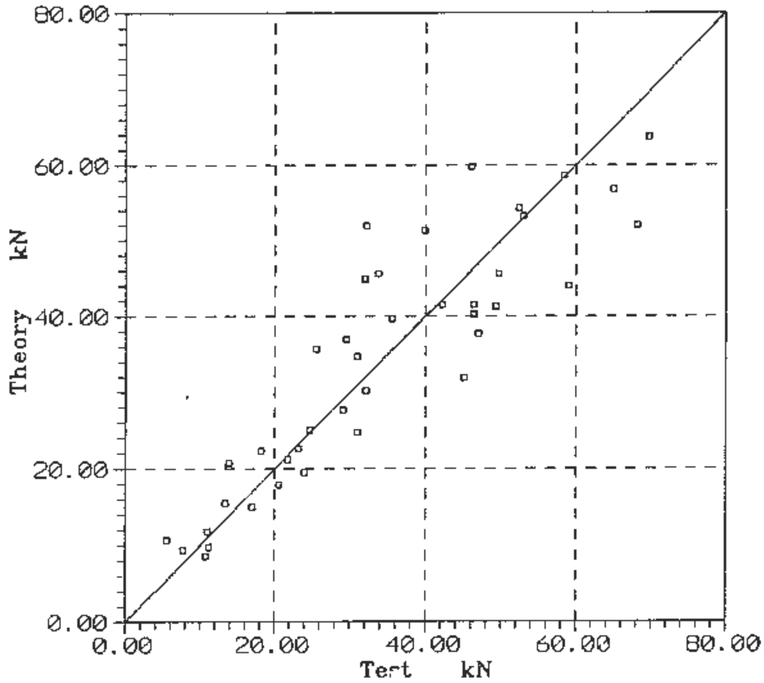

Figure 6: *The load obtained in the tests as a function of the theoretical load carrying capacity using (13)–(15).*

that the deformations in the bar can he quite large when the embedment length is larger than 10 times the diameter of the bar. In the case of long embedment lengths there will be large strains in the bar, and failure in the concrete near to the concrete edge will start for a load lower than the failure load. The strains at failure in this part will be large, reducing the value of the effectiveness factor ν, because concrete not is an ideally plastic material. Including l/d in the expression for the effectiveness factor this effect could be taken into account. However, for normal strength steel the failure for long embedment lengths will in most cases take place in the steel. In the tests from Brimnes & Harild /4/ used to develop (16) the steel quality was 12.9 for $l/d > 10$ to ensure failure in the concrete. The decreasing effect of the geometrical parameter l/d for large embedment lengths is not included here.

Eligehausen et al /5/ has investigated the load carrying capacity of drilled–in thread bars. The adhesive mortar was of the capsule–type based on polyester resin. On the basis of approximately 90 test results an expression is set up for the strength for $l/d \simeq 9$. Using the cylinder strength for the concrete the expression can he written

$$T = 0.92l^2\sqrt{f_c} = 74.5d^2\sqrt{f_c} \tag{17}$$

where for l or d in mm and f_c in MPa T is in N.

In figure 7 this expression, indicated by T_{CEB}, is shown together with the test results used. The expression (17) and the test results are also reported in Rehm et al /6/ and CEB 206 /7/.

Inserting $l/d = 9$ in (16) the expression can be written as

$$T = 0.63l^2\sqrt{f_c} = 51.1d^2\sqrt{f_c} \tag{18}$$

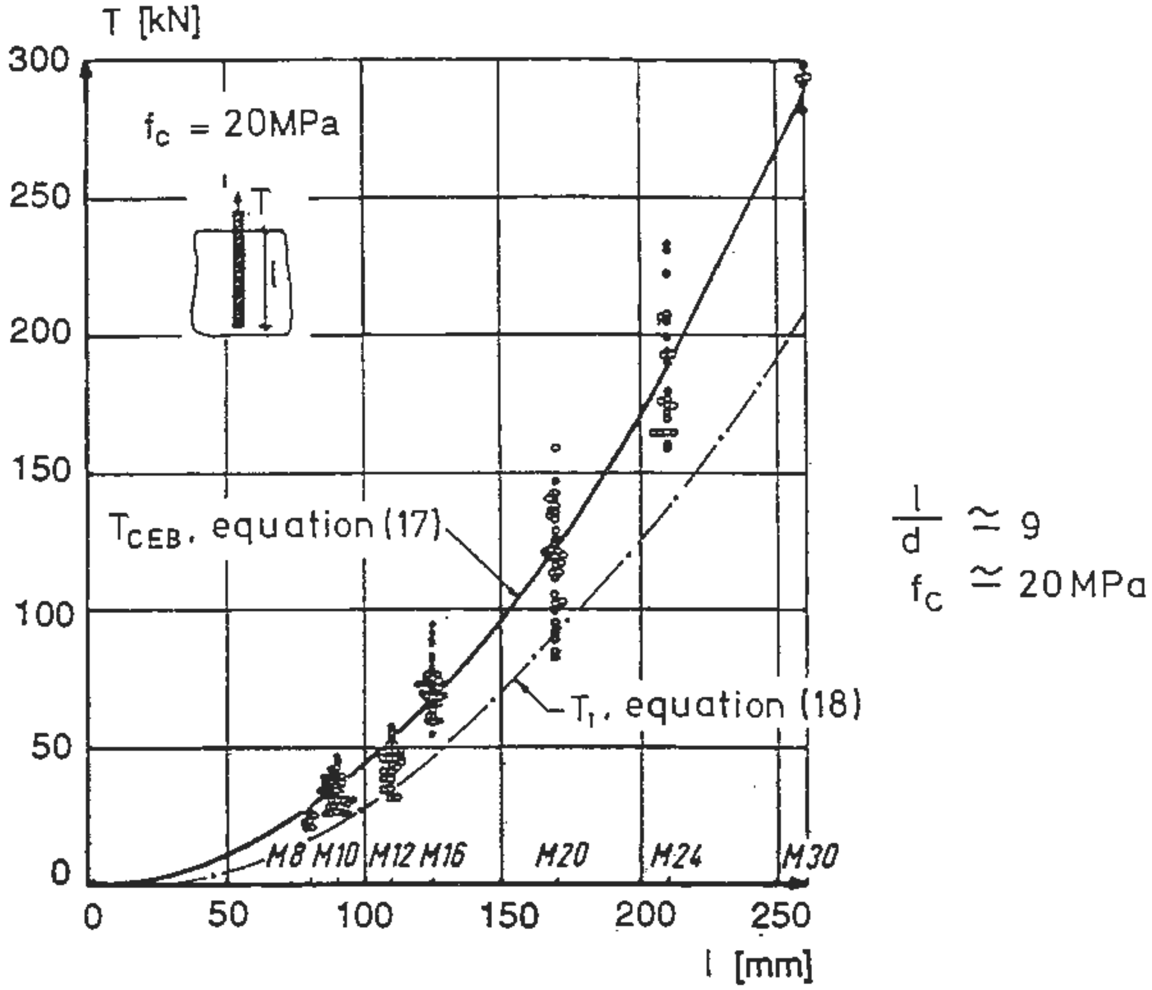

Figure 7: *Load T as a function of the embedment depth for test results and expression (17), T_{CEB}, from Eligehusen et al /5/, and expression (18) from this paper, T_1.*

d	mm	-	8	10	12	16	20	24	Mean
δ	mm	0	2	2	2	2	5	4	-
factor k	-	51.1	80	74	70	65	80	70	73

$$T = kd^2\sqrt{f_c} = 51.1d^2\left(1+\frac{\delta}{d}\right)^2\sqrt{f_c}$$

Table 1: *Values for the difference between drilling hole and bar diameter δ from HILTI /8/.*

Comparing (18) with (17) it is observed that the only difference is the value of the constant factor. In figure 7 expression (18) with $f_c = 20$ MPa is plotted. As can be seen, the load predicted by (18) is too small compared to (17) and the tests. This difference is mainly due to the fact that drilled–in, bonded anchors usually fail in the interface between concrete and grout. The failure surface is therefore different in the two cases. Including this effect expression (18) can be written

$$T = 51.1(d+\delta)^2\sqrt{f_c} \tag{19}$$

where δ is the difference in the diameter for the bar and the drilling hole. In table 1 values for δ for HILTI /8/ bonded anchor HVA, HAS are given.

As can be seen from table 1, the average of the constant factor taken account of the difference in diameter is approximately 73. Comparing with expression (17) it is seen that the difference between the two expressions is in this case reduced to approximately 2%. This indicates that the theoretical expression shown in this paper is valid also in the case of drilling in bonded anchors.

The expression for bonded anchors presented in CEB 206 /7/ found from tests is in principle identical with the expression shown here based on a theoretical approach. However there are at least two important differences. In the first place the CEB expression is only valid for the special case $l/d \simeq 9$. Secondly the safety level using the CEB expression is not the same as using the expression from this paper, because the effect that a high strength concrete is less plastic than a lower strength concrete not is considered in the empirical model. In the plastic based model the factor νf_c takes this effect into account. The effectiveness factor ν must be determined from the caracteristic value of $f_c = f_{ck}$ and f_c in νf_c as the design value of $f_c = f_{cd}$.

4 Summarized Comments

The theory of plasticity has been used to treat the load carrying capacity problem with cast-in-place thread bars (embedded bars) placed far from concrete edges parallel to the longitudinal direction of the bar.

The theoretical expression for the load carrying capacity is compared to test results, and satisfactory agreement is obtained. By making certain assumptions the theoretical expressions can be simplified into a very simple expression given in (16).

The results from the investigation given in this paper indicate that it is possible to treat the load carrying capacity problem of thread bars embedded in concrete by use of the theory of plasticity. However, many problems still have to be investigated before final design recommandations can be given. For instance tests with different diameters of the bars, ribbed reinforcing bars, drilled–in thread and reinforcing bars, located close to a concrete edge, more bars located close to each other and shear forces have to be investigated. But also the influence from temperature and dynamic load is important.

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