An energy based model for predicting the elastic tensile strain capacity of high performance Fiber Reinforced Composites (FRC)

by

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Abstract

In this paper a model is derived for predicting the increase in tensile strain capacity at first cracking of FRC with high volume fraction (i.e. typically 2% or higher) discontinuous fibers, randomly distributed throughout the matrix. The model is derived from energy principles; an approach similar to that used by Aveston Cooper and Kelly (ACK) for continuous FRC. Due to the increased complexity in obtaining the energy contribution, in discontinuous FRC, from fibers and matrix to overall energy absorption at first cracking (i.e. elastic limit) the two models are quite different.

The model developed in this paper predicts that the tensile strain capacity of a brittle cement-based matrix can be improved several fold by the addition of high volume fraction of fine and stiff fibers provided that the matrix exhibits high bond strength. This can been achieved in a DSP (Densified Small Particles) matrix which has a compressive strength greater than 150 MPa, and which can be processed with high fiber volume fraction.

The proposed model was validated from a limited experimental program in which beams were made out of DSP mortar containing 0%, 3%, 6%, 9% and 12% of short (6 mm) and fine (0.15 mm diameter) steel fibers and tested in flexure. Results showed that the elastic strain capacity increased from 150 microstrain at 0% fiber content to 470 microstrain at 12% fiber volume fraction.

Key words: concrete, modeling, FRC, elastic tensile strain capacity.

1. Introduction

In conventional FRC improvements in tensile strain capacity over that of the plain matrix is insignificant. Fibers are used mainly to improve the post-cracking ductility associated with the pull-out of the fibers bridging the failure crack.

With the advancement of high performance fiber reinforced cement-based composites improvements in total strain capacity (elastic and in-elastic) by a factor of 20 to 40 times over that of the plain matrix has been observed /1-6/. It is common for these composites that they contain large volume fraction of fibers (i.e. 20% or more in the case of SIFCON (Slurry Infiltrated Fiber Concrete)). However the mechanism for the tensile strain improvements of FRC containing discontinous fibers is not well understood.

In this study a model is derived from energy principles for predicting the improvement in elastic tensile strain capacity of FRC containing discontinous fibers, and the model is evaluated from a limited number of test results obtained in flexure on steel fiber reinforced DSP.

2. Background

There are few studies in the literature which have been concerned with elastic strain modeling of FRC with discontinuous fibers. This may be attributed to the small effect the conventional fiber volume fractions (of 0 to 2%) have on elastic strain capacity of the composite. From these studies, equations for predicting the elastic strain capacity of FRC are based on either mechanics of composite theory or energy relations during loading as introduced by Aveston Cooper and Kelly in 1971 / 7/.

There are two mechanics of composite-based predictions that have been widely used; the equation by Naaman /4, 5/ as shown in equation 1, and the expression by Swamy & Mangat /6, 8/ as shown in equation 2.

In Naaman's equation the cracking stress of the composite, σ_c, is given by:

$$\sigma_c = \sigma_m (1-V_f) + \alpha_1 \alpha_2 \tau V_f \; L/d \tag{1}$$

where α_1 is a bond coefficient representing the fraction of bond mobilized at matrix cracking, α_2 is the efficiency factor of fiber orientation in the uncracked state of the composite, τ is the bond strength, V_f is the fiber volume fraction, L is the fiber length, d is the fiber diameter, and σ_m is the matrix stress. The elastic strain can be obtained by dividing the elastic stress (equation 1) with the modulus of elasticity of the composite.

According to Swamy and Mangat:

$$\sigma_c = \sigma_m (1-V_f) + 0.82 \tau \; L/d \; V_f \tag{2}$$

The energy/ACK-based equations use the ACK /7/ model (equation 3), which was derived for composites with continuous fibers by applying a correction factor *k*. This factor is called the orientation factor which takes into account the random orientation of discontinuous fibers /9-12/. The ACK equation for the elastic strain is given as:

$$\varepsilon_{mu} = k \left\{ \frac{12 \tau \gamma_m E_f V_f^2}{E_c E_m^2 r V_m} \right\}^{\frac{1}{3}} \tag{3}$$

where τ is the bond strength, γ is the surface energy, E and V represent the moduli and volume fractions respectively; r is the fiber radius, and the subscripts m,c and f represent matrix, composite and fiber respectively. In the literature, the values of the orientation factor can vary from 0.5 /10/ to 0.27 /12/ within the same fiber systems. These values were reported for systems reinforced with glass fibers. In /13/ it was shown that *k* varied widely as well for FRC containing polypropylene, carbon, and steel fibers. It appears that the ACK equation cannot be used to predict the tensile strain capacity of FRC containing discontinuous fibers.

3. Model derivation

A model for predicting the elastic (cracking) strain of composites reinforced with discontinuous-short fibers is derived using the energy concepts similar to that used in the ACK model. This model was derived as part of a study for understanding the multiple cracking process in composites reinforced with discontinuous fibers /13, 14/. The multiple cracking phenomena can be explained by comparing the energy required to form a crack with the energy required to open that particular crack.

Energy changes due to the occurrence of the first crack are obtained by evaluating the different energy terms before and after the occurrence of a crack as shown in Figure 1. From the figure, it is evident that the major difference between the ACK model and the proposed model for discontinuous fiber composites is in the combination of the different energy terms associated with internal energy changes due to the occurrence of a crack. The ACK model combines all the energy terms at the elastic limit. In this study the terms associated with debonding and friction are separated according to the timing of their occurrence during the cracking process. This means that a condition of no debonding is assumed at the onset of cracking. It will be shown how the effect of partial debonding at first cracking can be taken into account as well.

The contribution of the following energy changes are considered just beyond the elastic range:

(1) The fracture energy of the matrix, which is the energy required to create new surfaces ($2\gamma_m V_m$); where γ_m is the matrix surface energy.

(2) The increase in the fiber strain energy, ΔU_f, as the result of the crack-bridging action of the fibers.

(3) The decrease in the matrix strain energy, ΔU_m, since the strain in the matrix at the crack face will be reduced to zero.

(4) The change in external work, $\Delta W_{external}$, due to the elongation of the fiber as the result of the crack-bridging action of the fibers.

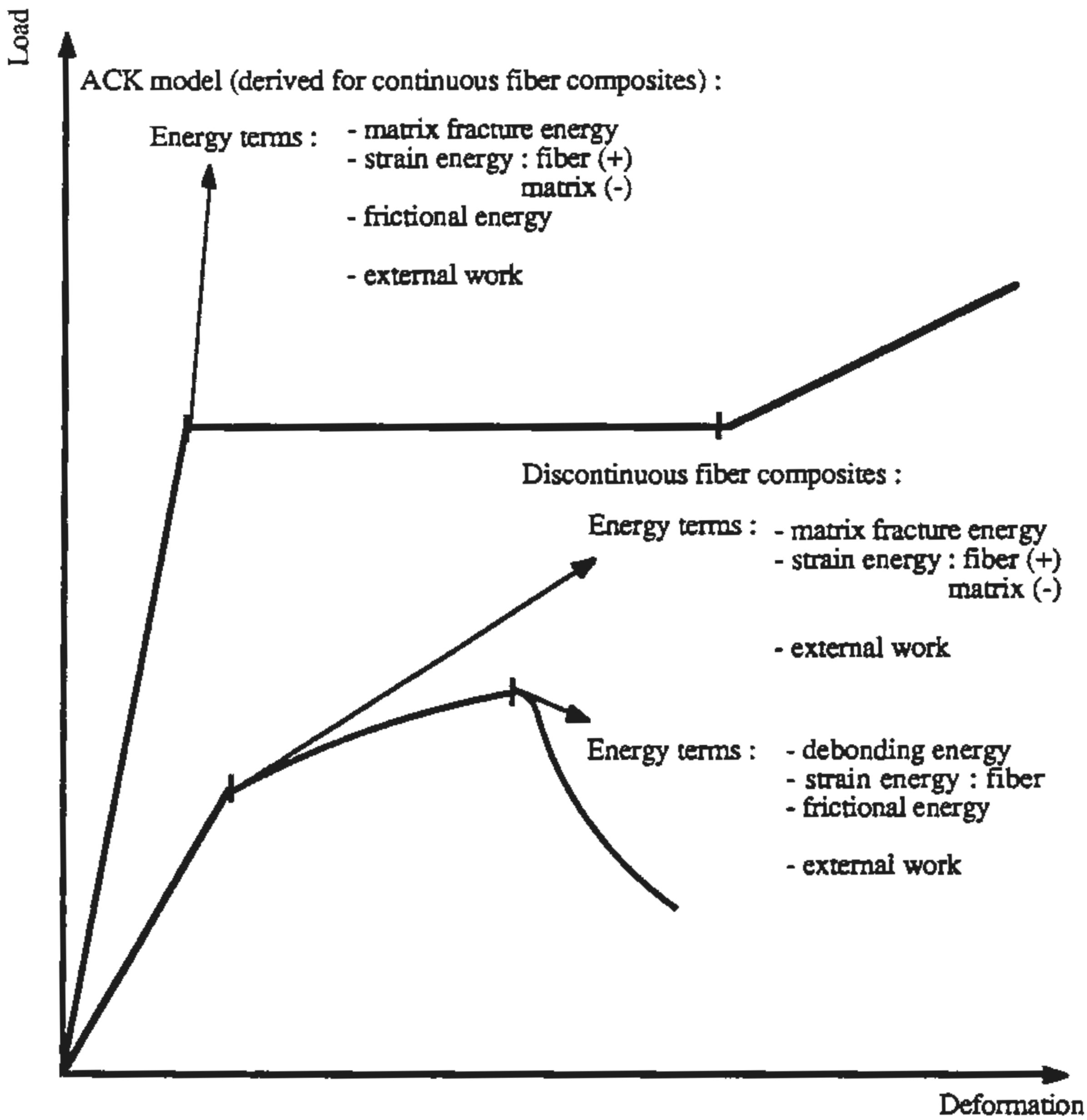

Figure 1 Different energy terms considered in the model derivation

The changes in the external work will be assumed to be equal to the changes in internal energy; the energy balance equation is given by :

$$\Delta W_{external} = 2\gamma_m V_m + \Delta U_f - \Delta U_m \qquad (4)$$

Other differences with the ACK model are associated with the nature of the problem such as the discontinuity of fibers and the randomness of the short fibers. The discontinuity will result in a more complex strain distribution in the composite and the randomness will result in a smaller volume fraction of fiber which effectively are bridging the crack. Some assumptions with regard to these problems will have to be made and they are discussed in the following section.

3.1 Assumptions:

The model was derived for a fiber reinforced composite under tensile loading (Figure 2). The assumptions used in the derivation of the model are:

3.1.1. No fiber debonding:

In the model derivation, the energy terms associated with debonding and friction were not included in the energy balance equation (equation 4) for the first cracking point. This means that a condition of no debonding is assumed at the onset of cracking. This assumption is reasonable since the stress-strain curve of FRC is always linear up to the first cracking point, i.e. there is no mismatch between fiber's and matrix's deformation. The validity of this assumption can be verified using expression from the shear-lag theory. If shear-lag analysis shows that some debonding has occurred, the effect of partial debonding can be included in the model as an approximation by adding the contribution of partial debonding energy in the model /13/.

3.1.2. Fiber length and transfer length:

A condition of short fibers is assumed, i.e. that the fibers are fully stretched. This means that the fibers will utilize all of their lengths to transfer the additional load, due to cracking, back to the matrix (i.e. transfer length is assumed to be equal to fiber length). Transfer lengths can be calculated from force equilibrium in a fiber after matrix cracking assuming frictional interfacial stress. The transfer length can not be incorporated directly in the derivation since it in turn is a function of the elastic strain (cracking strain) of the composite as shown in equations 5 and 6.

$$L_{tr} = \frac{d\,\sigma_f}{4\tau} \qquad (5)$$

where: d is the fiber diameter

τ is a constant frictional interfacial stress

and: σ_f is the fiber stress after cracking, given by :

$$\sigma_f = \varepsilon_{mu}\,(1+\alpha)\,E_f \qquad (6)$$

where
$$\alpha = \frac{E_m V_m}{E_f V_f} \qquad (7)$$

and ε_{mu} is the elastic strain of the composite.

By combining equations 5, 6 and 7 the transfer length is calculated from:

$$L_{tr} = \frac{d\,\varepsilon_{mu}\,(1+\alpha)\,E_f}{4\,\tau} \qquad (8)$$

It is not possible to solve equation 4 for the general case in which the transfer length is unknown. Thus solving for ε_{mu} becomes a non-linear problem since the transfer length is needed in the derivation of the elastic strain. In the first step the elastic strain is approximated assuming that the transfer length is equal to half the fiber length ($L_f/2$). In the second step a correction factor will be included in the equation to take into account the actual transfer length.

3.1.3 Strain in the composite, fiber, and matrix:

Just before the occurrence of the first crack the strain in the composite (ε_c) is assumed to be uniform with a magnitude of ε_{mu} (Figure 2).

Just before cracking

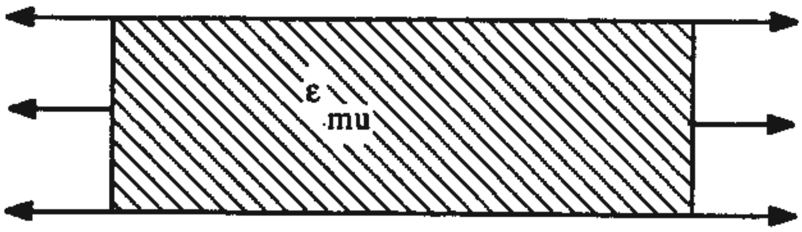

Figure 2 Strain in composite just before the occurrence of the first crack

Just after the occurrence of the first crack, it is assumed that strain localization does not take place in the composite. This means that the strain is the same throughout the composite at a certain distance away from the crack. However, the strain in the matrix and in the fiber in the area around the crack are altered by the presence of the crack (Figure 3). From the first assumption, the total length of this area will be equal to the fiber length. The strain in the matrix at the crack face is zero and it increases away from the crack face. The reverse trend is true for the fiber, its strain will be maximum at the crack face and decreases along its length, see Figure 3.

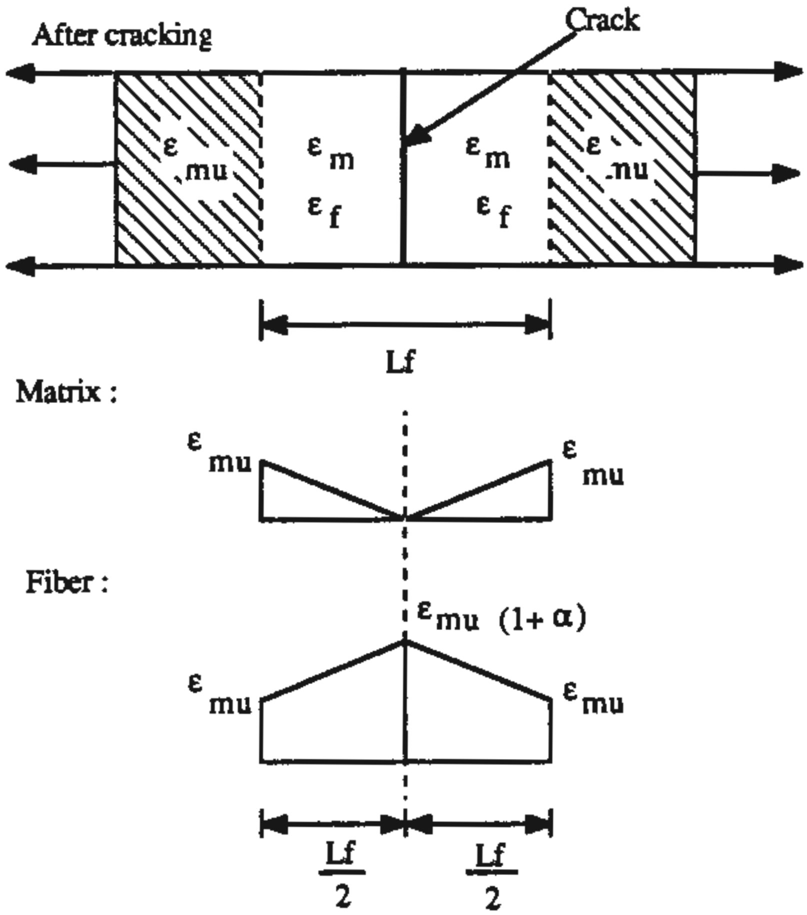

Figure 3 Strain in the composite, matrix, and fiber after the occurrence of the first crack

It should be pointed out that the fiber strain assumption shown in Figure 3 ignores both the non-linearity distribution of strain along the fiber length and the strain discontinuity at fiber ends. At fiber exit point, i.e. at the crack face, the strain in the fiber decreases from ε_{mu} (1+ α) to ε_{mu} at the fiber end due to the stress transfer process. However, in reality, this transfer process needs a certain length for the development of the interface shear stress between the fiber and the matrix from zero (at the crack face) to a certain value. This means the fiber stress will not decrease right away, instead it will maintain the strain at crack face, ε_{mu} (1+ α), along a certain length (Figure 4).

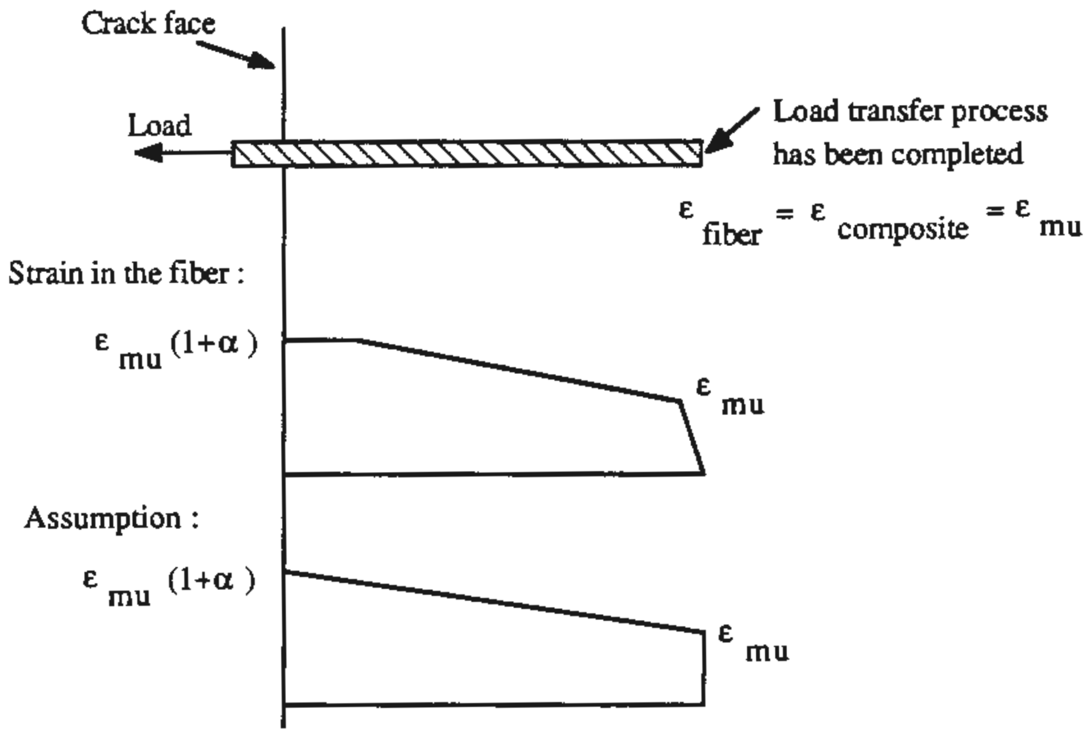

Figure 4 Strain in the fiber just after cracking

A similar strain discontinuity occurs also at the other end of the fiber, where the strain has to decrease from a certain value to zero along a certain length (Figure 4). In the assumption it is assumed that this length is negligible and that the stress transfer process has been terminated, which means that the strain at the fiber end is assumed to be the same as the strain in the composite and in the matrix which is ε_{mu}.

3.1.4. The effect of fiber embedment length:

The effect of different fiber embedment lengths is taken into account by assuming average strain values in the matrix and fiber. The strain in the matrix will vary from a value of ε_{mu} (for embedment length of $L_f/2$) to 0 (for embedment length of zero) which gives an average value of $\varepsilon_{mu}/2$. Similarly, the strain in the fiber will vary from

$\varepsilon_{mu}(1+\alpha)$ (for small values of embedment length) to ε_{mu} (for embedment length of $L_f/2$) which gives an average value of $\varepsilon_{mu}(2+\alpha)/2$.

As a summary, from assumptions 3.1.2 to 3.1.4 the following strain relationship is assumed in the model:

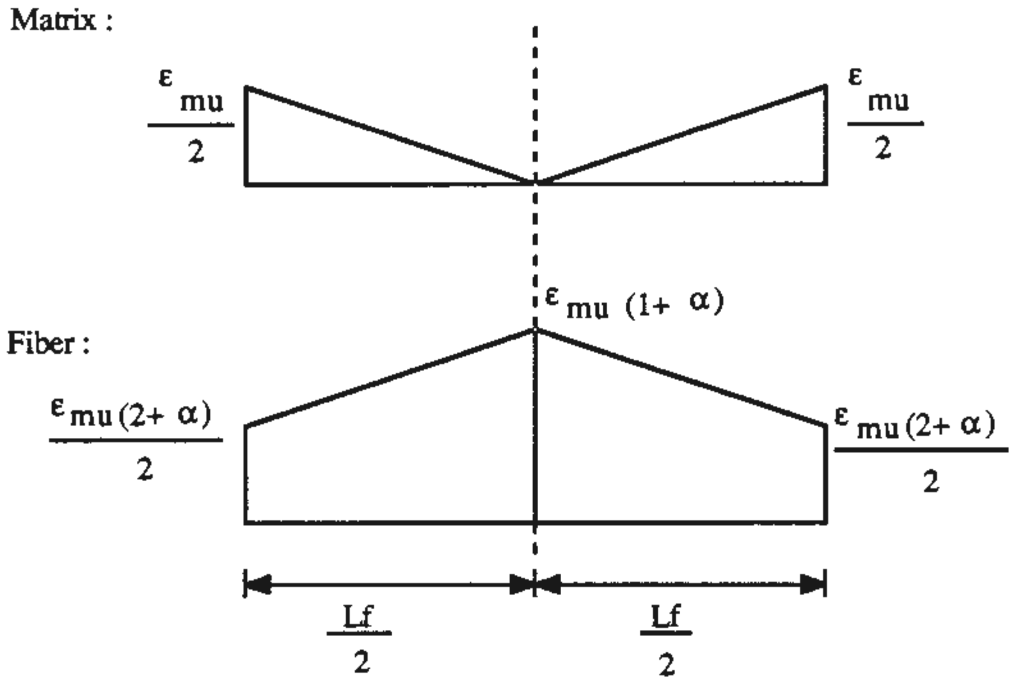

Figure 5 Matrix and fiber strain assumptions used in the model derivation

3.1.5. The effect of the randomness in fiber orientation and location to a crack plane:

The number of fibers effectively bridging a crack is half of the total number of fibers that may intercept the crack plane. This assumption is needed to take into account the random orientation of the fiber. Thus, the effective fiber volume fraction is half of the nominal fiber volume fraction /15, 16/. In this paper the effective fiber volume fraction is denoted by V_{ef}.

4. Energy terms evaluation

Using the assumptions in the previous section each of the energy terms in the energy balance equation (equation 4) can be evaluated:

Increase in the fiber strain energy (ΔU_f) :

It is the fiber strain energy after the occurrence of the first crack minus the strain energy before the occurrence of the crack. Using fiber strain assumption shown in Figure 5, the following expression was obtained:

$$\Delta U_f = [\int_0^{L_f/2} \frac{1}{2} E_f \varepsilon_f^2(x) dx - \int_0^{L_f/2} \frac{1}{2} E_f \varepsilon_{mu}^2 dx] 2 V_{ef} \quad (9)$$

where :

$$\varepsilon_f(x) = \varepsilon_{mu}(1+\alpha) - \frac{\alpha \varepsilon_{mu}}{L_f} x \quad (10)$$

Substituting expression 10 into expression 9 and carrying out the integration the following term is obtained:

$$\Delta U_f = \frac{1}{24} E_f V_{ef} L_f \alpha(18+7\alpha) \varepsilon_{mu}^2 \quad (11)$$

Decrease in the matrix strain energy (ΔU_m) :

It is the matrix strain energy after the occurrence of the first crack minus the strain energy before the occurrence of the crack. Using the fiber strain assumption shown in Figure 5, the following expression was obtained:

$$\Delta U_m = [\int_0^{L_f/2} \frac{1}{2} E_m \varepsilon_{mu}^2 dx - \int_0^{L_f/2} \frac{1}{2} E_m \varepsilon_m(x) dx] 2 V_m \quad (12)$$

where :

$$\varepsilon_m(x) = \frac{x}{L_f/2} \frac{\varepsilon_{mu}}{2} \quad (13)$$

ΔU_m is obtained by substituting expression 13 into expression 12 and performing the integration.

$$\Delta U_m = \frac{11}{24} E_m V_m L_f \varepsilon_{mu}^2 \quad (14)$$

The change in external work :

It is the change in external work associated with the change in length of the composite:

$$\Delta W_{external} = E_c \, \varepsilon_{mu} \, \Delta l \qquad (15)$$

where E_c is the modulus of elasticity of the composite, ε_{mu} is the elastic strain of the composite, and Δl is the length change of the composite due to fiber elongation.

$$\Delta l = \int_0^{L_f/2} [\, \varepsilon_f(x) - \varepsilon_{mu} \,] \, 2 \, dx \qquad (16)$$

where $\varepsilon_f(x)$ is given by equation 10.

The change in external work, shown in expression 17, was obtained using expressions 10, 15, and 16.

$$\Delta W_{external} = \frac{3}{4} E_c \, \alpha \, L_f \, \varepsilon_{mu}^2 \qquad (17)$$

5. **Elastic strain modelling:**

Step 1:

The equation for predicting the approximated elastic strain, ε_{mu}^*,(i.e. where it is assumed that the transfer length is equal to half the fiber length) can be obtained by substituting the energy terms in expressions 11, 14, and 17 into the energy balance equation as shown in equation 4. After rearranging the terms and solving for ε_{mu}^* the following equation is obtained :

$$\varepsilon_{mu}^* = \sqrt{\frac{2 \, \gamma_m \, V_m}{[\frac{3}{4} E_c - \frac{7}{24} E_f \, V_{ef} \, (1+\alpha)] \, \alpha \, L_f}} \qquad (18)$$

It is important to note that V_{ef} in equation 17 is the effective fiber volume fraction, i.e. half of the nominal value of fiber volume fraction.

Step 2:

Knowing the approximate value of ε_{mu}, the transfer length can be calculated using equation 8. A correction factor, ß, which is the ratio of the calculated transfer length

(L_{tr}) to the half-fiber length ($L_f/2$), is then used to include the effect of transfer length by multiplying the fiber length L_f with ß in equation 18. The rationale is that if a value of $L_f/2$ for transfer length as used in the model derivation is incorrect, a correction is needed. The implicit assumption is that this can be accomplished by simply correcting the term L_f in equation 18. This of course is a simplification.
Hence, β can be calculated as follows:

$$\beta = \frac{L_{tr}}{L_f/2} \quad (19)$$

Knowing the value of β, the approximated soultion for the elastic strain value (ε_{mu}) can be calculated using equation 20.

$$\varepsilon_{mu} \approx \sqrt{\frac{2\,\gamma_m\,V_m}{[\frac{3}{4}E_c - \frac{7}{24}E_f\,V_f\,(1+\alpha)]\,\alpha\,(\beta\,L_f)}} \quad (20)$$

5. Model validation

The model is evaluated using data for fiber reinforced DSP obtained in reference /13/.

The elastic strain values for different fiber volume fractions of the DSP system were calculated, using equations 18 and 20, and compared to experimental results (Figure 6). The inputs used are as follows: (1) $2\gamma_m$ value of 120 N/m which is obtained from fracture energy tests on matrix carried out in this study using notched beam specimens; (2) the geometric mean of bulk modulus was used for E_c with E_m value of 49,100 MPa (obtained from compression tests on matrix); (3) an E_f value of 200,000 MPa; and (4) L_f value of 6 mm.

The geometric mean of the upper and lower limits of bulk modulus /17, 18/ was used to estimate E_c at the different fiber volume fractions.

The predictions based on Naaman's model (equation 1) and Swamy&Mangat's model (equation 2) are also shown in Figure 6. For Naaman's model, an α_1 and α_2 values of 1 and 0.5 respectively were assumed. In the calculation, τ values of 5 MPa and 10 MPa were assumed for the DSP composite. The rationale is that, for steel fiber, the bond strength is about 5% of compressive strength of matrix material. Therefore, for

DSP matrix with compressive strength of 150 MPa, the bond strength is expected to be between 5 MPa and 10 MPa. The aspect ratio (L_f/d_f) is 40 (L_f and d_f values of 6 mm and 0.15 mm respectively).

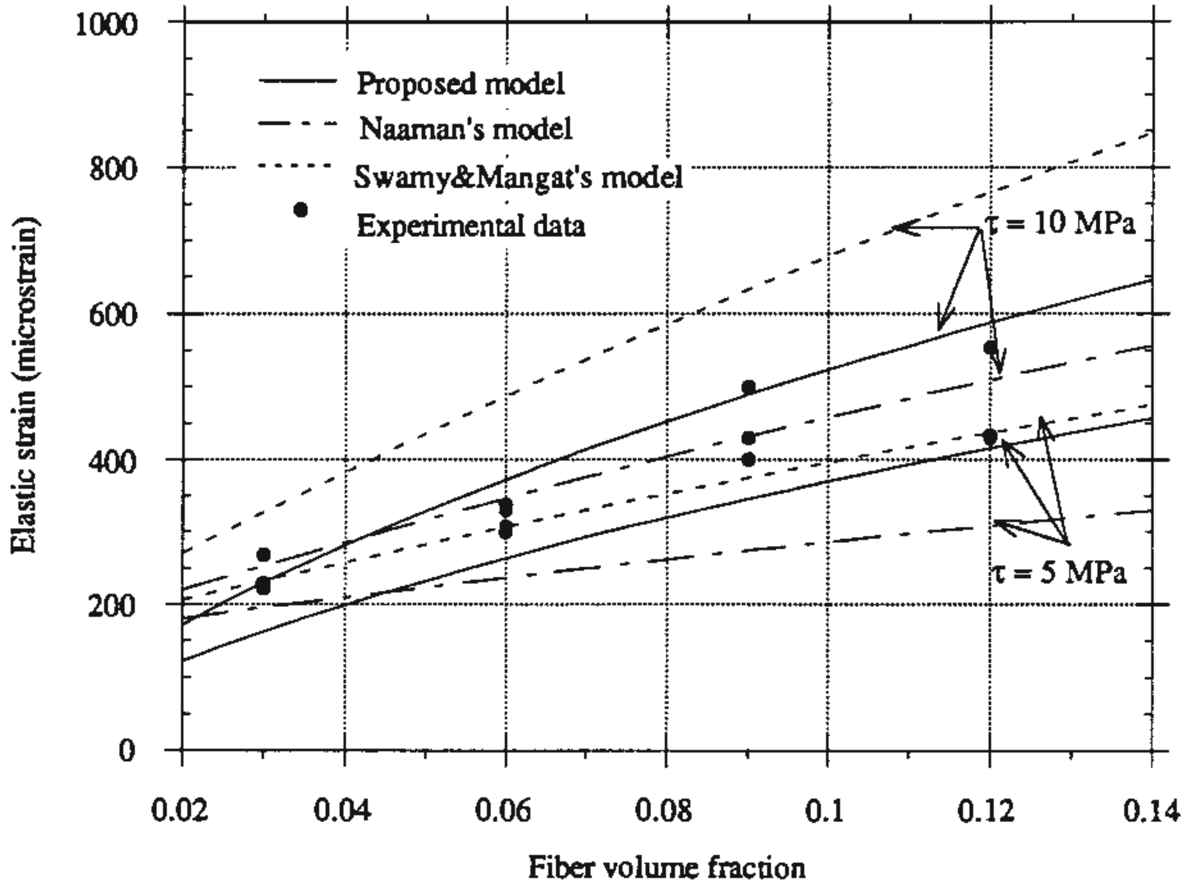

Figure 6. Comparison between experimental data and the proposed model, and the models by Naaman and Swamy&Mangat for fiber reinforced DSP

In general, there is good agreement between the strain values calculated using the proposed model and experimental data. Naaman's model (for τ value of 10 MPa) and Swamy&Mangat's model (for τ value of 5 MPa) also result in good agreement with the data.

Figure 6 shows that the elastic strain capacity increases with increasing fiber content. For V_f of 12% an average elastic strain value of 470 microstrain is experimentally obtained. Compared to the plain matrix this corresponds to an increase of almost

300%. This is a very important characteristic of fiber reinforced DSP in applications where crack-free condition becomes an important parameter.

An increase in scatter can be observed at 9% and 12% fiber volume fractions. This may be the result of incomplete compaction.

Also evident from Figure 6 is the effect of τ on the calculated elastic strain values. The proposed model predicts that an increase of τ from 5 MPa to 10 MPa will result in a 50% increase of elastic strain for high fiber volume fraction. This shows the importance of the interface properties on the elastic strain values.

6. The effect of fiber characteristics on elastic strain

Predictions based on the proposed model for fiber lengths ranging from 6 mm to 24 mm for DSP composite are shown in Figure 7. From the figure it can be seen that the predicted elastic strain increases with increasing fiber length, however, at a decreasing rate. Increasing the fiber length by twofold, from 6 mm to 12 mm, will result in an increase in elastic strain of about 16% at V_f of 12%; however, a fourfold increase in L_f (from 6 mm to 24 mm) will result in a 35% improvement in elastic strain capacity.

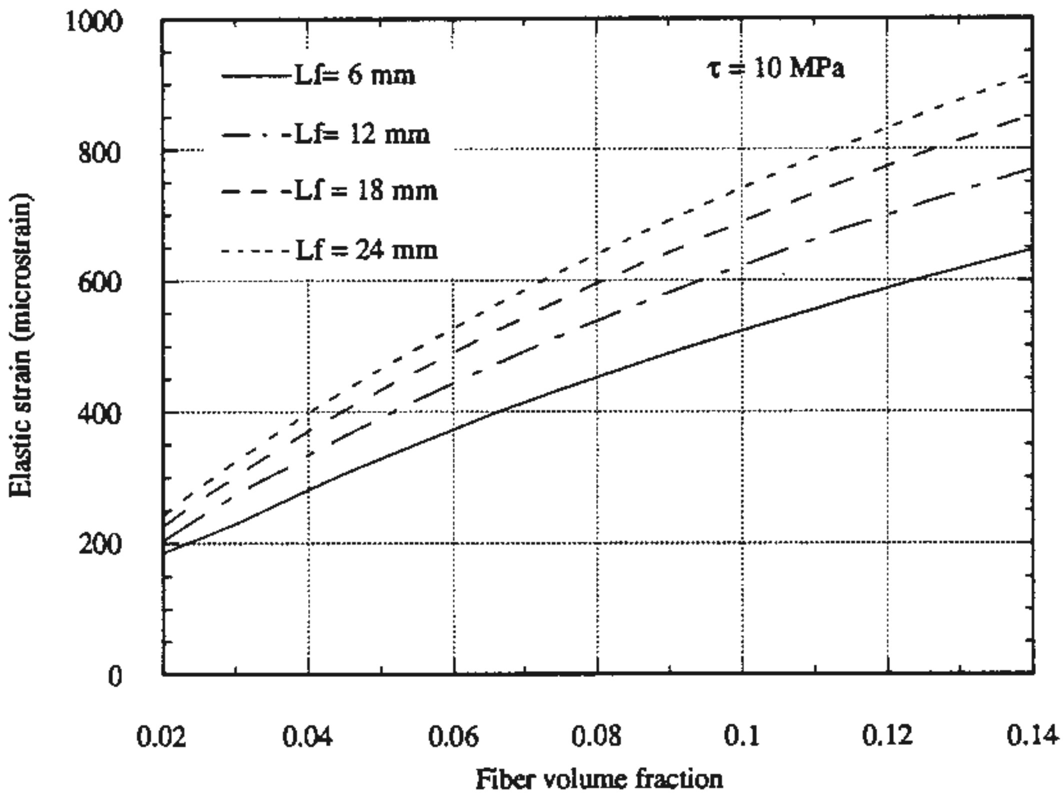

Figure 7 The effect of fiber length on elastic strain for DSP reinforced with small diameter (0.15 mm) steel fibers.

Predictions based on the proposed model for fiber diameters of 0.15 mm, 0.25 mm, and 0.5 mm are shown in Figure 8. The model predicts that the elastic strain values will increase with decreasing fiber diameter. This trend is also predicted by Naaman and Swamy&Mangat models, and by the ACK model.

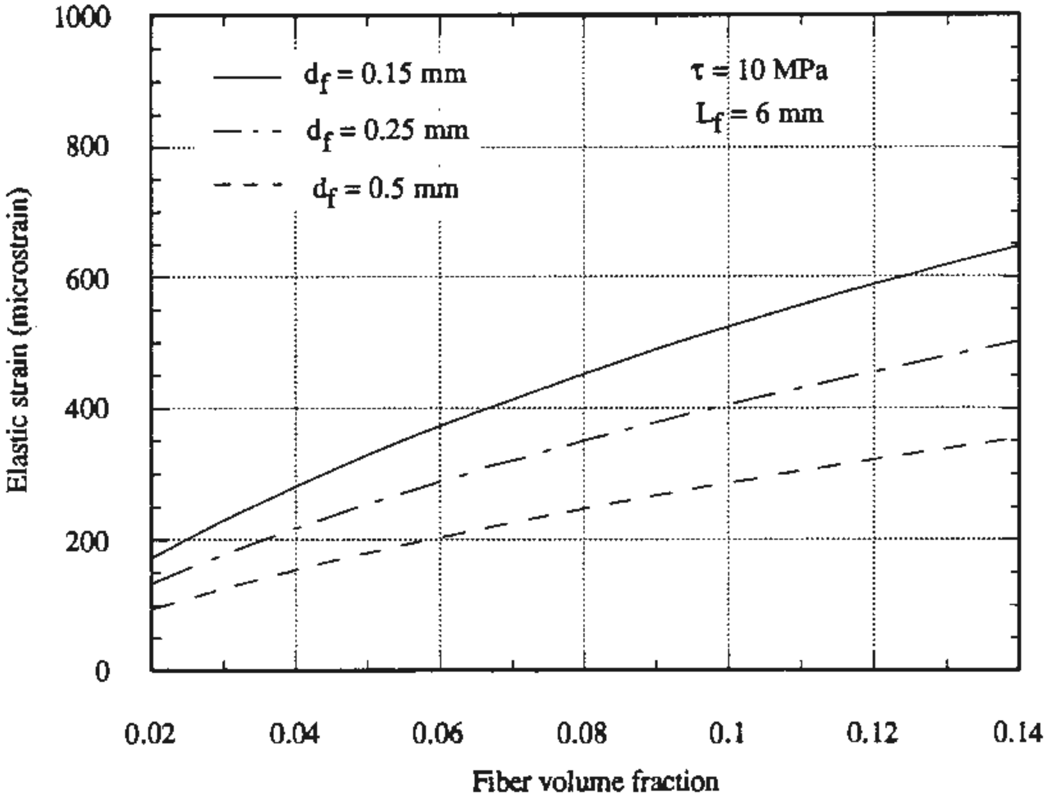

Figure 8. The effect of fiber diameter on elastic strain for fiber reinforced DSP

The effect of modulus of elasticity of fiber on elastic strain prediction is shown in Figure 9 for three modulus values, 200,000 MPa (steel fibers), 37,000 MPa (carbon fibers), and 10,000 MPa (polypropylene fibers) for fibers with a fiber length of 6 mm and a diameter of 0.15 mm. In practice, however, carbon fibers of the length and diameter used in the calculations may not be practical. However, some insight can be gained on the effect of fiber material on the performance of the composite. The model predicts that, for fiber reinforced DSP, the elastic strain values increases with increasing fiber stiffness. A similar trend is predicted also by the ACK model. The effect of fiber modulus on elastic strain values could not be predicted from the models by Naaman and by Swamy&Mangat.

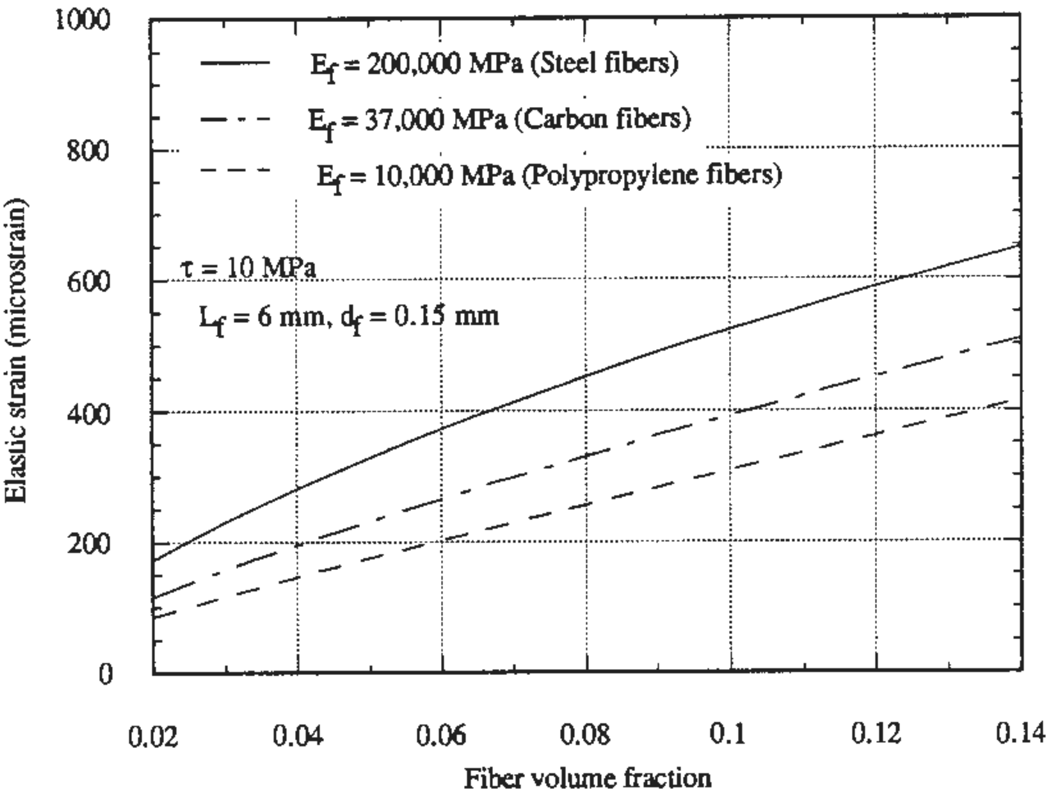

Figure 9. The effect of modulus elasticity of fiber on elastic strain for fiber reinforced DSP

7. Conclusions

1. An equation was developed from energy principles, for predicting the elastic tensile strain capacity of fiber reinforced composites containing high volume fraction of discontinuous fibers randomly distributed within the matrix. This equation agrees well with results from flexure tests obtained in an earlier study by the authors.

2. The model predicts that the tensile strain capacity of a brittle cement-based matrix can be improved several fold by the addition of high volume fraction (i.e. more than 2%) of fine and stiff fibers provided, that the matrix exhibits high bond strength.

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