Paper No. 12

THE APPLICATION OF THE NODAL FORCE CONCEPT IN YIELD-LINE ANALYSIS. An illustrative example

by K. CEDERWALL & A. LI

DISCUSSION

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The above paper presents an interesting discussion of the nodal force concept, and the authors should be commended for drawing attention to the issue. However, the writer does not agree with the authors' conclusion (p.53) that the nodal force approach is not applicable to the case at hand. On the contrary, the considered example constitutes a perfect example of the usefulness of the nodal force concept, when properly applied.

Consider a yield line pattern characterized by a number of geometrical parameters. As explained by the authors, the lowest upper bound (which does not necessarily constitute an exact solution in the sense of the theory of plasticity) can be derived in two ways. In the virtual work method the optimal values of the geometrical parameters are determined by minimization of the load parameter found from the work equation. In the equilibrium method the geometrical parameters are found by elimination between the equations of (moment) equilibrium. Thus each (independent) equilibrium equation corresponds to a geometrical degree of freedom for the yield line pattern, which is analogous to the role played by equilibrium equations in mechanics in general.

In addition to the external loads and the yield line moments the equilibrium equations contain so-called nodal forces at the intersections of yield lines with each other or with the free boundaries of the slab. As shown by Møllmann /4/ the nodal force formulae derived by Johansen /1/ are such that the equilibrium method gives the same solution as the virtual work method, i.e. the geometrical parameters determined by the equilibrium equations are the optimal values.

As mentioned by the authors (p. 50) this implies that a nodal force formula corresponds to a degree of freedom for the yield line pattern, i.e. the formulae are not valid at intersection points which are pre-determined by point loads, slab corners or symmetry. Like many other matters the late Prof. Johansen regarded this as self-evident, whence it is not too well explained in his work /1,3/, and he never bothered to correct the authors who misunderstood his nodal force concept on this point.

The considered yield line pattern, see Fig. 2, is defined by the parameters r, $x = \cot\alpha$ and $y = \cot\beta$, and the corresponding nodal forces are Q_1, Q_2 and Q_3, given by the expressions (2).

The corresponding three equilibrium equations have no solution for $a \neq 0$, and the authors regard this as a break-down of the nodal force approach.

However, the fact that a solution of the three equations is only possible for $a/r \to 0$ indicates that r shall be as large as possible, implying that the yield line (3) coincides with the clamped support. As correctly pointed out by the authors, this means that the corresponding nodal force formula is not valid. Indeed, since one degree of freedom has been removed from the intersection of yield lines (3) and (4), the nodal force Q_1 is a free variable, not given by the formula, thus replacing the lost freedom of the yield line pattern. The nodal forces Q_2 and Q_3, however, are still determined by the formulae, since they correspond to the free parameters x and y.

The two equilibrium equations for slab part II can now be written:

$$-mrx + m'(r+a) = rQ_2 - aQ_3$$

$$m(r+a) + m'(r+a) = rxQ_2 + (rx + (r+a)y)\,Q_3$$

Inserting the nodal force formulae (2), and introducing the notation $k = a/r$, this gives:

$$m'(1+2k)y = (2m+m')x \qquad (13)$$

$$(m+m')x^2 + m'(1+k)y^2 + m'xy = (m+m')(1+k) \qquad (14)$$

Equation (13) is identical with Eq. (11) derived by the authors by minimizing the load determined from the work equation, and by inserting into Eq. (14) we recover the authors' Eq. (12).

The part of the load P which is carried by the slab parts II is in vertical equilibrium with the nodal forces Q_2 and Q_3. Hence moment equilibrium for slab part I about the clamped edge gives:

$$P = 4(m+m')x + 2m'y \qquad (15)$$

This is somewhat more elegant than Eq. (9) found by the work method. Incidentally, Eq. (15) may be derived from Eq. (9) through elimination of k by means of Eqs. (13) and (14).

In conclusion it may be stated that the equilibrium method with a correct interpretation of the nodal force concept is certainly applicable to the considered case. The analysis gives the same result as the virtual work method, and the calculations are considerably simpler.

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Authors' reply to comments from
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The comments from M.W. Braestrup bear witness to a deep insight into the matter and we are very grateful. We are quite willing to rephrase ourselves and admit that this example does not illustrate a complete break down of the application of the nodal force concept but the situation could be described as a partial break down, where some of the nodal forces may be expressed by K.W. Johansen's expressions and some may not. If it is obvious which of the nodal forces that can be formulated according to the nodal force approach the problem can still be solved with the equilibrium method as demonstrated by M.W. Braestrup. In this case it was not quite clear for the authors from the beginning that for the true solution the yield line (3) coincides with the clamped support. This fact became obvious when we had derived the expressions in the virtual work method.

LIST OF MINI-SEMINARS IN THE PERIOD 1980-1990

Titles are quoted in the original language

1980

14. "Arbetsmiljø vid betongarbeten."
Date: 80-04-16
Place: Finska Betongföreningen och VTT,Tammerfors
Publication: Arbetsmiljøn - riskförebyggande åtgårder vid betongarbeten. VTT-symposium 14/1981

15. "Dynamisk Påkjente Konstruksjoner"
Date: 80-04-21
Place: SINTEF FCB, Trondheim
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Place: Aalborg Portland, Aalborg
Publication: CBL särtrykk nr 7

17. Armering och armeringsarbeten ergonomi
Date: 80-11-12
Place: CBI, Stockholm
Publication: CBI-rapport

1981

18. Ikkelineær analyse av armerte betongkonstruktoner
Date: 81-05-15
Place: NTH, Trondheim
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Publication: Condensed silica fume in concrete Inst. for Bygningsmateriallære, Rapport BML, 82610 februar 1982

1982

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Date: 82-10-28

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Place: CBI Stockholm

1984

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Publication: Ingen publisering på dette miniseminaret påtenkt.

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Publication: -

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Date: 90-11-20 - 90-11-20
Place: KTH, Stockholm
Publication: -

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