ANCHORAGE OF REINFORCEMENT IN CONCRETE STRUCTURES

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ABSTRACT

The anchorage of reinforcement in reinforced concrete structures is studied in terms of interaction theory, in which the bond between the concrete and the reinforcement is modelled by a bond-slip relationship, and the behaviour of the anchor units by a force-displacement relationship. The parameters for the models are determined statistically for each reinforcement and anchor unit type on the basis of pull-out test results. The interaction between the concrete and the reinforcement on a long anchorage length is solved numerically.

The effectiveness of the anchoring system is determined on the basis of slippage, anchorage length and the force anchored. The effect of bar size on bond behaviour proved to be significant. Hook anchors seem not to be as effective as assumed in building codes. Bond factor values and anchoring design formulas are proposed.

Key-words: concrete, reinforcing, steel, anchorage, bond, hook, cross joint, design

1. INTRODUCTION

The anchorage design of concrete reinforcement in Finnish building codes /1/ is based mainly on the present CEB-FIP Model Code 1978/3/. The main principles are:

1) The force anchored by the is linearly dependent on the anchorage length (l_b), circumference of the reinforcement (u_s) and tensile strength of the concrete (f_{ct}).

2) The bond properties of reinforcement is specified by the bond factor k_b.

3) The force anchored by a hook is specified by the length, which can be subtracted from the anchorage length of straight bar.

4) The force anchored by welded transverse bars is specified by the strength of the joint and the amount and sizes of the bars.

Finnish codes place reinforcing steels into four different bond classes: hot rolled ribbed bars (k_b = 2.4), cold worked ribbed bars (k_b = 2.0, altered to 2.4 in the last edition /2/), cold worked, indented bars (k_b = 1.1) and plain bars (k_b = 1.0). Hot rolled and cold worked ribbed bars are mostly used nowadays and are the most important reinforcing materials.

The aim of this work was to define bond factors for different reinforcing steel types and design formulae for hooks and welded transverse bars so that the equal safety in the ultimate limit state and behaviour in the service state can be achieve with different anchoring methods.

The basic information on anchoring properties was obtained from pull-out test results, which were used to construct mathematical models for anchoring behaviour. These models were then used to calculate the interaction between the concrete and the reinforcement, providing slip and force values in relation to anchorage length. The shear stress on the surface of the reinforcement and the active bond factor could be calculated from the force and anchorage length values /5,6/.

2. BOND STRESS-SLIP RELATIONSHIP

The bond stress-slip relationship can be described mathematically by four parts, presented by Eligehausen et. al. /4/ (fig. 1).

$$\tau = \tau_1 \left(\frac{s}{s_1}\right)^{\alpha} \quad , \text{ when } 0 \leq s \leq s_1$$

$$\tau = \tau_1 \quad , \text{ when } s_1 < s \leq s_2$$

$$\tau = \tau_1 + (\tau_1 + \tau_R) \frac{s-s_2}{s_2-s_3} \quad , \text{ when } s_2 < s \leq s_3$$

$$\tau = \tau_R \quad , \text{ when } s > s_3$$

where τ_1 is bond strength in massive concrete
τ_R bond value in repeated loading,
s slippage,
$s_{1,2,3}$ slip parameters.

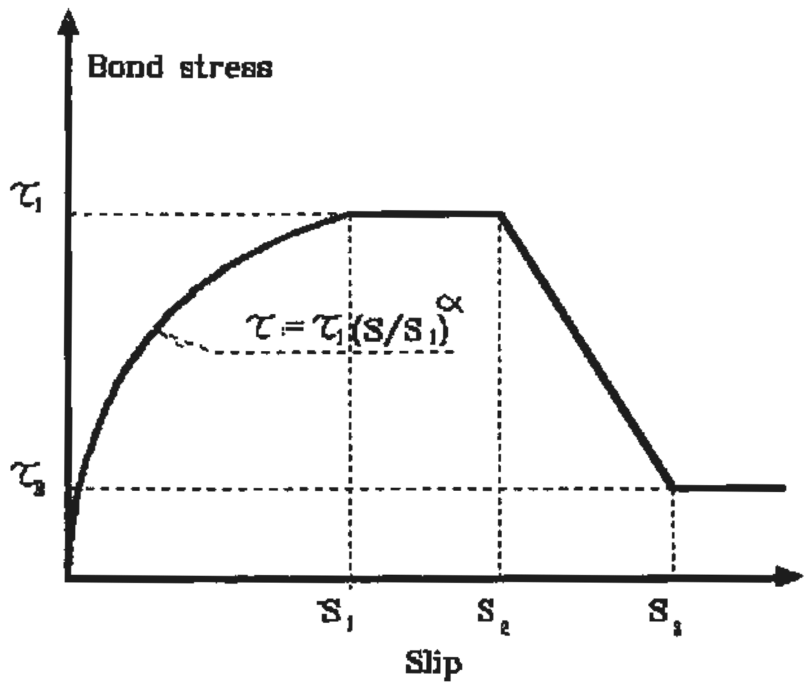

FIG. 1. Bond stress-slip relationship.

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EGENCIA DENMARK                               BOOKING REF: VCZ6FZ
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FLIGHT     D8 3083 - NORWEGIAN AIR INTERNATIONAL LTD       WED 04 APRIL 2018
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DEPARTURE: AALBORG, DK (AALBORG)                              04 APR 08:40
ARRIVAL:   COPENHAGEN, DK (KASTRUP), TERMINAL 3               04 APR 09:25
           FLIGHT BOOKING REF: D8/VCZ6FZ
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           MEAL:                     FOOD AND BEVERAGES FOR PURCHASE

NON STOP   AALBORG TO COPENHAGEN
           EQUIPMENT:                BOEING 737-800 (WINGLETS)


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           FREQUENT TRAVELLER:       D8-DY56328864 FOR BACH/ANNE METTE MRS
           MEAL:                     FOOD AND BEVERAGES FOR PURCHASE

NON STOP   COPENHAGEN TO AALBORG
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The model describes bond behaviour under monotonous loading in a case in which concrete cover does not split. In this study Only the first, increasing part was needed fo the present purpose.

3. BOND STRESS-SLIP PARAMETERS

The parameters of the nonlinear part τ_1, s_1 and α were determined statistically from the pull-out test results by non-linear regression analysis. The tests have been performed during the 1970's and 1980's at the Concrete and Silicate Laboratory of Tecnical Research Centre of Finland. The test arrangement is shown in Figure 2. The numbers of tests were 254 for hot rolled steel bars and 124 for cold worked steel bars.

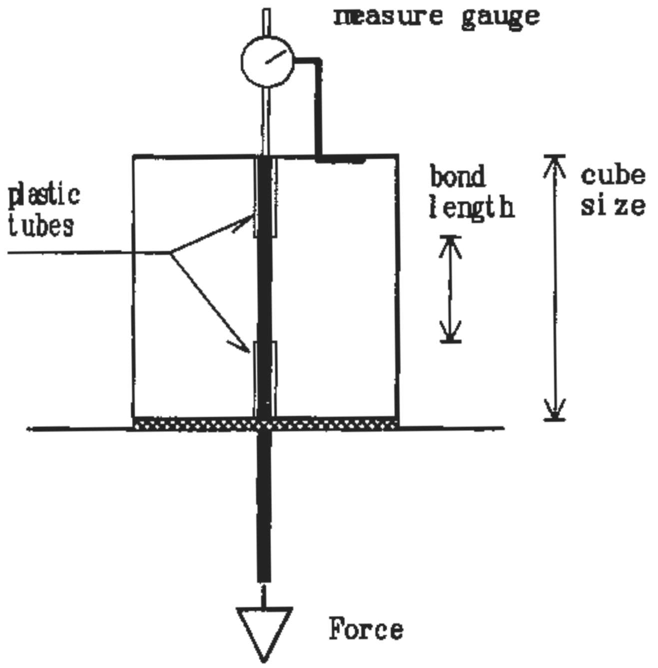

FIG. 2. Test arrangement in the pull-out test.

The fitting functions in the regression analysis were

$$\tau_1 = K\left[p_{11}\left(\frac{\alpha_{sb}}{0.05}\right)^{p_{12}}\left(\frac{K}{50}\right)^{p_{13}}\left(\frac{d_s}{10}\right)^{p_{14}} + 0,11\right]$$

$$s_1 = p_{21}\left(\frac{\alpha_{sb}}{0.05}\right)^{p_{22}}\left(\frac{K}{50}\right)^{p_{23}}\left(\frac{d_s}{10}\right)^{p_{24}}$$

$$\alpha = p_{31}\left(\frac{\alpha_{sb}}{0.05}\right)^{p_{32}}\left(\frac{K}{50}\right)^{p_{33}}\left(\frac{d_s}{10}\right)^{p_{34}}$$

where K is concrete cube strength (MPa)
α_{sb} rib face area
d_s bar diameter (mm)
p_{ij} parameters

The values obtained for the parameters in the case of hot rolled and cold worked ribbed steel bars are presented in Table 1.

Table 1. Bond stress-slip model parameters for hot rolled and cold worked bars.

Parameter	Reinforcement type	
	Hot rolled n= 254	Cold worked n= 124
p_{11}	0.304	0.194
p_{12}	0.191	0.154
p_{13}	-0.524	-0.558
R^2	0.74	0.296
p_{21}	1.351	0.490
p_{22}	-0.567	-0.278
p_{24}	0.460	0.531
R^2	0.57	0.418
p_{31}	0.222	0.221
p_{32}	-0.183	-0.162
p_{33}	0	0
p_{34}	0.199	0.124
R^2	0.967	0.956

Parameters p_{14}, p_{23} and p_{33} were set to zero in the last calculations because their values appeared earlier to be very small.

4. FORCE-SLIP RELATIONSHIP OF ANCHORING UNITS

The model for the increasing part of the force-slip relationship of anchoring units was basically the same as for the bond stress-slip relationship.

$$F = F_1 \left(\frac{s}{s_1}\right)^{\alpha}$$

The statistical data available from test results were rather limited. They are taken from reports /7,8,10/ and analyses of them are presented in /6 and 9/.

When developing design formulae, the basic forms of the formulae for F_1, s_1 and α were chosen first and the formulae were then "fitted" to the test results by means of coefficients.

In the case of a hook, the formula for F_1 with two coefficients was

$$F_1 = 12\ k_\beta\ K\ A_s \left(\frac{r}{d_s}\right)^{\rho},$$

where k_β is a coefficient depending on the position of the hook
- K cubic strength of concrete
- A_s cross sectional area of bar
- r radius of bend
- d_s diameter of bar
- ρ coefficient

The fitted values were 0.45 for coefficient ρ and 1.0, 0.75 and 0.63 for coefficient k_β depending on the position. The greatest value 1.0 applies to a vertical and horizontal bar with its hook bent upwards, 0.75 to a horizontal bar with its hook bent downwards and 0.63 to a vertical bar with its hook bent downwards.

The formula for α with fitted values is

$$\alpha = 0.48 \left(\frac{r}{d_s}\right)^{0.13} .$$

The value for parameter s_1 is chosen to be equal to 1 mm.

In the case of a welded transverse bar the basic formulae were

$$F_1 = 18\, k_{F\beta}\, k_{F\phi}\, K\, A_s ,$$

where $k_{F\beta}$ is a coefficient dependent on the position of the bar
$k_{F\phi}$ a coefficient dependent on the bar sizes $k_{F\phi} = d_T/d_L$
d_T diameter of the transverse bar
d_L diameter of the longitudinal bar

and

$$s_1 = (0.03\, d_L + 0.1)\, k_{s\beta}\, k_{s\phi}$$

where d_L is diameter of the longitudinal bar
$k_{s\beta}$ coefficient dependent on the position of the bar
$k_{s\phi}$ coefficient dependent on the bar sizes $k_{s\phi} = d_L/d_T$

The best fits gave values between 0.34 and 0.43 for exponent α. A constant value 0.4 was chosen for the later calculations.

Fitted values for coefficients $k_{F\beta}$ and $k_{s\beta}$ are given in Table 2.

Table 2. Force-slip model parameters for welded cross joints.

Position β (degree)	$k_{F\beta}$	$k_{s\beta}$
0	0.6	1.7
90	0.8	1.2
180	1.0	1.0

The position angle is specified as follows:

β = 0° denotes a vertical bar loaded downwards,

β = 90° denotes a horizontal bar,

β = 180° denotes a vertical bar loaded upwards,

The anchorage force will be restricted by any splitting of concrete cover, which can occur if the cover is not thick enough or if there is no transverse pressure. A method is presented in /6/ for adjusting the force-slip relationship and bond stress-slip relationship in the case of splitting.

5. CALCULATION RESULTS

The results of the interaction calculations can be represented in an anchorage length - slippage coordinate system by curves indicating a constant anchorage force. An example of such a curve is given in Figure 3, in which a force of 500 MPa * A_s is anchored by a straight, hot rolled ribbed bar without an end anchor.

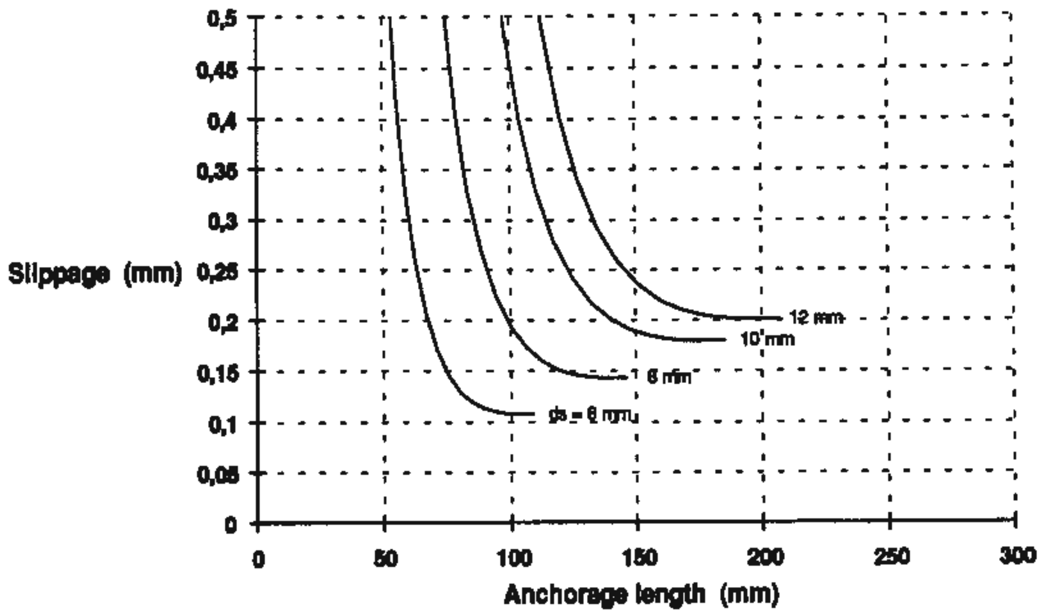

FIG. 3. Curves denoting a constant anchorage force of 500 MPaA_s for hot rolled, ribbed straight bars, bar sizes 6,8,10 and 12 mm. The strength of the concrete is 35 MPa.*

These groups of curves can be calculated for different
- steel types specified by the bond stress-slip model,
- concrete strength values, and
- end anchors.

Other two examples are presented in Figures 4 and 5, the first for a hook anchor and the second for a welded transverse bar anchor.

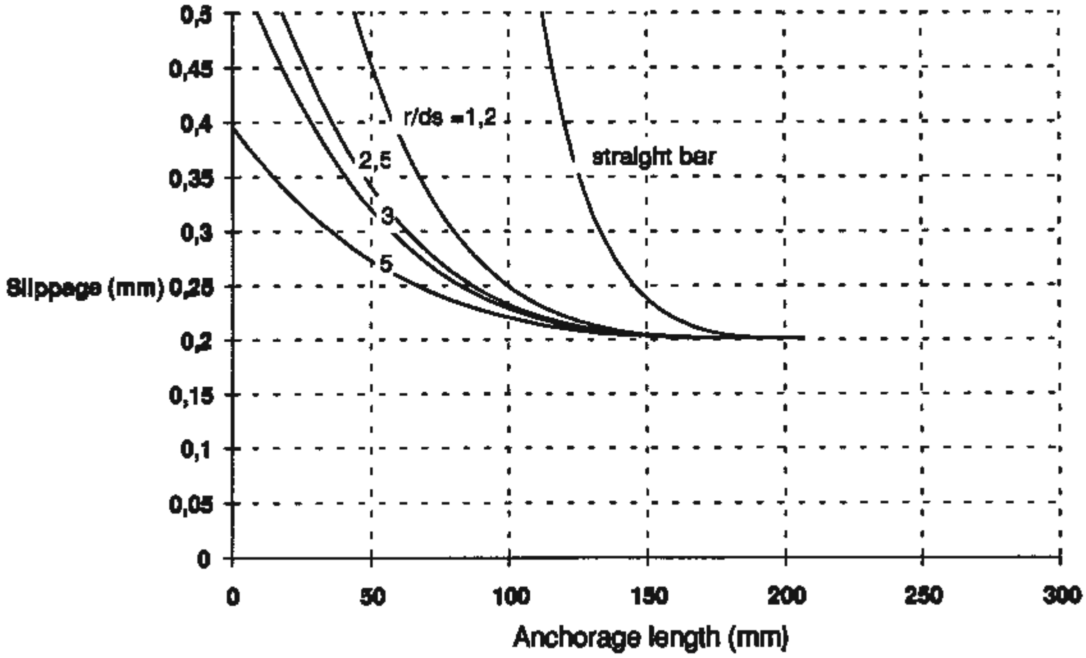

FIG. 4. Curves denoting a constant anchorage force of 500 MPaA_s for hot rolled ribbed bars (d_L= 12 mm) with a hook as the end anchor. The each curve represents a different radius of the bend. The strength of the concrete is 35 MPa.*

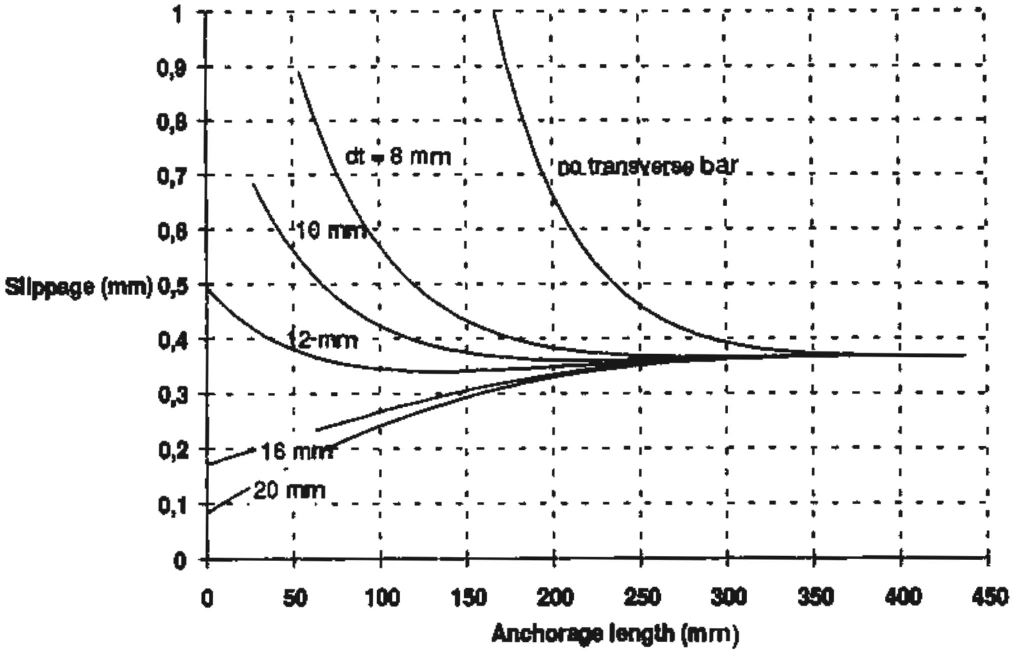

FIG. 5. Curves denoting a constant anchorage force of 500 MPaA_s for hot rolled ribbed bars (d_L= 12 mm) with a transverse bar as the end anchor. The each curve represents a different size of transverse bar(d_T), The strength of the concrete is 35 MPa.*

The curves for the straight bars allow the bond factors k_b to be calculated for a given value of the slippage on the loaded end.

$$k_b = \frac{F_b(s)}{u_s\ L_b(s)\ f_{ct}} ,$$

where $F_b(s)$ is force anchored with slippage s
$L_b(s)$ anchorage length corresponding to force $F_b(s)$
u_s circumference of bars
f_{ct} tensile strength of concrete

The curves, which can be used to determine the effect of given anchor units, denote the length by, which the anchor shortens the anchorage length of a pure bond. The effect of the anchor is thus the vertical distance between the curves with and without anchor (Fig. 4 and

5). The calculation system can also give the forces of anchors as an output.

The slippage of the loaded end of the bar is the most important factor when evaluating the effectiveness of an anchoring system. Different anchoring systems are equally effective if they can anchor the same force with same length and the same slippage at the loaded end.

Since it is difficult to determine the absolute value of the slippage that can be allowed in structures, the examination must be carried out using several levels of slippage.

Values of k_b for hot rolled ribbed steel bars are shown in Figure 6.

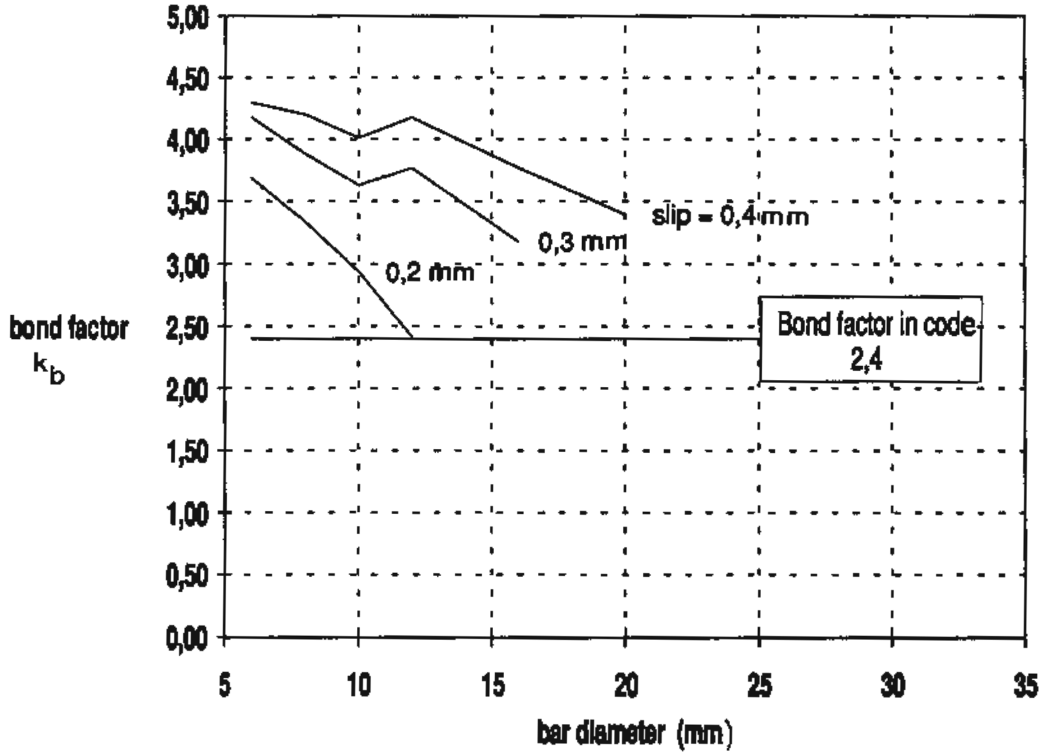

FIG. 6. Bond factors for hot rolled ribbed straight bars, The anchored force is 500 MPa A_s, and the strength of the concrete 35 MPa.*

6. DISCUSSION AND CONCLUSIONS

The method used here is best applied for comparing the behaviour of different anchoring systems, where upon the results can be used to determine bond factors for new reinforcement types or to derive design formulae for new anchoring systems. A certain caution should be exercised regarding the use of absolute values for bond factors, as the pull-out test does not correspond exactly to the anchoring behaviour of a real structure.

Despite the limitations the results reveal some important facts. The slippage of the reinforcement that can be allowed in the critical cross-section is the decisive factor when studying the effectiveness of an anchoring system. The value of the permitted slippage should be low enough that it can be achieced in the structure even in its ultimate limit state, and it should also be lower than the limit of the bond stress-slip model. The author proposes a limit between 0.3 and 0.5 mm.

Another important fact is the effect of bar size on anchoring behaviour. The fact that the increase in rib face area with bar size is not sufficient compared to the increase in cross-sectional area means that a structure reinforced by thick bars whould need much greater deformation to reach the same anchorage force than a structure reinforced by thin bars with an equal sum of cross-sectional areas.

The bond properties of Finnish cold worked ribbed bars appeared to be slightly poorer than those of hot rolled bars compared in terms of bond stress-slip relationships, the difference being 5 - 13 %. When the bars are compared in terms of a longer anchorage length and the effect of bar size is considered, the situation looks different, however. The calculated bond factors for both bar types are far above the design value of 2.4 when the bar size is less than 12 mm, hot rolled bars having slightly higher values. Cold worked bars are manufactured only in sizes 4 - 12 mm, whereas hot rolled bars are available up to size 32 mm. Because of the bar size effect there is no technical basis for the lower bond factor value sett for cold worked ribbed bars.

It was found here that the effectiveness of hook anchorage is far below the level assumed in the building codes. Where these codes maintain that anchorage length can be reduced by 15 d_s in the normal case by, the corresponding values derived from the present calculations would be 2 - 10 d_s depending on bar size. This calls for a new design formula to be recommended. The force anchored by a hook can be calculated from

$$F_h = 7\ k_\beta\ f_{cd}\ A_s \sqrt{\frac{r}{d_s}}$$

where k_β is a coefficient dependent on the position of the hook ($0.63 \leq k_\beta \leq 1.0$)
f_{cd} design strength of concrete
A_s cross-sectional area of bar
r radius of bend
d_s diameter of bar

The force anchored by the hook will develop at the point of intersection of the hook and straight bar, so that the space taken up by the hook reduces the effectiveness of its anchorage.

The corresponding design formula for a welded transverse bar would be

$$F_{rd} = 20\ k_{F\beta}\ f_{cd}\ A_s\ \frac{d_T}{d_L}\ ,$$

$$F_{rd} \leq k_{br}\ F_o \sqrt{n}$$

where $k_{F\beta}$ is a coefficient dependent on the position of the bars, Table 2 , ($0.6 \leq k_{F\beta} \leq 1.0$)
k_{br} a coefficient dependent on the strengthening effect of the concrete on the joint (= 1.35)
d_T diameter of transverse bar
d_L diameter of longitudinal bar
F_o strength of joint tested in air.

The force anchored by the transverse bar will develop in the joint. The space that the transverse bar anchor needs is quite small compared to the hook anchor. Although there must be uncracked concrete in front of the transverse bar on the loaded side, the force exerted by the bond on this straight bar length can be added to the anchorage force of the transverse bar

It has not been possible here to take into consideration the effects of the casting process, e.g. the compactibility of the concrete and the compacting methods used or the location of the reinforcement in the structure, because of a lack of test results. Further research is needed to increase our knowledge concerning the effects of concrete properties on anchoring behaviour and the effects of anchoring behaviour on the structure.

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