A STATISTICAL APPROACH TO THE SERVICE LIFE DESIGN OF CONCRETE STRUCTURES

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ABSTRACT

Methods for evaluating the technical service life of a building or a part of it are reviewed. The service life can often be expressed by means of a mathematical model where descriptive variables include parameters of materials and conditions and structural dimensions. The scatter of service life can be estimated by means of the scatter of the variables affecting it. A log-normal distribution is recommended as the assumed distribution of service life. Derived as examples are the probability density functions and cumulative probability functions of service life with respect to frost resistance and corrosion of reinforcement. The distributions have been compared to those obtained by a simulation process.

Keywords: service life, frost resistance, corrosion of reinforcement

1 GENERAL THEORY

1.1 Definition of service life

Normally, buildings or their component parts have a finite service life. Many factors cause the serviceability of buildings to diminish with time, leading finally to a situation in which a complete repair or rebuilding is necessary.

Service life expectations are needed when planning new building construction or the maintenance and repair of the old construction base. Service life expectations facilitate the comparision of different design alternatives, and allow economical aspects to be taken into account. Comparative calculations include not only the direct construction costs but also the costs of maintenance and repair.

Service life is the period of time during which a building or a structure meets the requirements set for it. Both the requirements and the quality of buildings or component parts may change with time.

The requirements can be 1) technical 2) functional or 3) economic. Accordingly we can talk about a technical, functional or economic service life.

In this context only the technical service life is treated. The technical service life is determined on the basis of the technical requirements set for the structure. The technical quality of the structure is descended due to weathering and corrosion taking place in the building materials.

At the first stage weathering and corrosion affect the protective concrete layer nearest to the exposed surface with the following consequences:

1) the interaction between the concrete and the reinforcement is weakened,
2) fitting and installation attachments are loosened,
3) the comfort and serviceability are reduced, and
4) the aesthetic appearance is impaired.

Naturally, at a later stage corrosion and weathering also affect the bearing capacity of the structure. However, very often the other above mentioned properties are more decisive for the termination of the technical service life.

1.2 Mathematical service life models

The propagation of damage is often expressed as a mathematical model in which descriptive variables include parameters of materials and conditions influencing the durability and structural dimensions.

A change in a certain important technical property can often be used as the measure of damage. The service life is determined according to the point in time when the examined property reaches the maximum or minimum permissible limit value.

When creating this type of mathematical model, certain restrictions, choices, eliminations and simplifications must be made, but in such a way that the accuracy of the model is not essentially lost.

Service life is generally presented in the following form:

$$t_l = t_l(x_1,x_2, \ldots x_n) \qquad (1)$$

Where t_l is the service life (usually years) and
x_i a parameter of the material or condition or a structural dimension (i = 1...n)

Formula (1) is most often constructed such that the mean value of service life is obtained by inserting the mean values for variables x_i into formula (1).

1.3 Standard deviation of service life

Service life is a random quantity often with a very wide scatter. The scatter of service life can be estimated from the scatter of the variables affecting it /1/:

$$\sigma^2\ (t_l) = \sum_{i=1}^{n} \{\frac{\delta t_l}{\delta x_i}\ \sigma(x_i)\}^2 \qquad (2)$$

where $\sigma(t_l)$ is the standard deviation of service life,
$\sigma(x_i)$ the standard deviation of variable x_i,
$\frac{\delta t_l}{\delta x_i}$ the partial derivative of service life in relation to variable x_i and
n the number of variables.

In formula (2) the mean values for each variable are inserted into the partial derivative expressions.

1.4 Distribution and probability of service life

Generally service life cannot be described with a normal distribution because negative values for the service life would then be possible. Negative values can be avoided by using log-normal distribution which means that the service life distribution is considered to be normal on a logarithmic time scale. Obviously log-normal distribution is also more correct in many cases.

If the service life distribution is log-normal with a mean value $\mu(t_l)$ and with standard deviation $\sigma(t_l)$, so function $Y=\ln(t_l)$ is normally distributed and its mean value and standard deviation are obtained from the following formulae /1/:

$$\sigma^2(Y) = \ln(1 + (\frac{\sigma(t_l)}{\mu(t_l)})^2) \qquad (3)$$

$$\mu(Y) = \ln\ \mu(t_l)-\tfrac{1}{2}\ \sigma^2(Y) \qquad (4)$$

The probability of the service life being shorter than a given time t is as follows:

$$P\{t_l < t\} = P\{\ln\ t_l < \ln\ t\} = P\{Y < \ln\ t\} = \phi(-\beta) \quad (5)$$

where $\beta = \dfrac{\mu(Y) - \ln t}{\sigma(Y)}$ and

ϕ is the cumulative density function of the (0,1)-normal distribution

2 PREDICTION OF SERVICE LIFE IN RELATION TO FROST RESISTANCE

The service life of a concrete structure in relation to frost resistance is influenced by external conditions and the quality of the concrete. The severity of the external conditions depends on e.g. the moisture content, temperature changes and the possible exposure to deicing salts (chlorides). The type of concrete in relation to frost resistance depends on e.g. its water/cement ratio, air content and binder type. Frost damage may occur either as weathering of surface or as gradual weakening of the material.

For design purposes the service life model with respect to frost resistance can be expressed as follows:

$$t_1 = k_e \cdot k_c \cdot A(a) \cdot F(w/c) \qquad (6)$$

where k_e is the environmental coefficient
k_c the binder coefficient
$A(a)$ the function dependent on the air content of concrete a, and
$F(w/c)$ the function dependent on the water-cement ratio of concrete, w/c.

In the service life calculations presented below a simplified formula (7) is used for the frost resistance /2, 3/.

$$t_1 = 1.41 \cdot k_e \cdot \frac{a}{(w/c - 0.3)} \qquad (7)$$

where a is the air content of concrete, %,
w/c the water-cement ratio of the concrete and
k_e the environmental coefficient.

The binder coefficient has not been taken into account in formula (7) because knowledge is still lacking concerning the effects of cement replacements on frost resistance. The formula is applicable only to Portland cement concrete.

The following mean values can be given to the environmental coefficient k_e.

- edge beams	1.5
- bridge piers in sea water	1.5
- bridge beams in fresh water	3.5
- dams, power plant structures	3.5
- balconies, steps	5
- facades generally	8

In Table 1 a choice has been made for the mean values and standard deviations of problem variables.

The air content of concrete is assumed to be 5% when k_e is 1.5 and 4% when k_e is 5 or higher.

Table 1. Problem variables and their stochastic properties.

Variable	Mean (μ)	Coefficient of variance (σ/μ)
Water-cement ratio w/c	0.4, 0.5, 0.6, 0.7	0.1
Air content of concrete a , %	4, 5	0.5
Environmental coefficient k_e	1.5, 5	0.4

In Figures 1 and 2 the probability density functions and the cumulative probability density functions of service life are presented with the above assumptions. The probability density function expresses the probability of the service life ending during a given 5 year period ($t-5 < t_1 < t$). The cumulative probability function shows the probability of service life being shorter than a given time t. When a designer wants to plan a structure in such a way that the target service life will be achieved with an accepted risk, he can read the required w/c ratio directly from Figure 2.

EXAMPLE
What is the required w/c ratio of concrete when the target service life of an edge beam of a bridge is 30 years with the risk of 25%?

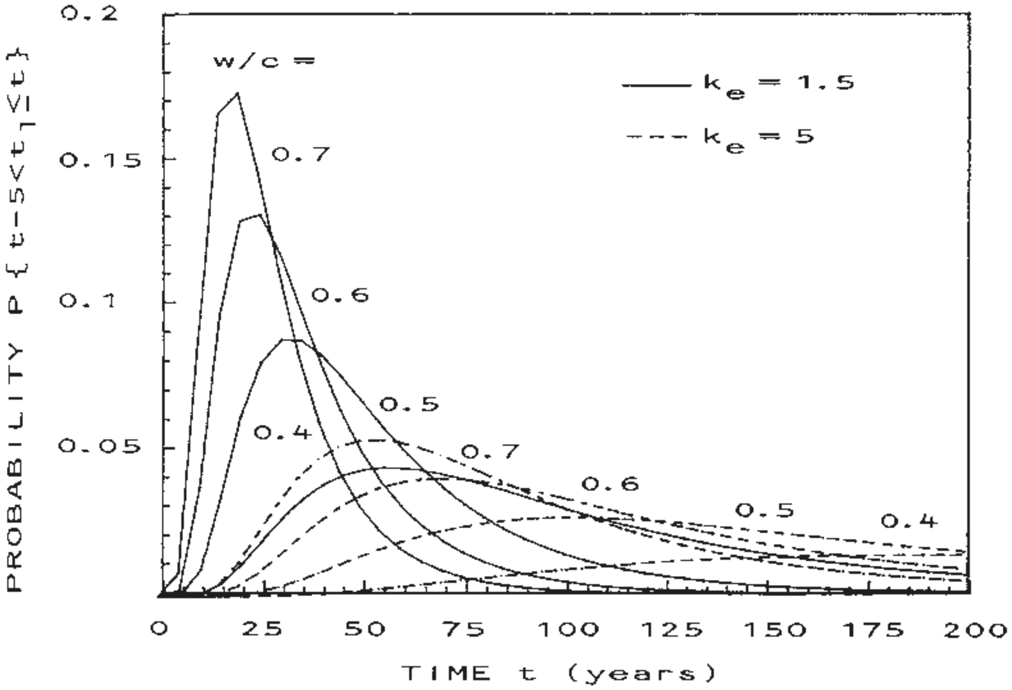
ANSWER: According to Figure 2, w/c = 0.5.

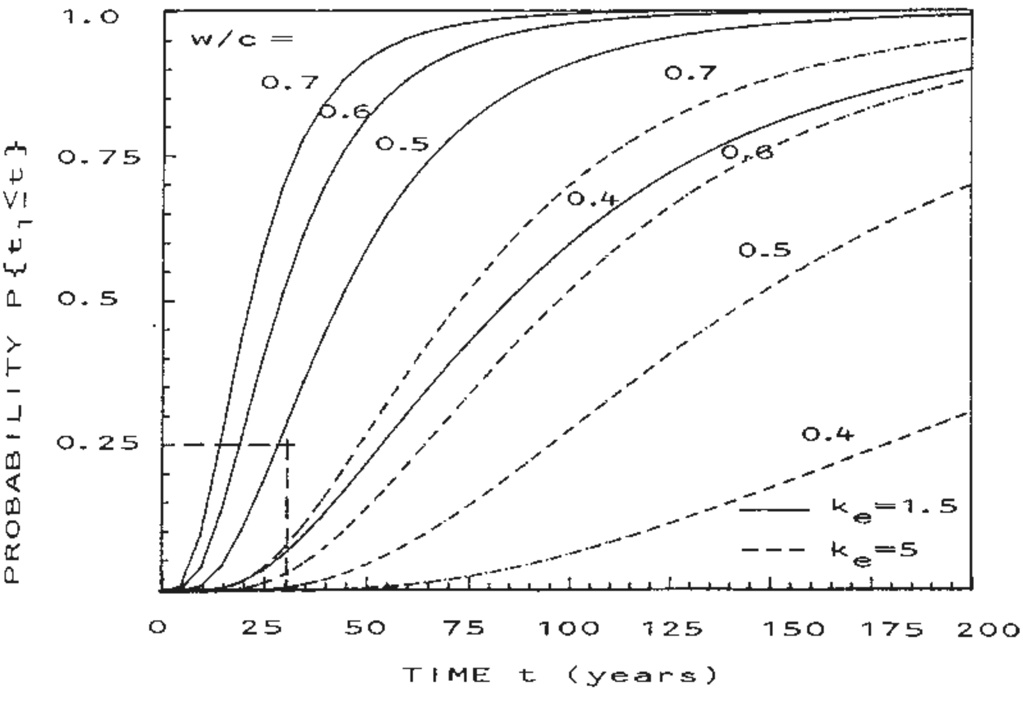
3 PREDICTION OF SERVICE LIFE IN RELATION TO CORROSION OF THE REINFORCEMENT

The service life of a concrete structure in terms of corrosion of the reinforcement depends on the ambient conditions, concrete quality and thickness of the concrete cover. Chloride salts also affect corrosion, but structures susceptible to the effects of chlorides are not dealt with here. The study focuses on structures in normal weather conditions where concrete becomes carbonated due to the effect of carbon dioxide in the air and allows the initiation of corrosion in the reinforcement.

The rate of carbonation is strongly dependent on the moisture content of the concrete. If the concrete is sheltered from rain (as under a bridge or balcony), the rate of carbonation is higher than on exposed surfaces.

Quality factors of concrete influencing the carbonation rate include density and binder type. The density depends mainly on the water-cement ratio of the concrete. If





Figs. 1 and 2. Probability density functions (top) and cumulative probability density functions (bottom) of service life with respect to frost resistance when the environmental coefficient is 1.5 or 5.

byproducts such as fly ash and blast furnace slag are used in concrete as cement replacements a reduced water-cement ratio should be used with a coefficient of efficiency of about 0.3 for fly ash and about 0.5 for slag.

An approximative formula (8) is used to calculate the service life in relation to the corrosion of the reinforcement /4/.

$$t_1 = k_e \cdot \frac{100\ C^2}{\{26(w/c - 0,3)^2 + 1,6\}^2} + k_a \cdot C \qquad (8)$$

where k_e is the environmental coefficient
k_a the coefficient of active corrosion time
C the thickness of the concrete cover (cm) and
w/c the w/c ratio or the reduced w/c ratio of concrete

The first term of the formula expresses the time needed for the initiation of corrosion (carbonation time of the concrete cover) and the latter the active corrosion time up to the time when the concrete cover spalls due to corrosion.

The following values can be given to the environmental coefficient:

k_e = 1.0 when the structure is sheltered from rain
k_e = 1.75 in average weather conditions, e.g. in facades
k_e = 2.5 when the structure is exposed to rain

The coefficient of active corrosion time is normally assumed to be $k_a = 7$.

In Table 2 a choice has been made for the mean values and standard deviations of problem variables.

Table 2. Problem variables and their stochastic properties.

Variable	Mean (μ)	Coefficient of variance (σ/μ)
Water-cement ratio w/c	0.4, 0.5, 0.6, 0.7	0.1
Environmental coefficient k_e	1.75	0.25
Coefficient of active corrosion k_a	7	0.75
Concrete cover C, cm	2.5	0.25

Figures 3 and 4 show the probability density functions and cumulative probability functions calculated for different w/c ratios, assuming the thickness of the concrete cover to be 25 mm.

Figure 3 shows the probability of the service life ending during a given 5 year period ($t-5 < t_1 < t$). Figure 4 shows the probability of the service life being shorter than a given time t. The required water-cement ratio of the concrete can be read from Figure 4 as a function of the target service life and the accepted risk.

EXAMPLE
The thickness of the concrete cover in a balcony slab is 25 mm. What is the required water-cement ratio of concrete when the target service life is 70 years with the risk of 25%?

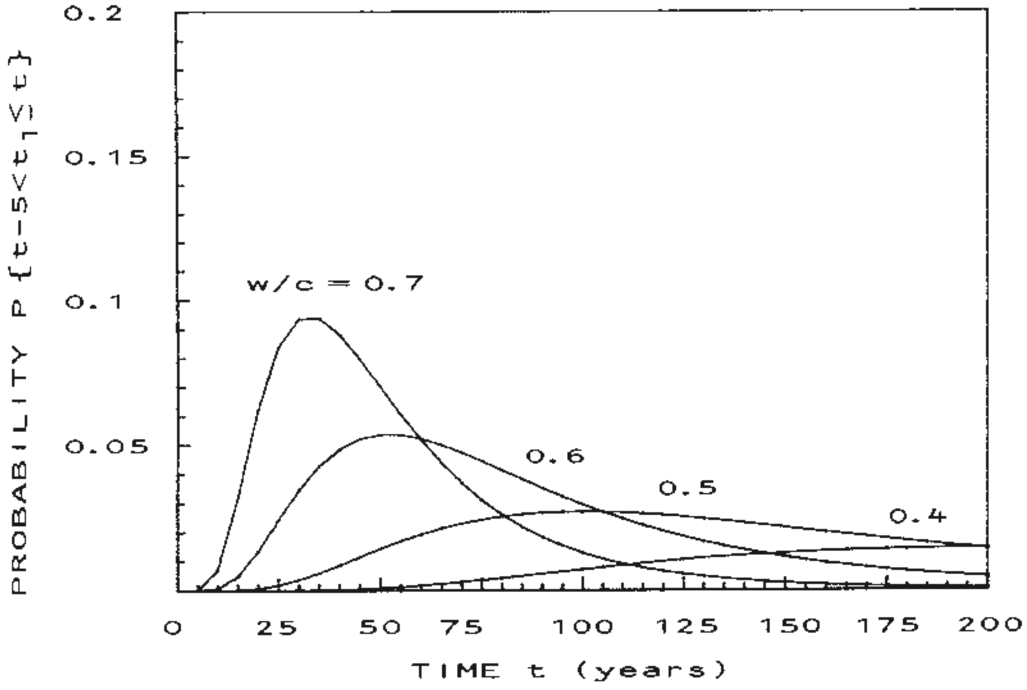
ANSWER: w/c= 0.54 in accordance with Figure 4.

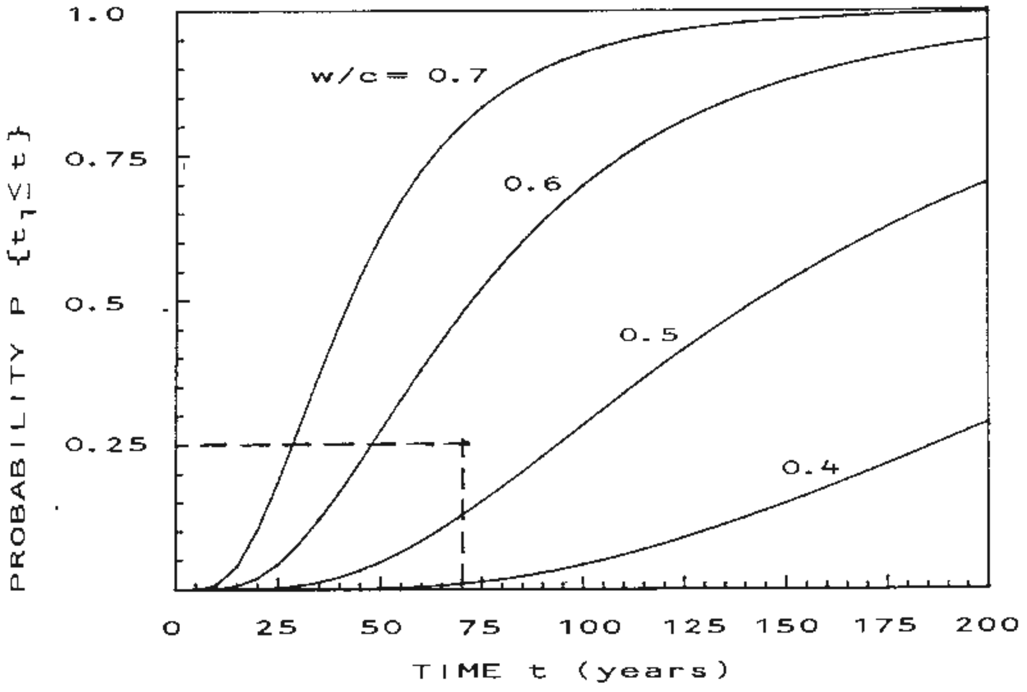
4 COMPARISON WITH THE EXACT SOLUTION

The statistical method of calculation described above is not exact due to the assumption of a log-normal service life distribution and the approximative method used to evaluate the standard deviation. The exact distribution can be studied using a simulation process.

In a simulation process no assumption is made for the type of service life distribution. However, the distributions of the parameters in the service life model must be known. By giving random values to the parameters (distributed in the assumed way) and counting the relative numbers of cases shared in each time interval, the whole distribution of service life can be obtained.

In Table 3 service life probabilities have been calculated using a special simulation process code ISPUD /5/. These values have been compared with those obtained by the 'log-normal method'. In the simulation process the distribution of all parameters in the service life models have been assumed to be log-normal. The value of the water-cement ratio is assumed to be 0.6. Otherwise the stochastic properties of the variables are as presented in Tables 1 and 2.





Figs. 3 and 4. Probability density functions (top) and cumulative probability density functons (bottom) of service life with respect to corrosion of reinforcement when the thickness of the concrete cover is 25 mm.

Table 3. Comparison of assumed log-normal service life distribution to the distribution obtained via a simulation process.

Frost-resistance *)			Corrosion of reinforcement **)		
time years	log-norm. approxim.	simul. process	time years	log-norm. approxim.	simul. process
0	0	0	0	0	0
10	0.040	0.044	20	0.019	0.016
20	0.267	0.270	40	0.170	0.147
30	0.517	0.501	60	0.381	0.340
40	0.697	0.676	80	0.563	0.504
50	0.811	0.789	100	0.698	0.628
60	0.881	0.857	120	0.791	0.724
70	0.924	0.905	140	0.855	0.784
80	0.951	0.920	160	0.899	0.840
90	0.968	0.930	180	0.928	0.880
100	0.979	0.947	200	0.949	0.895

*) refers to formula (7).
**) refers to formula (8).

The probabilities obtained using the log-normal approximation are usually slightly higher than those obtained by the simulation process. However, in normal cases the accuracy of the log-normal approximation can be considered satisfactory.

5 CONLUSIONS

The following conclusions can be made:

1) The service life of concrete structures can be predicted with the aid of mathematical models.

2) A statistical treatment of service life models is necessary due to the wide scatter of service life.

3) It is possible to design concrete structures that meet a specified target service life. Graphic figures can be used in simple design.

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