PUNCHING OF REINFORCED CONCRETE SLABS: CODE RULES, PLASTIC ANALYSIS, TEST RESULTS

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ABSTRACT

The safety against punching failure is normally checked by comparing a concrete strength parameter with a nominal shear stress, defined as the load divided by a measure of the slab depth and the length of a control perimeter around the loaded area. The resulting strength formula for axisymmetric punching can also be derived as a plastic upper bound solution corresponding to a conical failure surface.

The influences of different parameters are assessed on the basis of 66 tests on slabs with square and circular loading, which are compared with various code predictions. It appears that there are large variations in safety level and consistency of the codes.

In particular it is noted that the present Danish concrete code is non-conservative, and a revision proposal is presented.

Key-words: Plasticity, Punching, Reinforced Concrete, Slabs.

1. INTRODUCTION

Punching shear failure occurs in reinforced or prestressed concrete slabs subjected to concentrated loading, e.g. from impact (shelters), wheel loads (bridge decks) or column supports (flat slabs). The failure is characterized by the punching out of a piece of the slab, bounded by a doubly curved surface between the edge of the loaded area and the opposite slab face, see FIG. 1. This entails little or no activation of the main slab reinforcement, whence the punching shear failure load may be significantly lower than the flexural capacity of the slab. Furthermore, the failure may be both sudden and brittle.

In most building codes the safety against punching is checked by comparison of a nominal shear stress with a concrete strength parameter. The nominal shear stress is found by dividing the resulting load within a control perimeter by a concrete area which is the product of the control perimeter length and a measure of the slab thickness. The control perimeter is placed at a distance from the loaded area which is a multiplum of the slab depth. Although the majority of codes agree on the basic procedure, there is considerable differences concerning the specification of the control perimeter and the corresponding concrete strength parameter.

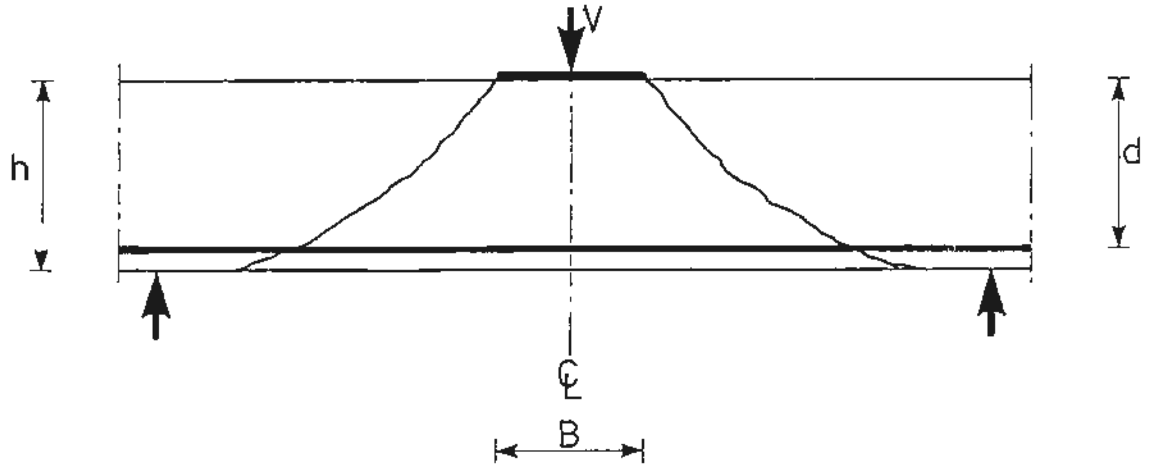

FIG. 1 Punching shear failure of reinforced concrete slab

The present paper initially discusses central punching, i.e. the load is applied far from slab edges or supports. Neither shear reinforcement nor lateral restraints are assumed to be present. A number of code rules are reviewed, and related to a simplified plastic analysis of axisymmetric punching failure. The code predictions are compared with the results of 66 punching tests with circular and square loads, on the basis of which the influence of each individual parameter is assessed. Finally, a proposal for revision of the Danish concrete code is presented.

2. ULTIMATE LOAD EXPRESSIONS

The treatment of punching shear described in the preceding section leads to the following general expression for the punching resistance for a circular loading:

$$V = \pi \ (B + 2\beta t) t \tau_c \tag{1}$$

where

V : predicted strength
B : load diameter
t : slab thickness measure (h or d)
τ_c : concrete strength parameter
β : constant factor

The strength parameter may include correction factors, accounting for the influence of slab reinforcement or absolute slab depth (size effect).

Some building codes explicitly introduce a "control surface", defined as a cylindrical section through the slab, following the control perimeter. This is unfortunate, since it gives the impression that the punching failure is related to shearing on a cylindrical surface, which it is not, see FIG. 1. It also leads to absurdities when the slab depth is not constant (column capitals, drop panels, loads applied in recesses, etc.).

In the formulation of equation (1) care has been taken to separate the control perimeter and the slab depth. Their product constitutes a nominal concrete area, which represents the doubly curved failure surface and should not be construed as a physical section in the slab. In cases of variable slab depth rules must be made to ensure a sensible depth measure, e.g. referring to the projection on the slab normal of the surface spanning between the edge of the loaded area and the control perimeter on the opposite slab face.

The sections below review punching strength predictions from European, American, British and Danish building codes, and equation (1) is compared with an upper bound plastic solution for punching.

2.1 CEB-FIP Model Code

In the MC 78 /7/, /8/, prepared by Comité Euro-International du Béton and Fédération Internationale de la Précontrainte, the punching resistance for circular loading is given as:

$$V = \pi(B + d)d \cdot 1.6(1.6 - d)(1 + 50r)0.035f_c^{2/3} \qquad (2)$$

Here d (in m) is the effective depth to the reinforcement of the slab section, r is the geometrical reinforcement ratio, and f_c (in MPa) is the concrete cylinder strength.

Equation (2) is of the same form as (1), with:

$t = d$
$\tau_c = 1.6(1.6 - d)(1 + 50r)0.035f_c^{2/3}$
$\beta = 0.5$

The control perimeter is placed at a distance from the loaded area which equals half the effective slab depth, and this implies that corners on the load contour corresponds to circular arcs on the control perimeter. Thus for a square load of side length B the factor $\pi(B + d)$ is replaced by $(4B + \pi d)$

For orthogonal reinforcement d is inserted as the geometrical mean of the effective depths to the two bar directions, and r is defined as the arithmetrical mean of the corresponding reinforcement ratios.

The factor (1.6 - d) in equation (2) is replaced by 1.0 for $d \geq 0.6$ m. Also the range of influence of the reinforcement ratio is limited; in the Model Code proper /7/ the maximum value that can be inserted into (2) is $r = 0.008$, whereas in the Compléments /8/ the limit is extended to $r = 0.02$.

2.2 Eurocode 2

The Final Draft of EC2 /9/ determines the punching resistance for circular loading by the expression:

$$V = \pi(B + 3d)d(1.6 - d)(1.2 + 40r)0.035f_c^{2/3} \tag{3}$$

Equation (3) is of the same form as (1), with:

$$\begin{aligned} t &= d \\ \tau_c &= (1.6 - d)(1.2 + 40r)0.035f_c^{2/3} \\ \beta &= 1.5 \end{aligned}$$

The main difference from the CEB-FIP Model Code is that the control perimeter is placed at the distance 1.5d from the loaded area, and the strength parameter is reduced by the factor 1.6. The effective depth d and the reinforcement ratio r are defined as in MC 78, and the control perimeter has rounded corners. Thus for a square load of side length B the factor $\pi(B + 3d)$ is replaced by $(4B + 3\pi d)$.

As in MC 78 the maximum depth affecting equation (3) is $d = 0.6$ m, whereas the upper limit for the reinforcement ratio is $r = 0.015$.

2.3 American Concrete Code

By the ACI 318-83 /2/, /3/ the punching resistance for circular loading is calculated as:

$$V = \pi(B + d)d \cdot 0.332\sqrt{f_c} \tag{4}$$

Equation (4) is of the same form as (1), with:

$$\begin{aligned} t &= d \\ \tau_c &= 0.332\sqrt{f_c} \\ \beta &= 0.5 \end{aligned}$$

As in MC 78 the control perimeter is placed at the distance 0.5d, but it is allowed to be polygonal. Thus for a square load of side length B the factor $\pi(B + d)$ is replaced by $4(B + d)$. For orthogonal reinforcement d is defined as the geometrical mean of the two effective depths.

2.4 British Building Code

In the recently revised BS 8110 /5/ the punching resistance for circular loading is found from the expression:

$$V = 4(B + 3d)d \cdot 0.29 \sqrt[4]{0.5/d} \sqrt[3]{100rf_c} \qquad (5)$$

Equation (5) is of the same form as (1), with:

$$t = d$$
$$\tau_c = 0.29 \ (4/\pi) \sqrt[4]{0.5/d} \sqrt[3]{100rf_c}$$
$$\beta = 1.5$$

The control perimeter is always rectangular with a minimum distance of 1.5d to the loaded area. Thus a square load is treated the same way as a load on the inscribed circle. For orthogonal reinforcement the geometrical mean is inserted for both the effective depth d and the reinforcement ratio r.

The upper limit for the beneficial influence of the reinforcement ratio is r = 0.03.

2.5 Danish Concrete Code

From the 3rd edition of DS 411 /10/ it appears that the punching resistance for circular loading can be determined as:

$$V = \pi(B + 2d)h\sqrt{0.1f_c} \qquad (6)$$

Equation (6) is of the same form as (1), with:

$$t = h$$
$$\tau_c = \sqrt{0.1f_c} = 0.316\sqrt{f_c}$$
$$\beta = d/h$$

The control perimeter is placed at the distance d from the load, but the nominal concrete area is found by multiplication by the total slab depth h. For polygonal loaded areas it is common practice to assume a polygonal control perimeter. Thus for a square load of side length B the factor $\pi(B + 2d)$ is replaced by $(4B + 2\pi d)$.

2.6 Plastic analysis

The punching shear strength can be determined by means of the theory of plasticity if the concrete is assumed to be a rigid, perfectly plastic material. An upper bound solution was derived by Braestrup & al. /6/, cf. also Nielsen /14/. The failure is assumed to be concentrated in a rotationally symmetric surface with a generatrix running from the edge of the loaded area to the opposite slab face, see FIG. 1. The concrete in the failure surface is in a plane state of deformation, the relative displacement rate being perpendicular to the slab. The concrete is assumed to be described by the modified Coulomb failure criterion as yield condition with the associated flow rule. An upper bound for the punching load is determined by equating the rate of external work done by the load with the rate of internal work dissipated in the failure mechanism. The shape of the failure surface is optimized so as to minimize the upper bound.

The rigid-plastic constitutive model for concrete is characterized by three parameters:

- Angle of internal friction ϕ
- Uniaxial compressive strength νf_c
- Uniaxial tensile strength ρf_c

The quantities ν and ρ are called effectiveness factors /14/.

The effectiveness factor ν is primarily a measure of concrete ductility, and comparison with tests shows that the effective concrete strength νf_c is approximately proportional to the square root of the cylinder strength, i.e. $\nu = K/\sqrt{f_c}$.

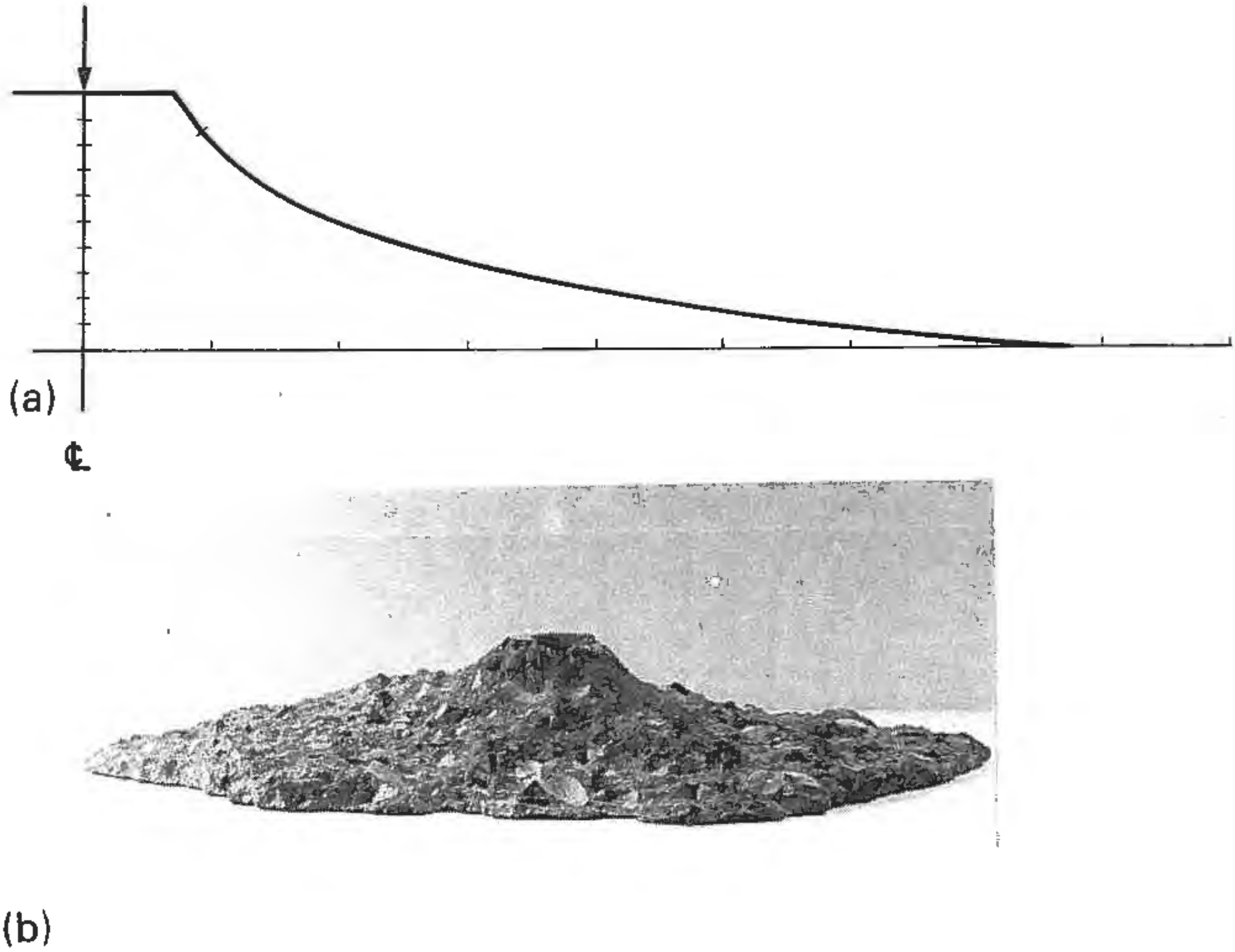

FIG. 2. Punching failure surface generatrix
a) Plastic solution b) Experimental profile

The optimal failure surface generatrix is found by variational calculus to be a catenary, possibly combined with a tangent inclined at the angle ϕ with the slab normal, see FIG. 2a. If the concrete tensile strength is neglected ($\rho = 0$) the optimal failure surface will reach as far as the slab supports. An extent of the failure surface which agrees with test results requires the introduction of a finite, albeit very small, tensile strength. An experimentally produced failure profile is shown in FIG. 2b, whereas FIG. 2a shows the failure surface generatrix corresponding to the same slab thickness and load diameter, theoretically determined with $\phi = 37°$ and $\rho = 1/400$.

The corresponding upper bound cannot be given explicitly, since it requires the solution of a transcendental equation, but curves for the ultimate load as a function of geometrical and physical parameters can be generated by a simple numerical procedure /6/.

An approximate expression for the punching capacity is obtained by assuming that the failure surface is a truncated cone, with the generatrix at the angle $\alpha \leq \pi/2 - \phi$ to the slab plane. This results in the upper bound:

$$V = \pi(B + h\cot\alpha)h \, \frac{1 - \cos\alpha}{2\sin\alpha} \, K\sqrt{f_c} \qquad (7)$$

Here the tensile concrete strength is neglected, and the effective compressive strength is inserted as $\nu f_c = K\sqrt{f_c}$.

Equation (7) is of the same form as (1), with:

$$t = h$$

$$\tau_c = \frac{1 - \cos\alpha}{2\sin\alpha} \, K\sqrt{f_c}$$

$$\beta = 0.5 \cot\alpha$$

Thus the control perimeter approach to punching analysis corresponds to an upper bound plastic solution to the slab punching problem.

3. EXPERIMENTAL EVIDENCE

Punching shear tests from the literature, involving circular and square loaded areas, are used as experimental background for an assessment of the various expressions for the punching resistance. The tests were introduced in a state-of-the-art report by Regan & Braestrup /15/, where the selection criteria are described. In particular it should be noted that the minimum slab thickness h is 100 mm, and that the maximum load size B (diameter or side length) is 3.5d. The database contains 29 tests with circular loading and 37 tests with square loads. (The database in /15/ comprises 68 tests, of which 2 have been excluded because they involve rectangular loads).

The comparison is made by calculation of a strength factor v defined as the ratio between the observed ultimate load V_u and the capacity V determined by the expression under consideration. Thus the average value v corresponds to a safety factor over and above the safety level corresponding to the partial coefficients on load and strength parameters, whereas the scatter of the values describes the consistency of the expression.

For the five code formulae described above the 66 test results give the following average values and coefficients of variation:

MC 78 : $v_m = 2.22 \pm 20\%$
EC 2 : $v_e = 1.98 \pm 14\%$
ACI 318-83 : $v_a = 1.47 \pm 15\%$
BS 8110 : $v_b = 1.00 \pm 9\%$
DS 411 : $v_o = 0.88 \pm 14\%$

It appears that the CEB-FIP Model Code is the most conservative and the least consistent. The best agreement with tests is found with the British code, which is tied up with the fact that it is calibrated against experimental evidence which is largely identical with the considered database. Finally it is evident that the Danish code is unconservative for punching, the strength being overestimated by (1.00 - 0.88)/0.88 = 14%.

In fairness it should be noted that the above values do not faithfully reflect the code predictions, because the upper limits on the influence of the slab depth and reinforcement ratio have not been invoked, which will affect the strength factors for MC 78, EC 2 and BS 8110. The upper limits have been omitted to facilitate the investigation of the influence of the individual parameters.

In the following FIGS. 3 - 7 the factors $v = V_u/V$ between experimentally observed failure loads and predicted capacities according to the code formulae are plotted against a) load size to slab depth ratio B/h, b) slab depth h, c) concrete strength f_c, and d) reinforcement ratio r. Each plot provides an assessment of the influence of the parameter in relation to the code formula. An increasing trend of the point set indicates that the prediction underestimates a positive influence of an increase in the value of the considered parameter (or overestimates a negative influence). Note, however, that this conclusion is indicative only, since the database is not sufficiently large to justify the assumption that the various parameters are independent. With this in mind the influences of the individual parameters are reviewed below.

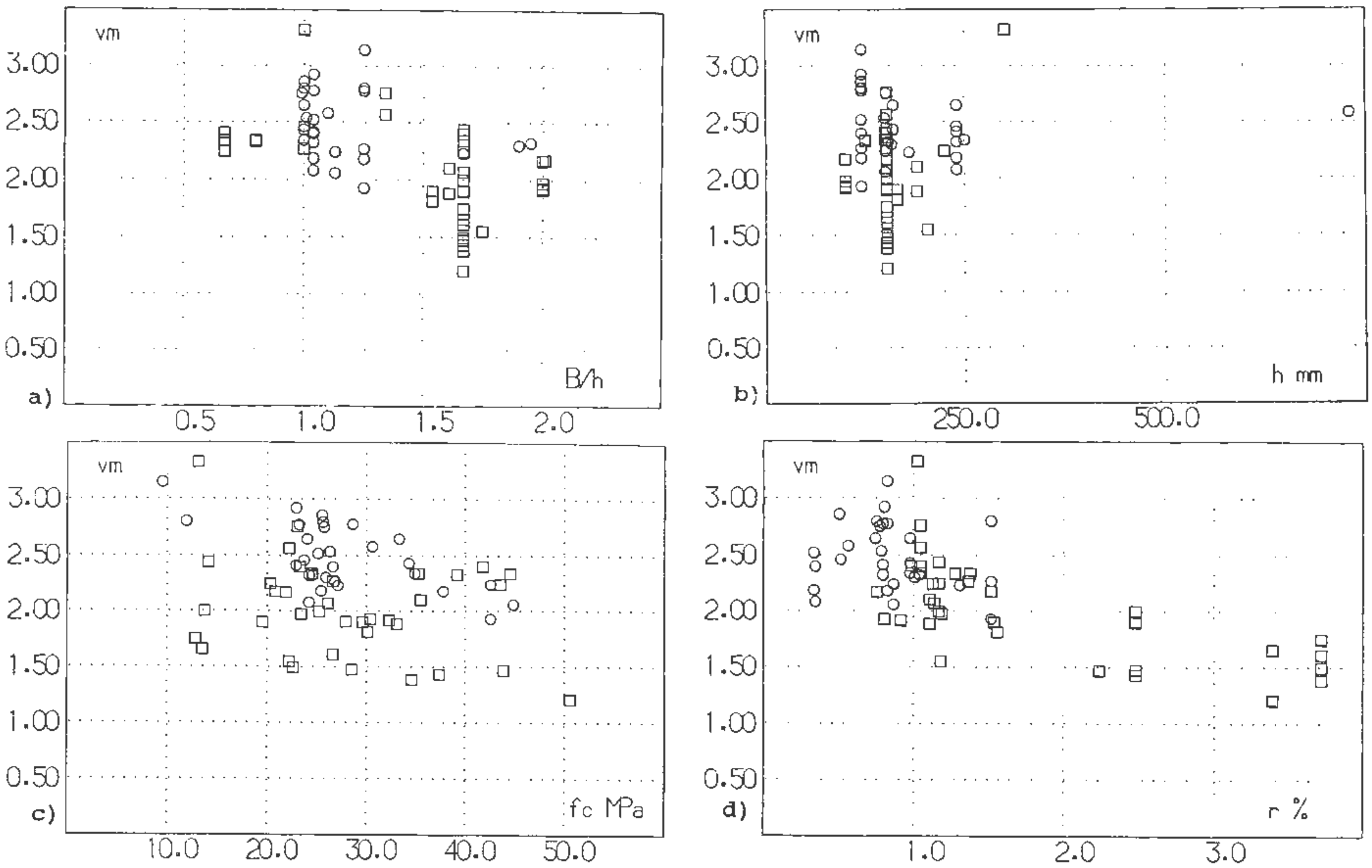

FIG. 3. Strength factors v_m for MC78

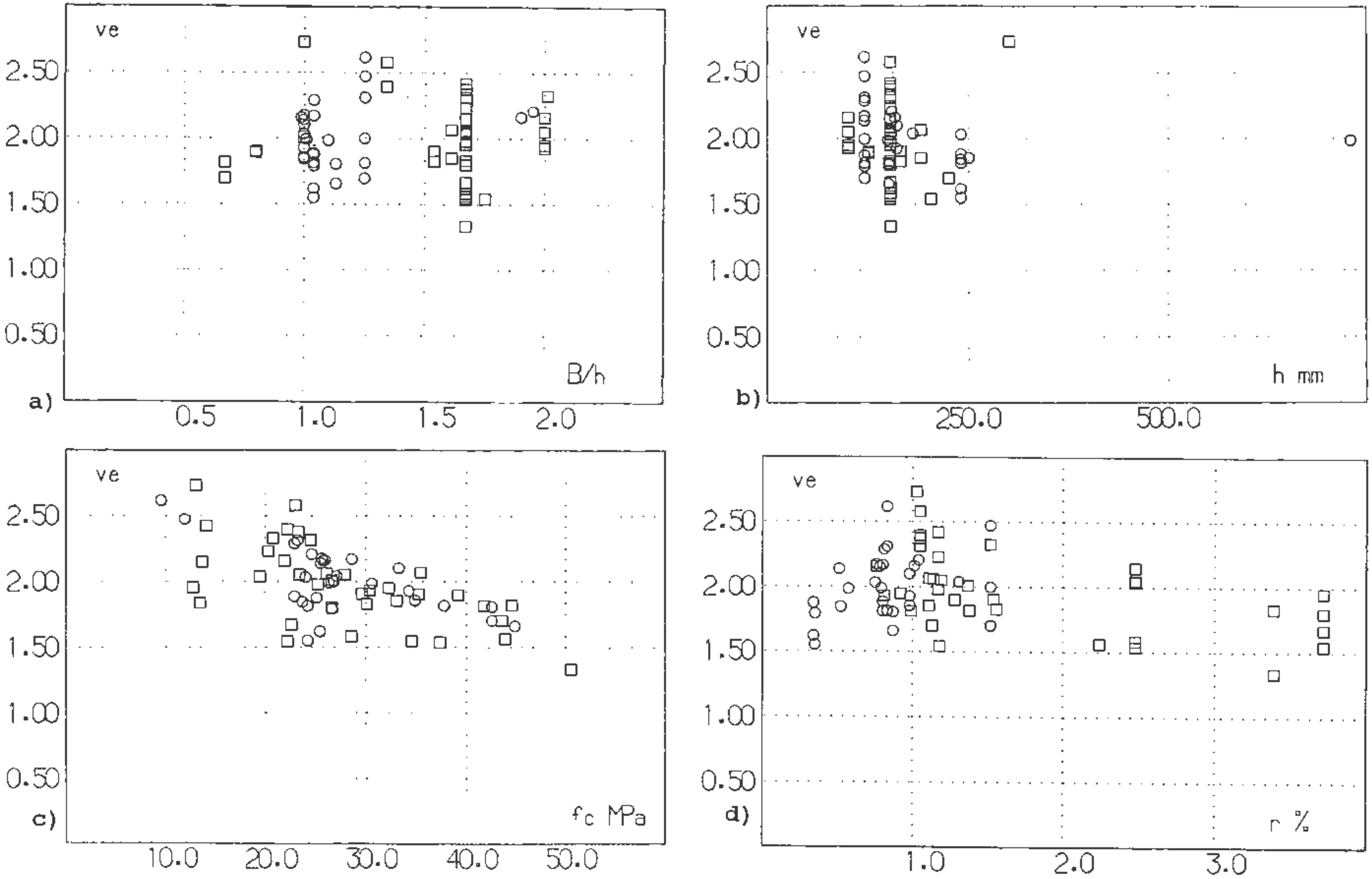

FIG. 4. Strength factors v_e for EC2

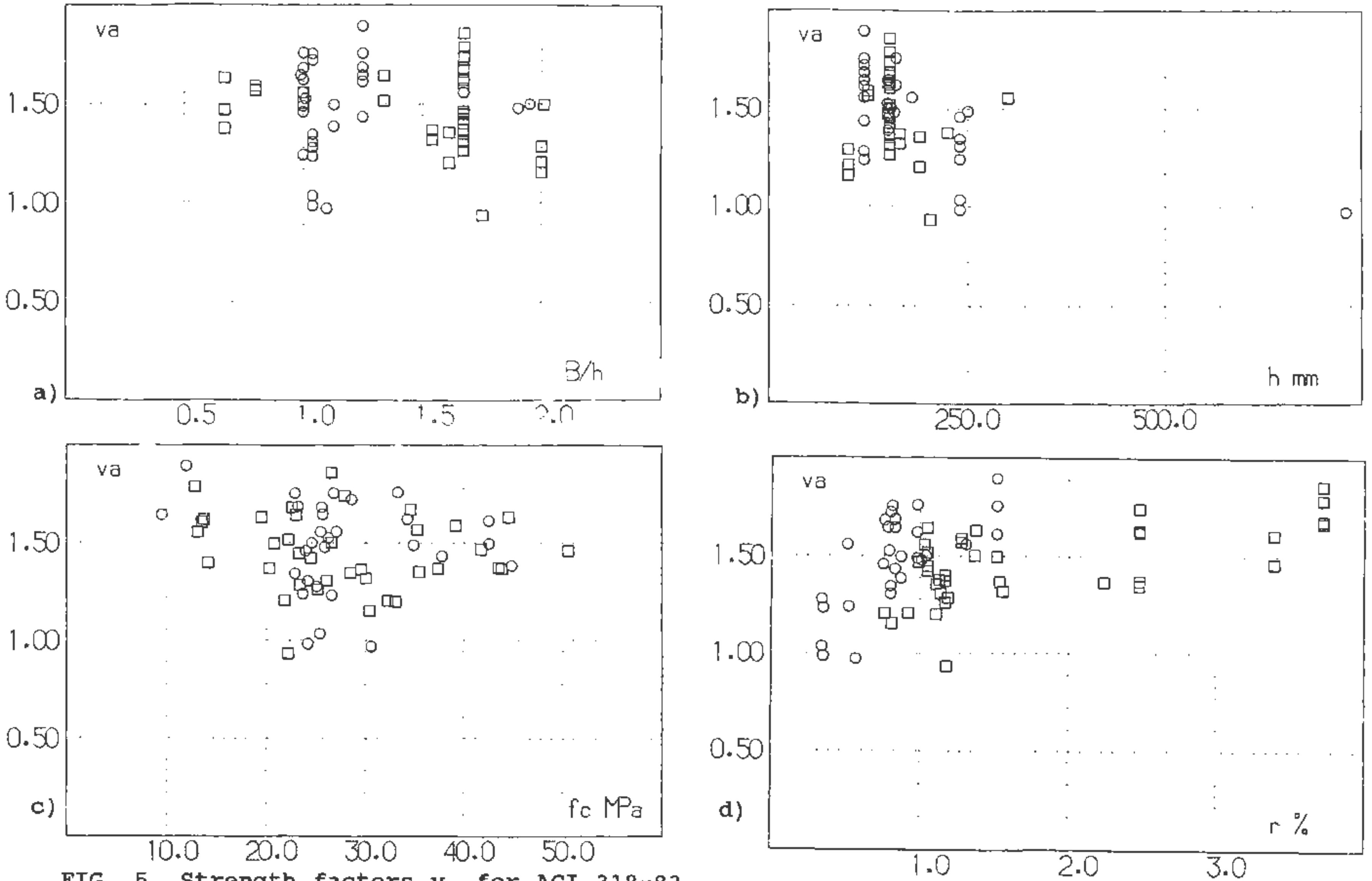

FIG. 5. Strength factors v_a for ACI 318-83

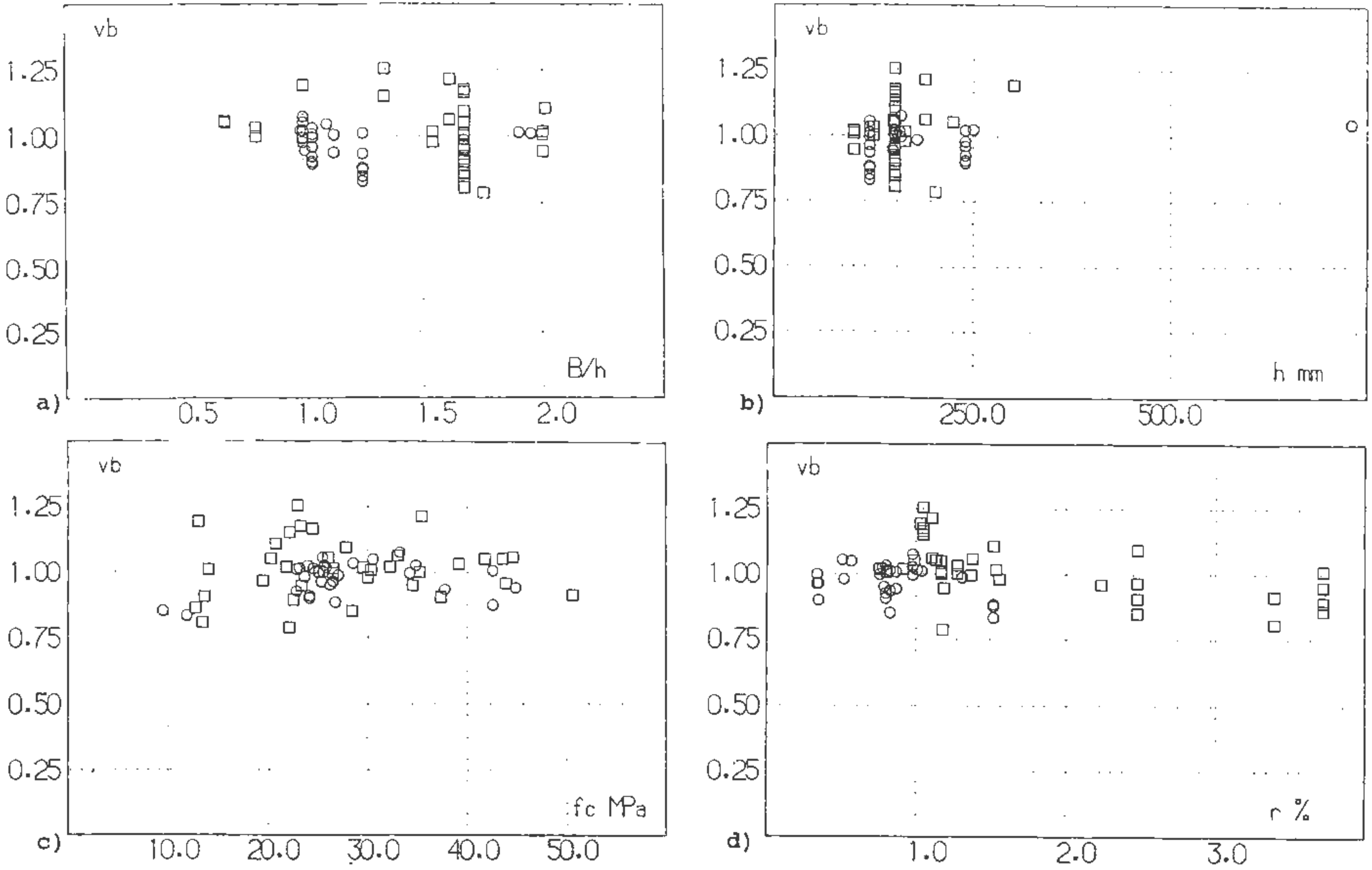

FIG. 6. Strength factors v_b for BS8110

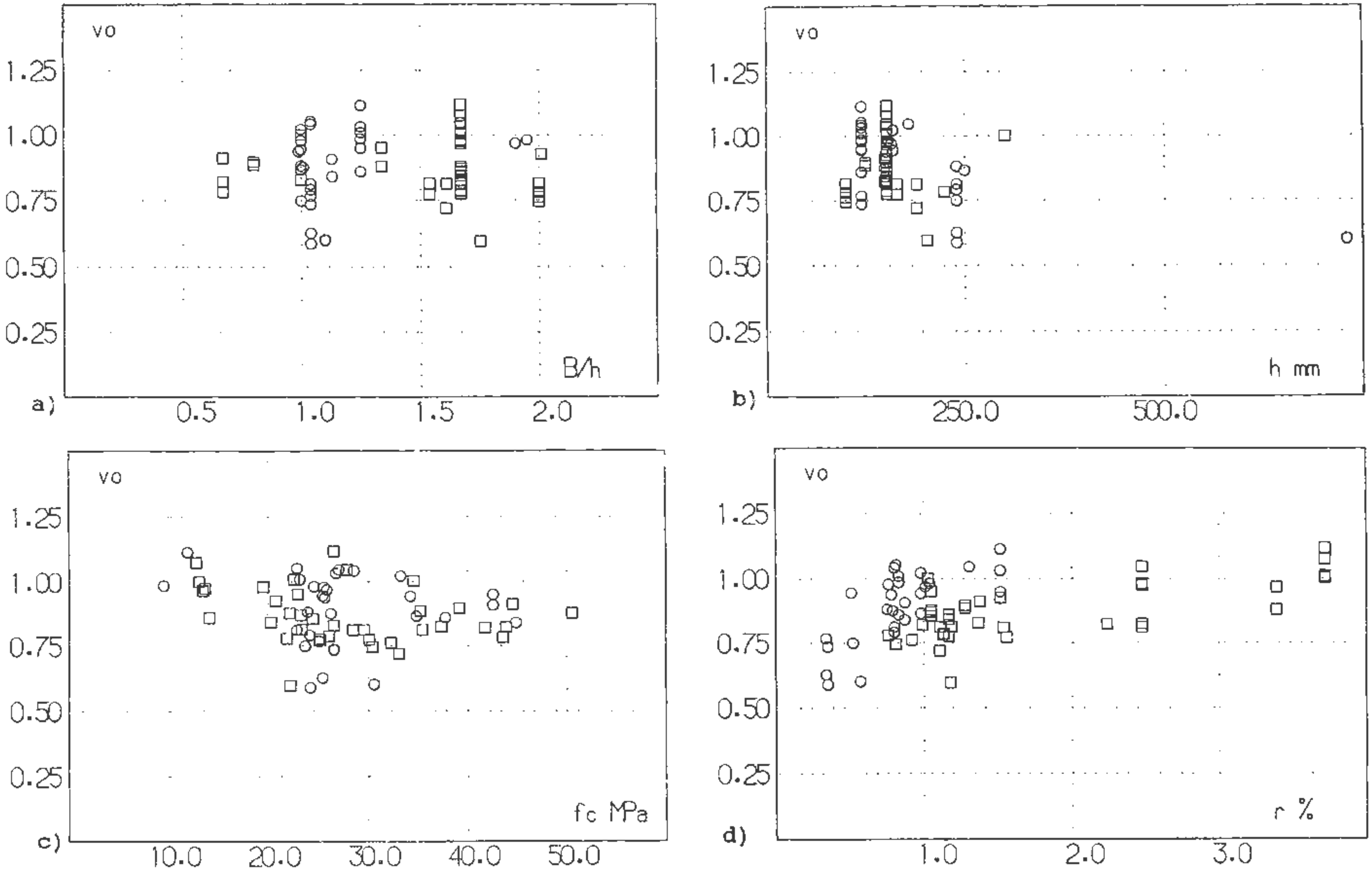

FIG. 7. Strength factors v_o for DS411

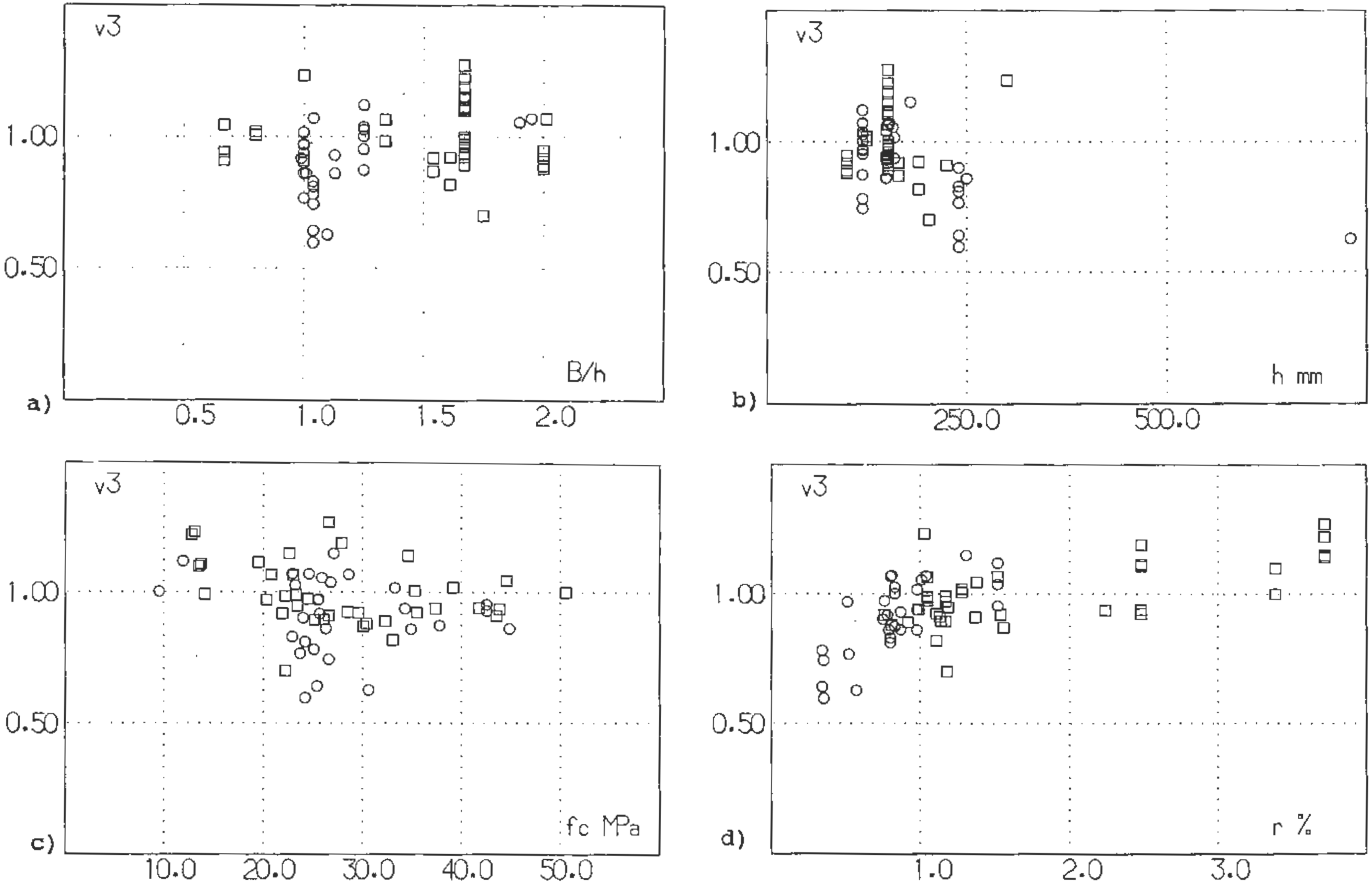

FIG. 8. Strength factors v_3 for DS411, revision proposal

3.1 Control perimeter

The main difference between the various code rules concerns the distance from the loaded area to the control perimeter. The distance 0.5d is used by many codes, including MC 78 and ACI 318-83, but the latter allows a polygonal control perimeter, whereas the former insists on rounded corners for a polygonal loaded area.

The larger distance 1.5d was introduced by BS 8110, and has recently been adopted by the draft EC 2, but whereas the former uses a rectangular control perimeter in all cases, the latter requires rounded corners as in MC 78.

The intermediate distance 1.0d is adopted by DS 411, which also allows polygonal control perimeters.

The closer the control perimeter is placed to the loaded area, the larger is the influence of the load size B relative to the slab depth. Consequently, the strength factors have been plotted against the ratio B/h. It is noted that calculations after MC 78 and ACI 318-83 give relatively lower formal shear stresses at large load sizes. In the Model Code this is compensated by a reduction of the strength parameter for B > 3.5d. The present American code contains no such provision, but a proposed revision /4/ introduces a reduction in strength for large loads, which for circular loading will be effective for B > 5.4d. However, the effect is noticeable over the entire range of load sizes, see FIGS. 3a and 5a.

The draft EC 2, BS 8110 and DS 411 all give consistent results, in the sense that the safety level is largely independent of the ratio B/h, see FIGS. 4a, 6a and 7a. It may be concluded that the distance 0.5d to the control perimeter is too small, whereas both 1.0d and 1.5d are satisfactory.

The influence of the shape of the control perimeter is investigated by separating the results of the database into tests with square and circular loads. The corresponding average values and variation coefficients for the strength factor are as follows:

Load Shape	Square	Circular
No. of tests	37	29
MC 78	2.02 ± 21%	2.48 ± 12%
EC 2	1.97 ± 16%	2.00 ± 13%
ACI 318-83	1.45 ± 13%	1.48 ± 16%
BS 8110	1.01 ± 11%	0.97 ± 7%
DS 411	0.87 ± 13%	0.89 ± 16%

The individual values are plotted in FIGS. 3 - 7 using square, respectively circular, symbols.

If the average value turns out to be lower for square loading than for circular it indicates that - everything else being equal - the adopted control perimeter is too large for square loading (or too small for circular loading). This might be used to assess the consistency of the various combinations of control perimeters for square and circular loading, viz.:

- square with rounded corners/circle (MC 78, EC 2)
- square/circle (ACI 318-83, DS 411)
- square/square (BS 8110)

However, the differences are drowned by the variation of the other parameters. Thus we note that square loading is overrepresented among the slabs with high reinforcement ratios, which tends to reduce the strength factors for MC 78, see FIG. 3d, cf. also Section 3.4 below.

The analysis may be repeated, including only tests with roughly similar parameters (20 MPa $\leq f_c \leq$ 40 MPa, 100 mm $\leq h \leq$ 250 mm, 0.3% $\leq r \leq$ 2.0%), resulting in a reduced database of 21 slabs with square loading and 23 with circular. That does not improve the picture, however, the differences between the mean values are simply small compared with the standard deviations, making it impossible to conclude which definition of the control perimeter is the most consistent.

3.2 Slab thickness

The slab thickness is represented by the effective depth to the reinforcement in all the considered codes, except DS 411 which uses the total thickness of the slab. That is largely a matter of taste, and the difference is insignificant.

However, the magnitude of the slab depth may be used to investigate the influence of specimen size, wherefore the strength factors are plotted against h. Apart from a single test with h = 730 mm most thicknesses are in the interval 100 mm $\leq h \leq$ 250 mm, for which reason the plots give no clear picture of the influence of this parameter. Indications are, however, that there is a reduction in safety level for large slab depths, which is not reflected by the ACI 318-83 and DS 411 formulae, that take no account of size, see FIGS. 5b and 7b.

Reduction factors are introduced into MC 78, EC 2 and BS 8110, the two former being identical in this respect. In both cases the reduction seems to slightly overestimate the size effect, cf. FIGS. 3b, 4b and 6b. In MC 78/EC 2 this is corrected by the provision that the reduction is only effective for h < 600 m, a modification which is not considered in the plots.

3.3 Concrete strength

The strength factors are plotted against the concrete cylinder strength f_c in order to assess the appropriateness of the adopted concrete strength parameters. The plots indicate that MC 78 and EC 2 overestimate the influence of concrete strength by assuming that the ultimate load is proportional with $f_c^{2/3}$, see

FIGS. 3c and 4c. On the other hand, consistent results are obtained by assuming proportionality with $\sqrt{f_c}$, as ACI 318-83 and DS 411, see FIGS. 5c and 7c, or with $f_c^{1/3}$ as BS 8110, see FIG. 6c.

3.3 Reinforcement

The influence of the reinforcement is investigated by plotting the strength factors against the reinforcement ratio r. Neither ACI 318-83 nor DS 411 take any account of slab reinforcement, which results in a slightly higher safety level for increasing amount of reinforcement, see FIGS. 5d and 7d. A strength increase as a function of the reinforcement ratio is introduced into MC 78, EC 2 and BS 8110, and in all cases the positive influence of reinforcement seems to be slightly overestimated, see FIGS. 3d, 4d and 6d. This is counteracted by the various upper limits for the beneficial effect of the reinforcement, which are not considered in the plots.

4. REVISION OF THE DANISH CONCRETE CODE

As stated above, DS 411 is non-conservative, i.e. the safety level for punching is lower than for other types of failure. Adopting the philosophy that the safety should be given by the design values of loads and material strengths, the mean value of the strength factors should be 1.0, as is the case with BS 8110. Furthermore, the treatment of punching in the Danish code consists of a few lines of commentary text, which may seem rather modest in view of the complexity and practical importance of the problem, even considering the Danish code tradition for brevity.

The current text in DS 411, 3rd edition, March 1984, is directly carried over from the 2nd edition, December 1973. Already in the seventies it was realized that the code was non-conservative, cf. Hess & al. /13/, but a revision was postponed until a later date. In July 1987 a working group was established with the task to prepare a proposal for revision of DS 411 on various points, including punching.

The revision proposal, issued for public comments in 1989, is apart from central punching concerned with the following aspects, not treated in the 3rd edition of DS 411:

- variable slab thickness
- edges and corners
- holes in the slab
- shear reinforcement

The proposal is reviewed in the sections below.

4.1 Nominal shear stress

As shown in Section 3.1 consistent results are obtained with a control perimeter in a distance from the loaded area which equals the slab depth, so DS 411 is upheld in this respect. However, it is proposed that the effective depth d be replaced by the slab thickness h, since it does not appear reasonable that the punching capacity should depend upon the position of the reinforcement, when the latter does not otherwise appear in the strength formula, cf. below. At any rate, it is not likely that an increase in concrete cover - everything else being equal - should reduce the loadcarrying capacity. An empirical relationship established by Alexander & Simmons /1/ shows a significant trend in the opposite direction. For the same reason, the total slab thickness h is maintained as the slab depth measure used to determine the nominal shear stress.

Concerning the shape of the control perimeter, it is taken to be rectangular in all cases, as in BS 8110. As shown in Section 3.1 it makes no discernable difference, and it leads to considerable practical simplifications, particularly for loads near holes or free edges.

Insertion of the above modifications into equation (6) leads to the following expression for the punching strength for a square load of side length B:

$$V = 4(B + 2h)h \sqrt{0.1\, f_c} \qquad \text{(square)} \qquad (8a)$$

Analysis of the 37 tests with square loading yields the following mean value and coefficient of variation for the strength factor V_u/V:

$$v_1 = 0.76 \pm 12\% \qquad \text{(square)}$$

Application of (8a) to the 29 tests with circular loading yields:

$$v_1 = 0.62 \pm 15\% \qquad \text{(circular)}$$

It appears that the safety level is lower for circular than for square loading, which shows that the control perimeter is too large if a circular loaded area is replaced by the circumscribed square. Specifying the inscribed square, we find:

$$V = 4(B\sqrt{2}/2 + 2h)h \sqrt{0.1\, f_c} \qquad \text{(circular)} \qquad (8b)$$

resulting in the strength factor:

$$v_2 = 0.69 \pm 16\%$$

This is more consistent with the square loads, albeit still somewhat less conservative. Other reasonable possibilities, such as the square with the samme area (side length $B\pi/4$) or with the same circumference (side length $B\sqrt{\pi}/2$) would give intermediate values. It is therefore generally proposed that the control perimeter for non-rectangular loaded areas be determined from an inscribed rectangle.

4.2 Strength parameter

The punching capacity according to DS 411 does not depend upon the slab reinforcement, and this is maintained, although it is somewhat unconservative for small reinforcement ratios, cf. Section 3.4. Neither is it proposed to introduce a correction factor to account for the size effect, although it is somewhat unconservative for large slab thicknesses, cf. Section 3.2. Consequently, the strength parameter is assumed to depend upon the concrete grade alone.

As shown in Section 3.3 it leads to consistent results to assume the punching capacity to be proportional with $\sqrt{f_c}$, as in the present DS 411. However, the strength parameter $\sqrt{0.1\ f_c}$ is identified with the tensile concrete strength, which is unfortunate since failure by punching is no tensile failure.

Instead it is proposed to introduce the effective concrete strength νf_c, where the effectiveness factor is inserted as $\nu = 2/\sqrt{f_c}$, which is the same value proposed for a revision of the DS 411 code clause on beams and slabs without shear reinforcement, cf. Feddersen /11/.

For the total database of 66 tests, the formulae (8a) and (8b) yield the strength factor:

$$v_2 = 0.73 \pm 15\%$$

This implies that the predicted nominal shear stress at punching failure is:

$$\tau = 0.73\ \sqrt{0.1\ f_c}$$

Assuming the strength parameter to be proportional to νf_c, we find:

$$\tau_c = K_1 \nu f_c = 2K_1\ \sqrt{f_c}$$

The factor K_1 is found by putting $\tau = \tau_c$:

$$K_1 = \frac{1}{2}\ 0.73\ \sqrt{0.1} = 0.1154 \approx 0.12$$

Thus

$$\tau_c = 2K_1\ \sqrt{f_c} = 0.24\ \sqrt{f_c}$$

This in the same critical shear stress which is proposed for beams and slabs without shear reinforcement /11/, except for correction factors accounting for the influence of longitudinal reinforcement and arching action.

4.3 Axisymmetric punching

The proposal for a revision of DS 411 described in the two preceding sections results in the following formula for the punching resistance with circular loading:

$$V = 4(B\sqrt{2}/2 + 2h)h \cdot 0.24\ \sqrt{f_c} \tag{9}$$

Equation (9) is of the same form as (1), with:

$$t = h$$

$$\tau_c = \frac{0.48}{\pi}\ \sqrt{2f_c} = 0.216\ \sqrt{f_c}$$

$$\beta = \sqrt{2}$$

Comparison with equation (7) shows that the prediction corresponds to the plastic upper bound with:

$$\cot\alpha = 2\sqrt{2} = 2.83$$

$$K = \frac{0.48}{\pi}\ \sqrt{2} \cdot \frac{2\sin\alpha}{1-\cos\alpha} = 2.52$$

Thus the revision proposal corresponds to the upper bound solution for a conical failure surface at the inclination $\cot\alpha = 2\sqrt{2}$, corresponding to $\alpha = 19.5°$, and an effectiveness factor $\nu = 2.52/\sqrt{f_c}$.

4.4 Non-circular loading

The proposed revision leads to the following expression for the punching strength of a slab subjected to a rectangular load with side lengths B and L:

$$V = 2(B + L + 4h)0.24\ \sqrt{f_c} \tag{10}$$

If the loaded area is non-rectangular then B and L are the dimensions of an inscribed rectangle.

The strength factors V_u/V calculated by equation (10) for the 66 tests with square and circular loading are plotted in FIGS. 8 against the parameters B/h, h, f_c and r. Mean value and coefficient of variation are found to be:

$$v_3 = 0.96 \pm 15\%$$

This confirms that the revision proposal gives a reasonably safe and consistent evaluation of the punching capacity of slabs.

To assess the validity of the revision proposal for rectangular loaded areas, equation (10) is applied to a test series reported by Hawkins, Fallsen & Hinojosa /12/. The series comprises 9 square slabs, loaded along the edge and supported on rectangular columns with varying aspect ratios. The strength factors are plotted in FIG. 9 against the parameter L/B, the mean value and coefficient of variation being:

$$v_4 = 0.73 \pm 7\%$$

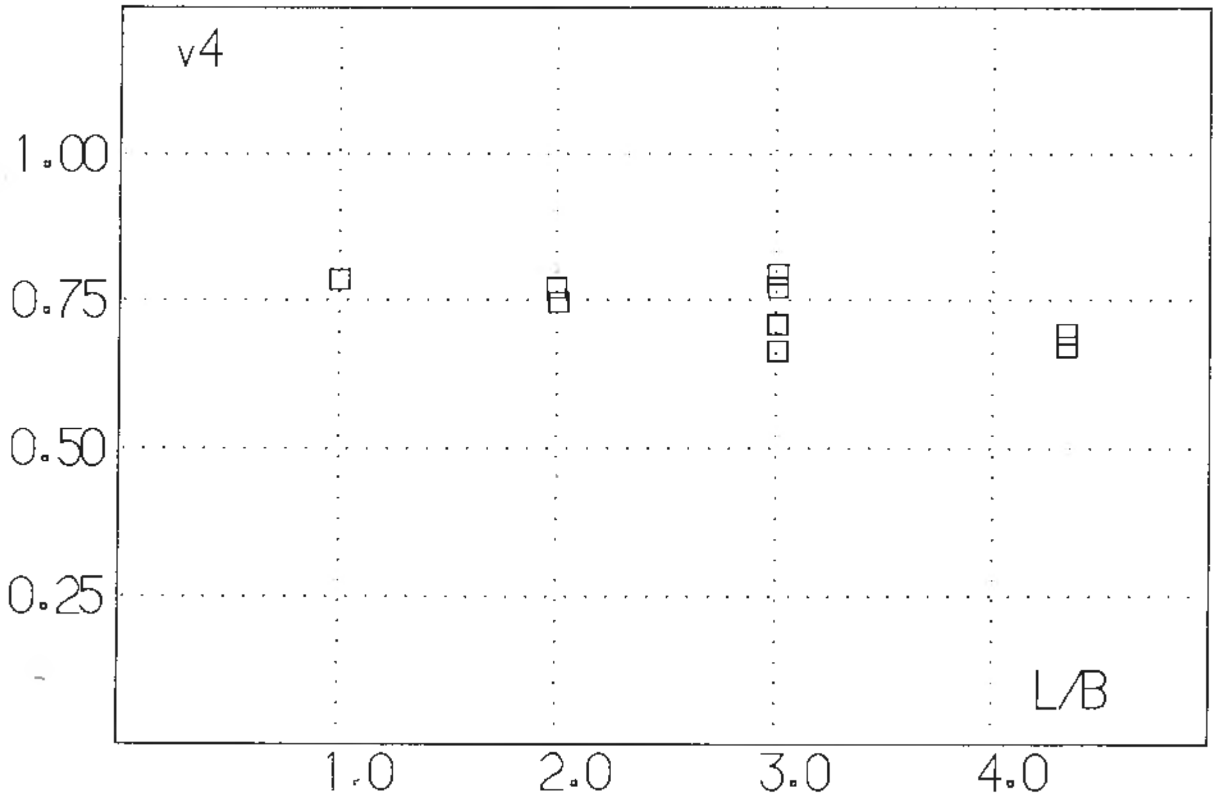

FIG. 9 Strength factors for DS 411, Revision Proposal, plotted against rectangular load aspect ratio

The mean is somewhat low, but there is no significant systematic variation with the aspect ratio.

4.5 Variable slab depth

When the above approach is generalized to cases where the depth of the slab is not constant, it is reasonable to regard the failure surface as being represented by a conoid with generatrices at 45° to the slab plane, and spanning between a loaded area and a control perimeter on the opposite slab face. The concrete area used to define the nominal shear stress is found as the product of the control perimeter and the (average) projection of the conoid on the slab normal.

To identify the most critical situation, the nominal concrete area should be minimized, and it may be necessary to consider different combinations of control perimeters and loaded areas. In FIG. 10 is shown a case with three control perimeters u_1, u_2, u_3 and the corresponding slab depths h_1, h_2, h_3.

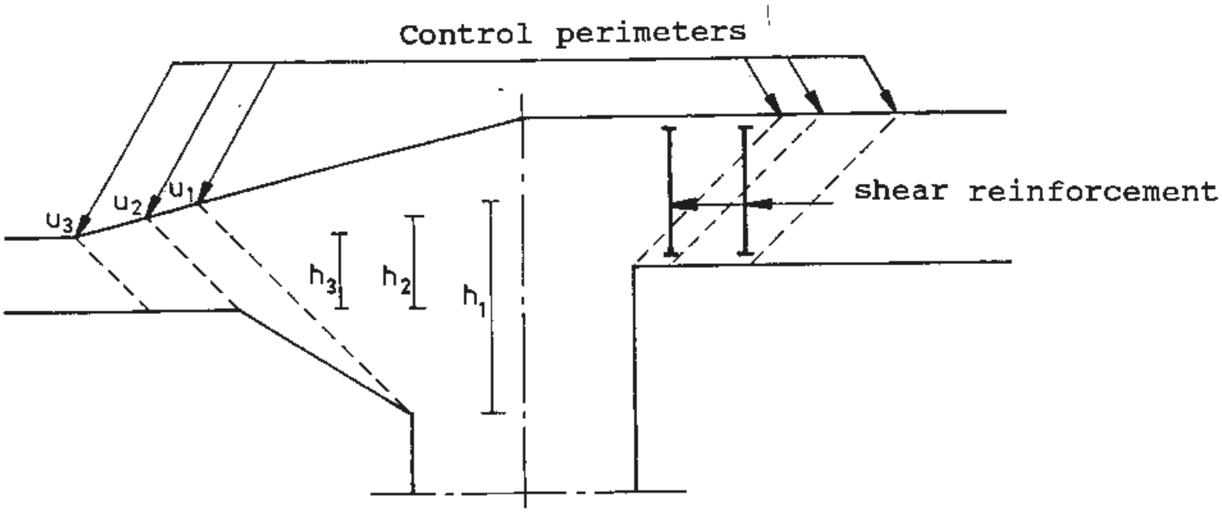

FIG. 10 Control perimeters in case of variable slab depth or shear reinforcement

Note that the perimeters u_1 and u_2 are defined by the column, respectively the column capital, which if non-rectangular are replaced by inscribed rectangles, whereas u_3 is defined by the discontinuity of the opposite slab face, which may have any (convex) shape in the slab plane.

4.6 Free edges

If the load is close to a free edge of the slab, the failure surface may reach the edge. This is proposed to be accounted for by a reduction of the control perimeter, which is defined as the shortest curve in the slab enclosing the rectangle in the distance h from the loaded area.

By punching at edge or corner columns it is likely that the normal load is accompanied by a bending moment, and most codes have special rules for excentric punching. It is, however, Danish practice to regard the punching capacity as independent of any co-existing moment action, which is assumed to be resisted by the slab reinforcement. This approach is maintained in the revision proposal, whereas the other investigated codes treat loads near slab edges as excentric punching. Thus ACI 318-83 prescribes a rather complicated calculation of a maximum nominal shear stress based upon the statical moment of a cylindrical control surface at the distance 0.5d.

The MC 78 Compléments /8/ also allow a uniform distribution of the nominal shear stress on a reduced control surface, which reaches the slab edge. The determination is, however, complicated due to the rounded corners. The proposed rectangular shape of the primary control perimeter considerably simplifies the determination of alternative, shorter perimeters in the vicinity of free edges. As mentioned in Section 2 the control perimeter is an empirical quantity, used to define a nominal shear stress.

Consequently, it is not reasonable to undertake any detailed calculation of control perimeters and shear stress distributions.

The so-called punching failure at an edge column is in reality a torsional failure of the edge zone, and the revision proposal stresses the need for reinforcement against torsional boundary moments.

4.7 Holes in the slab

Most codes reduce the length of the control perimeter by the polar projection of holes in the vicinity of the load, but the maximum distance within which holes should be considered varies from 5d to 10h. It is proposed to simplify the treatment by considering a hole outside the primary control perimeter (at the distance h) as a free edge. Thus the control perimeter is modified to include the hole if and only if it reduces the length. For holes within the control perimeter it is proposed to reduce the length by the polar projection.

4.8 Shear reinforcement

Building codes normally allow an increase of the punching capacity by the provision of shear reinforcement within a specified region around the load, but different rules apply to the calculation of the corresponding concrete contribution and the maximum total strength. Although not stated explicitly in DS 411 it is normal Danish practice to add the yield force of shear reinforcement within the control perimeter. This is formalized in the revision proposal, but an upper limit of 100% is introduced for the strength increase.

The shear reinforcement must be fully anchored, and it may be necessary to check control perimeters corresponding to failure surfaces wholly or partly outside the shear reinforcement, see FIG. 10.

4.9 Supported slab edges

An increase in punching strength is obtained if the opposite slab face is supported in the immediate vicinity of the load /15/. This is not taken into account in any of the considered codes, and not in the revision proposal either. It would be fairly simple, however, to introduce a correction factor, like the one describing arching action in beams and slabs without shear reinforcement /11/.

NOTATION

B : Diameter of circular load or (smaller) side length of square (rectangular) load

d : Effective depth to reinforcement of slab (m)

f_c : Concrete cylinder strength (MPa)

h : Slab debth

L : Longer side length of rectangular load

r : Geometrical reinforcement ratio

t : Slab thickness measure

V : Calculated punching capacity

V_u : Observed punching load

α : Inclination of conical failure surface

β : Factor defining distance to control perimeter

ν : Effectiveness factor (compressive strength)

ρ : Effectiveness factor (tensile strength)

τ : Nominal shear stress

τ_c : Concrete strength parameter (MPa)

ϕ : Angle of internal friction for concrete

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