SHEAR CAPACITY OF COMPOSITE PRESTRESSED CONCRETE BEAMS

Krister CEDERWALL, Professor
Division of Concrete Structures, Chalmers University of Technology, Gothenburg, Sweden

Elzbieta SARAN, Researcher
Division of Concrete Structures, Chalmers University of Technology, Gothenburg, Sweden

The results of experimental investigations of the shear capacity of composite prestressed concrete I-beams are summarized. The test specimens consist of the precast and prestressed lower part of the beams and upper part reinforced and cast in situ, with the joint between the parts in the web. A series of 16 full-scale beams was tested, including 4 non-prestressed reference beams (2 homogeneous and 2 composite) and 12 prestressed beams (3 homogeneous and 9 composite). The amount of prestress was varied. On the basis of the test results the relevance to composite beams of the equation in the Swedish Code BBK-79 for shear capacity of homogeneous prestressed concrete beams is discussed. The test series indicate a slight overestimation of the beneficial influence of prestressing, greater for homogeneous beams than for composite beams. It can be observed that composite beams can be treated with the same approach as homogeneous beams, however more sophisticated approaches are recommended.

Keywords: beam, composite, prestressed, concrete, shear capacity

1. INTRODUCTION AND BACKGROUND

In most building codes for concrete structures, the Swedish code /1/ included, the shear capacity of prestressed members are evaluated with the truss analogy incorporating an empirical correction term, the so-called concrete contribution. This added shear capacity V_c is often assumed to correspond to the shear at the beginning of shear cracking.

For non-prestressed members the ultimate shear capacity, V_u is estimated from the expression:

$$V_u = V_s + V_c$$

where V_s is the capacity according to the truss analogy (the contribution of the shear reinforcement) and with the inclination Θ of the concrete compressive struts generally limited to values close to $\Theta = 45^0$.

The expression (1) is often referred to as some kind of an addition theory. For prestressed members the concrete contribution V_c is favorably affected by the prestress force P_e and the increase V_p attributed to the prestress force is formulated as:

$$V_p = M_0 \,/\, a \qquad (2)$$

where

$$a = M_d \,/\, V_d$$

In fully prestressed members the influence of the prestressing force is substantial, and shear reinforcement is as a rule only needed in a low degree.

These described design recommendations are experimentally verified for homogeneous beams. In the last decades there has been an extensive use of composite concrete beams in Sweden, where the lower part of the beam elements are prestressed, prefabricated and delivered to the building site where they are united with an upper part concreted in situ. A good example of this kind of combined prefabricated and cast in situ components is presented in Fig.1.

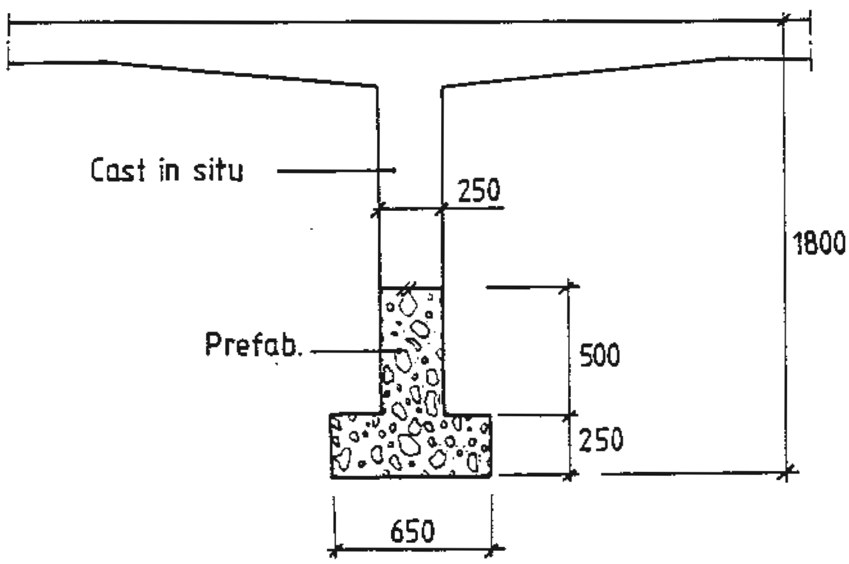

Fig. 1 Composite beam consisting of a prefabricated lower part and a cast in situ upper part. Span = 30 m. Valhalla skating-rink, Gothenburg.

The high strength and prestressed bottom part is manufactured at a factory some 30 km from Gothenburg and transported by lorry to the building site. For these kind of composite members the Swedish code gives no guidance concerning the effect of prestressing on the shear capacity. For the structure mentioned above the designer chose to neglect any favorable effects and the amount of shear reinforcement was rather massive.

To get more insight into this matter a test series was planned and today is almost completed. The main parameters studied have been:

(1) Concrete strengths in upper and lower parts of the composite beam

(2) Prestressing forces (full prestressing, partial prestressing and no prestressing)

The level of the joint has in most test beams been arranged as close as possible to the bottom flange and this test series is not sufficient to illuminate the effect of the joint level.

The joint was intended to be made to correspond to natural surface roughness. The amount of shear reinforcement was rather small, which agreed with the practice for homogeneous beams. The detailing of the joint reinforcement varied in the different test beams, but in most cases this reinforcement was sufficient to prevent a premature joint failure, thus the strength of the joint is not treated as a main parameter.

The lay-out of the test series and some preliminary evaluations of the test results are presented in this paper. It is moreover the intention of the authors to present later a more refined analysis of the test beams using the modified compression field theory /2/ and an extended version of this theory /3/.

2. TEST PROGRAM

The test beams reported here were all, with one exception, designed with a cross-section according to Fig. 2.

The test series comprises 16 beams some of which were non prestressed and used as reference beams (B7, B9, B10 and B16). The 16 beams are presented in Table 1 below. It was originally intended to include beams of different sectional geometries, but this was later on omitted, and only one beam B5 was made as a typical I-section with the same lay-out of the top flange as the bottom flange. This beam is therefore not considered as belonging to the population.

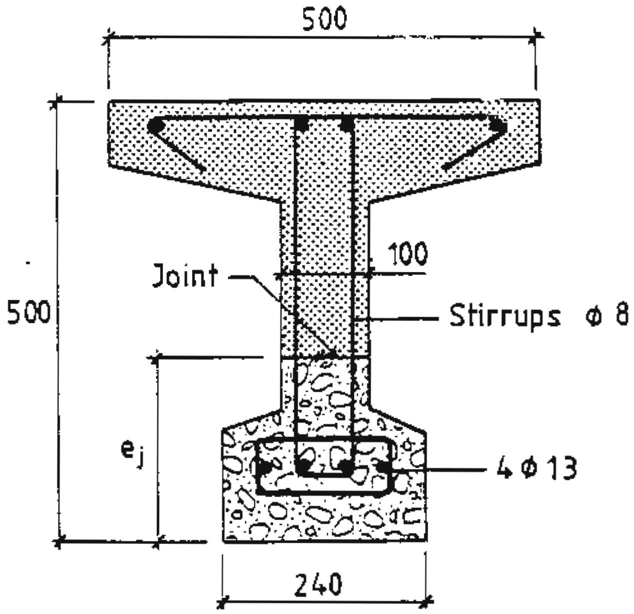

Fig. 2. Beam section - the level of the joint was chosen to e_j = 175 mm for all beams except B11 and B14 where e_j = 350 mm and 200 mm respectively

Table 1. Description of the test beams

Beam (1)	Type (2)	Pres. (3)	Joint (4)	Stir. (5)	Comments (6)
B1	Ho	FP	-	A	
B2	Ho	FP	-	A	
B3	Co	FP	Low	A+B	
B4	Co	FP	Low	A+B	
B5	Different geometry				Not ev.
B6	Co	PP	Low	A+B	
B7	Ho	NP	-	A	Reference
B8	Co	PP	Low	A	
B9	Co	NP	Low	A	Reference
B1o	Ho	NP	-	A	Reference
B11	Co	FP	High	A	JF.Not ev.
B12	Co	FP	Low	A	
B13	Co	FP	Low	A	
B14	Co	FP	Mid	A	
B15	Co	FP	Low	A	JF.Not ev.
B16	Co	NP	Low	A	Reference

All test beams were provided with a minimum shear reinforcement marked A in Table 1 and in Fig. 3. Shear reinforcement is placed with a spacing s = 500 mm, not allowing it to be considered as statically effective, but the inclination of the critical shear cracks and the following analysis of the test-beams show without a doubt that this shear reinforcement is fully effective. In order to avoid joint failure some of the beams were fitted with extra joint reinforcement marked B. This extra reinforcement extended just above the joint in order to affect the shear failure mechanism as little as possible. These beams B3, B4 and B6 must however be treated carefully and possibly omitted if they deviate from the general pattern. As is also to be seen from Fig. 3 all beams were fitted with splitting reinforcement close to the beam ends.

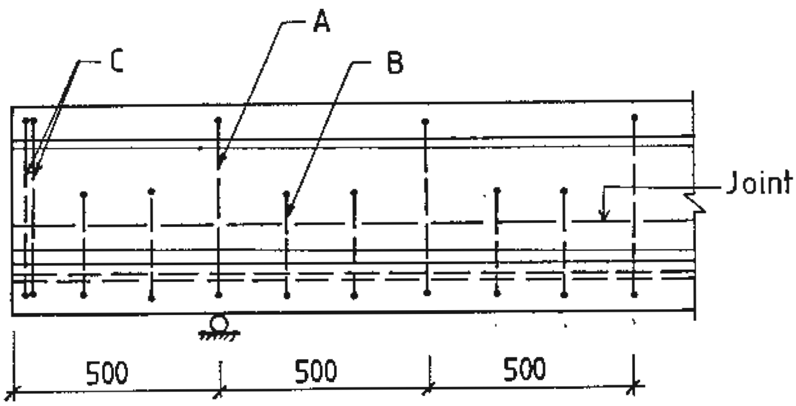

Fig. 3 Detailing of transverse reinforcement. A= Two-legged stirrups as ordinary minimum shear reinforcement: ϕ8, f_{sy}`=530 MPa for B1-9 and $f_{sy}$`=480 MPa for B10-16, B= two legged stirrups: ϕ8 as extra joint reinforcement, C= two stirrups ϕ8 as splitting reinforcement

For the beams B11 and B15 a premature joint failure (JF) occurred and they are therefore not analyzed in this context. Some of the beams have been cast in one operation and are marked Ho (homogeneous) in Table 1. For the composite beams (Co) the joint level is indicated as low, middle or high. The amount of prestress has been kept on three levels, namely: full prestressing (FP), partial prestressing (PP) and no prestressing (NP). The main reinforcement has for all beams consisted of four strands ϕ13 mm (f_{sy} / f_{su} = 1680 MPa / 1920 MPa) and the effective prestressing forces amounted to about 400 kN in the FP- case and to about 200 kN in the PP- case. More precise values are given below together with the analysis of the test results. The concrete strength for the different components of the beams are also presented below.

3. SECTIONAL PROPERTIES OF THE TEST BEAMS

For the theoretical evaluation of the influence of the prestressing forces upon the shear capacity the sectional properties of the test beams with regard to different moduli of elasticity for the different components must be calculated. For this purpose the original section is transformed to an idealized "transformed" section according to Fig. 4. Sectional data for the geometrical section and for the three joint levels are given below in Table 2. For the composite beams the sectional data for the transformed section can be calculated from:

$$A_{tr} = A_b + \alpha * A_t$$

Table 2. Sectional data for the three joint levels of the test beams

Joint e_j mm	A_b mm^2*10^4	e_b mm	J_b mm^4*10^8	A_t mm^2*10^4	e_t mm	I_t mm^4*10^8
175	3.675	422	0.78	7.25	101	5.46
200	3.925	415	1.07	7.00	93	4.36
350	5.425	363	5.28	5.50	57	0.71

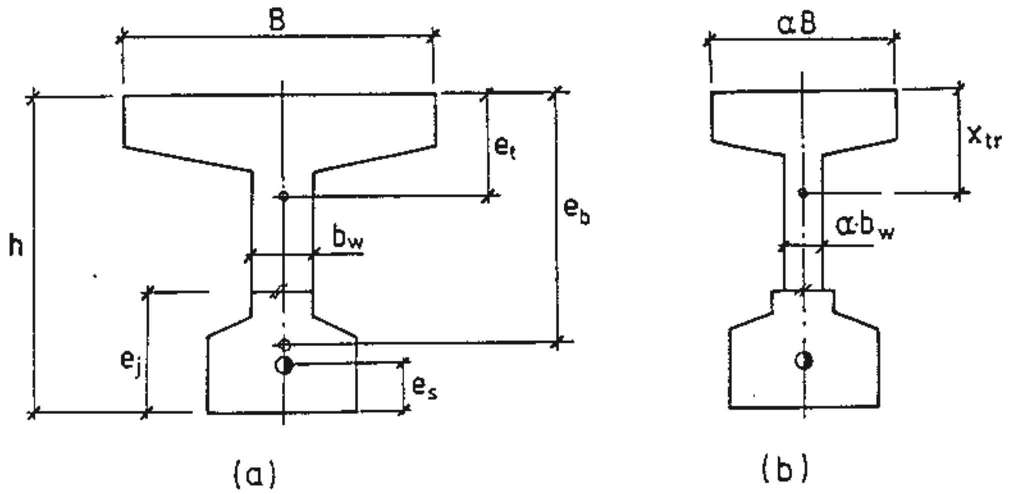

Fig. 4. Geometrical section (a) and transformed section (b) for composite test beams: h = 500 mm, B = 500 mm, b_w =100mm, e_s = 80 mm, α = relation between moduli of elasticities for the components.

$$x_{tr} = (A_b*e_b + \alpha\ A_t*e_t)/A_{tr}$$

$$I_{tr} = I_b+A_b*(e_b-x_{tr})^2+\alpha\ *I_t+\alpha\ *A_t*(x_{tr}-e_t)^2$$

The ratio α of the moduli of elasticities for the components of the composite beams has been evaluated as:

$$\alpha = \sqrt{f_{cct} / f_{ccb}}$$

The modulus of elasticity is assumed to be proportional to the square root of the compressive strength in agreement with general experience /4/. The sectional data corresponding to the transformed sections of the prestressed composite beams are summarized in Table 3 below. A value of $\alpha = 1$ corresponds to either a homogeneous section or a composite section with the same concrete strength in the bottom and the top part.

Table 3. Data for the transformed sections

Beam	f_{cct} MPa	f_{ccb} MPa	α	A_{tr} $mm^2 * 10^4$	x_{tr} mm	I_{tr} $mm^4 * 10^8$
B1	73.8	73.8	1.00	10.93	209	31.36
B2	40.7	40.7	1.00	10.93	209	31.36
B3	38.2	53.0	0.85	9.84	221	29.14
B4	14.6	84.8	0.41	6.65	278	19.95
B6	12.8	73.2	0.42	6.72	277	20.23
B8	25.8	79.3	0.57	7.81	252	23.94
B12	23.5	81.4	0.54	7.59	256	23.26
B13	73.6	76.0	0.98	10.78	210	31.09
B14	37.0	82.5	0.67	8.62	240	26.15

4. TEST ARRANGEMENTS AND TEST EQUIPMENT

All test beams were made with a total length of 5.0 m and tested freely supported with a distance between the supports of 4.0 m according to Fig. 5.

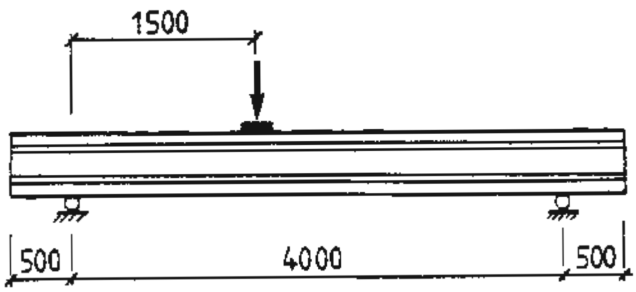

Fig. 5. Loading arrangements.

The arrangement corresponds to an overhang of 0.5 m, which according to our previous experience with these kind of strands is enough to give full transfer of the prestressing force and it could be relied upon that the full prestressing force was active over the span even if inclined cracks crossed the tension zone close to the support. The load was applied 1.5 m from the nearest support and both supports consisted of rollers because of the fact that the test rig consists of a stiff frame and the loading jack can be considered as immovable.

During loading the vertical load, the deformations, and strains in top and bottom fibers were continuously registered in a x-y plotter and a logger. The crack pattern was registered at certain load levels and indicator clocks were used to observe the beginning of the bond slip. As an extra measure the entire failure process was video-taped.

The accuracy of recorded data was checked by calibration tests and measuring errors seemed to be smaller than one or two percent, and it can be definitely stated that any deviations between experimental and theoretical results are not caused by insufficient measuring technique.

5. EVALUATION OF THE REFERENCE BEAMS

The four reference beams B7, B9, B10 and B16 in Table 1 are all non-prestressed, but reinforced with the same four strands as the rest of the test series. They are also fitted with the minimum stirrup arrangement according to alternative A described above. This alternative, however, provides two stirrups C at the end of the beam and one stirrup close to the support and this makes it plausible that a premature triggering of the shear failure due to a breakdown of the dowel force capacity will not take place. These matters will be further discussed in the concluding remarks.

The failure mechanism for all of the four reference beams can be described by the crack pattern of the beam B9 presented in Fig. 6.

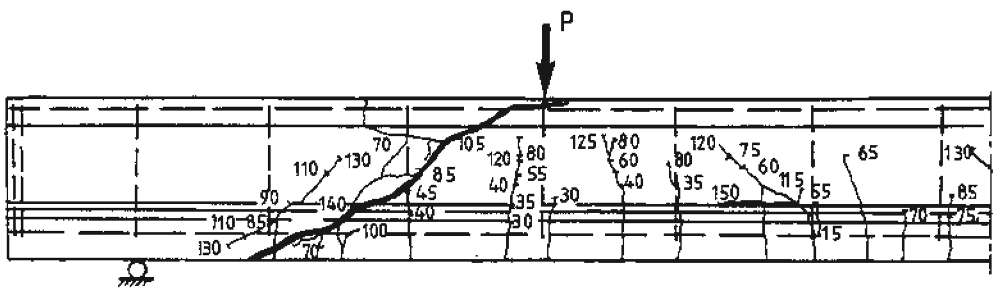

Fig. 6. Crack pattern at failure for beam B9

It is to be seen that the final critical crack crosses one stirrup at about middepth of the beam and that a breakdown of the compressive zone is the final cause of the failure. Splitting of the tension zone due to dowel forces does not seem to be of remarkable magnitude.

In agreement with earlier investigations on smaller test-beams /5/ the shear capacity V_u of the reference beams was evaluated in a first alternative as:

$$V_u = V_c = k_1 * b_w * d * \sqrt{f_{cct}} \quad (3)$$

and in second alternative as:

$$V_u = V_c + V_s = k_2 * b_w * d * \sqrt{f_{cct}} + V_s \quad (4)$$

In the first alternative no account has been taken of the shear reinforcement and in the second alternative the term V_s corresponds to the capacity of one double legged stirrup ϕ 8.

In the evaluation no account has been taken of the concrete strength of the bottom part, and this can be seen as a consequence of the short discussion above concerning dowel forces in the tension zone.
The results of the evaluation are summarized in Table 4 below.

Table 4. Evaluation of test beams

Beam	V_u kN	f_{cct} MPa	k_1	k_2
B7	144	52.0	0.44	0.28
B9	100	27.3	0.46	0.21
B10	137	55.4	0.41	0.27
B16	107	40.7	0.38	0.21
		Mean values	0.42	0.24

From the evaluation of the reference beams it is seen that none of the used evaluation formulas can be excluded and both are used for the evaluation of the prestressed beams in the next chapter. The variation of the test results seems moreover to fall within normal limits according to our experience from other test series.

6. EVALUATION OF PRESTRESSED HOMOGENEOUS AND COMPOSITE TEST BEAMS

The beneficial influence of the prestressing force, in the following termed V_p, has been evaluated from the test beams by the formula:

$$V_u = V_c + V_s + V_p \quad (5)$$

where in accordance with the previous chapter $V_s = 0$ in alternative (1) and $V_s = 53$ kN or 48 kN in alternative (2). The details of the evaluation are given in Table 5 below. The failure mechanism for all of the beams treated in this context are similar and can be illustrated for instance by test beam B13 in Fig. 7.

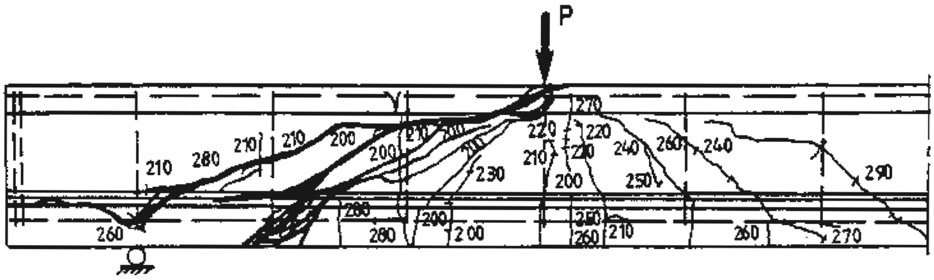

Fig. 7. Crack pattern of the test beam B13 at failure

In principle the failure mechanism and the final cause of failure seems to be the same as for the non-prestressed beams, but more inclined cracks of almost equal magnitude occur, and also a tendency to engage more of the dowel force capacity seems obvious. The reinforcement detailing seems, however, sufficient to prevent a premature breakdown of the dowel force capacity.

In column (7) of Table 5, the theoretically evaluated increase V_p is presented.

Table 5. Evaluation of prestressed homogeneous beams (B1, B2) and prestressed composite beams (B3, B4, B6, B12, B13, B14)

Beam (1)	P_e kN (2)	f_{cct} MPa (3)	V_u kN (4)	V_p alt(1) kN (5)	V_p alt(2) kN (6)	V_p calc kN (7)
B1	388	73.8	204	41	58	88
B2	377	40.7	181	60	59	85
B3	370	38.2	200	91	85	70
B4	385	14.6	141	74	50	63
B6	212	12.8	119	55	30	35
B8	185	25.8	137	47	33	35
B12	402	23.5	190	105	94	69
B13	425	73.6	215	63	80	83
B14	411	37.0	197	89	88	81

For both homogeneous and composite beams we have used the expression:

$$V_p = M_0 / a$$

according to equation (2) in the introductory chapter. For all the test beams the shear span was a = 1.5 m. For the homogeneous beams:

$$M_0 = P_e * \left[\frac{I_{tr}}{A_{tr}*(h-x_{tr})} + d - x_{tr} \right] \quad (6)$$

and for the composite beams:

$$M_0 = P_e * \frac{I_{tr}}{I_b*(h-x_{tr})} * \left[\frac{I_b}{A_b} + e_s* \ (h-e_b-e_s) \right] \quad (7)$$

The sectional data are given in Tables 2 and 3 above.

Finally, in order to decide if alternative (1) or alternative (2) is to be used for conclusions the ratio between columns (7) and (5) and between columns (7) and (6) are put together below:

Table 6.

Beam	B1	B2	B3	B4	B6	B8	B12	B13	B14
Alt(1) (7):(5)	2.15	1.42	0.77	0.85	0.64	0.74	0.66	1.32	0.91
Alt(2) (7):(6)	1.52	1.44	0.82	1.26	1.17	1.06	0.73	1.04	0.92

The alternative (1) corresponds to a mean value of 1.05 and a standard deviation of 0.497 and alternative (2) corresponds to a mean value of 1.11 and a standard deviation of 0.268. In the following therefore the evaluation according to alternative (2) is considered to be more reliable and this is also indicated by the crack pattern at failure.

The beams B3, B4 and B6 were fitted with extra joint reinforcement, and the question was whether it had any influence on the failure load. These beams seem, however, not to deviate from the rest of the test population and they are not excluded. The general conclusions that can be drawn from this test series when alternative (1) is excluded are summarized in the concluding chapter below.

7. SUMMARY AND CONCLUSIONS

In order to investigate whether it is possible to treat composite prestressed concrete beams in the same way as homogeneous beams a series of 16 beams were tested. The series included four non-prestressed beams as reference beams. Of the prestressed beams two were homogeneous and the rest composite. The amount of prestress also varied. The conclusion that can be drawn are:

(a) The test series indicate that a slight overestimation of the beneficial influence of prestressing is obtained using the rough code recommendations, and this overestimation seems to be greater for homogeneous beams than for composite beams. In the near future it will be necessary to use more physically correct design methods to estimate the shear capacity, as is indicated by the relatively high value of the standard deviation for the test results compared with the theoretical values.

(b) It is obvious from the test series that composite beams can be treated with the same approach as homogeneous beams, and these tests indicate that this results in the same reliability. These findings of course presuppose that the joint is properly designed. In the calculations of M_0 (the decompression bending moment) for composite beams, the attention must be paid to the fact that only the bottom part is prestressed. Differential shrinkage between the components can cause stresses, but these stresses should mainly concern the detailing of the joint.
It should also be emphasized that all test beams were made so that the critical crack did not cross the tension zone on the transmission length. To take this measure to prevent disturbance of the transmission zone can, however, be considered good practice, and is thus recommended.

(c) Finally (which is already commented upon), the general impression from this test series is the insufficiency of the existing design rules. It therefore also seems necessary to evaluate these tests with a more sophisticated approach, as for instance with the modified compression field theory. This is also the intentions of the authors.

NOTATION

a	= shear span from support to concentrated load
A_b	= area of prefabricated component
A_t	= area of cast in situ component
A_{tr}	= transformed section area
B	= beam width
b_w	= thickness of web
d	= effective depth of beam
e_b	= distance from top to gravity centre of bottom component
e_j	= level of joint
e_s	= distance from bottom to centroid of longitudinal tension reinforcement
e_t	= distance from top to gravity centre of top component
f_{ccb}	= mean compressive strength of bottom part concrete of composite beam (cubes 150*150 mm)
f_{cct}	= mean compressive strength of the top part concrete of composite beam (cubes 150*150 mm)
f_{su}	= ultimate strength of prestressed reinforcement
f_{sy}	= yield strength of prestressed reinforcement
f_{sy}'	= yield strength of non-prestressed reinforcement

h	= total depth of beam
I_b	= moment of inertia of bottom component
I_t	= moment of inertia of top component
I_{tr}	= moment of inertia of transformed section
k_1, k_2	= constants referring to eq.(3) and (4)
M_0	= decompression bending moment in the most strained section
M_d	= design bending moment
P_e	= effective prestress force at section after allowance for all prestress losses
s	= spacing of stirrups
V_c	= shear strength provided by concrete
V_d	= design shear force
V_p	= shear strength due to effective prestress force
V_s	= shear strength provided by shear reinforcement
V_u	= failure shear force obtained in test
x_{tr}	= distance from top fiber to gravity centre for transformed section
α	= relation between moduli of elasticities for components

REFERENCES

/1/ BBK.79. "Bestämmelser för betongkonstruktioner" Stockholm 1979, (Recommendations for concrete structures).

/2/ Vecchio, F. J. and Collins, M. P., "The Modified Compression Field Theory for Reinforced Concrete Elements Subjected to Shear", ACJ Journal, Proceedings V.83, No 2, Mar.-Aug. 1986, pp. 219-231.

/3/ Öberg, S., "Shear Capacity of Concrete Beams with Post-Tensioned Shear Reinforcement", PhD dissertation submitted to Civil Engineering Department,Chalmers University Of Technology.

/4/ Granholm, H., "A General Flexural Theory of Reinforced Concrete", John Wiley & Sons, 1965.

/5/ Hedman, O., Losberg, A., "Design of Concrete Structures with Regard to Shear Forces", Translation into English of the Paper published in "Nordisk Betong", No 5, 1975, pp. 19-29.