MINIMUM REINFORCEMENT FOR CRACK CONTROL

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SYNOPSIS

Minimum reinforcement, the reinforcement initiating a second crack after the first one has appeared in a strained concrete structure, is discussed under consideration given the influence from reinforcement on the effective tensile strength and the concrete strength variation along the reinforcement.

Key words: Crack control. Effective tensile strength. Minimum reinforcement.

Some part of a reinforced concrete structure under tension may crack. A minimum reinforcement for crack control may initiate a second crack after some additional strain. The problem here is to find that reinforcement.

Provided the word section is meaningsful in describing the situation the first crack will appear in the weakest section, weak e.g. because of low tensile strength. It will be initiated by

$$\sigma_{ct} = f_{ct,o} \tag{1}$$

It opens under

$$\sigma_{ct,ef} = f_{ct,o} (1 + \kappa\rho) \tag{2}$$

where κ is a coefficient depending probably on concrete characteristics as d_g, w/c, and ε_{cs} as well as on τ_b. The physical meaning of what is expressed by κ could be a greater contribution in early stages from force in the reinforcement crossing the crack than from the decrease in concrete stress during opening of the crack. Figures are given in /Ho 86/, slightly different from those hereafter depending on differences in the definition of ρ.

So, from /Fa 69/	$f_{ct,o}$ = 1,49 MPa;	κ = 55
/Ho 72/	1,02	80
/Ja 85/, long time	1,56	30
/Ja 85/, short time	1,47	30

/Ha 77/ in his first four tests found $f_{ct,o}$ = 2,87 MPa and κ = 8. Corresponding for the last three ones were $f_{ct,o}$ = 3,29 MPa and κ = 14. These tests, however, were extreme with ρ up to about 70 ^{o}/oo. ρ changed to A_s/A_c gives $f_{ct,o}$ = 2,85 and 3,13 MPa respectively and κ = 10 and 17.

The test series, thus quoted, were performed for other reasons than finding $f_{ct,o}$ and κ. In these respects, therefore, some inaccuracy may be accepted. To find precisely the moment for crack opening in a reinforced member may be difficult.

A special series with 100 splitting tests therefore has been executed with specimens ø 125 to 127 mm and t 60 to 136 mm, drilled from concrete pipes, reinforced with ρ = 2,91 to 5,03 ^{o}/oo, and with d_g = 12 mm. Splitting leads to a sudden and easily observable crack opening. The result was

$$f_{spl} = \frac{2\,F_u}{\pi\,A_c} = 4{,}35\ (1 + 42\,\rho)\ \text{MPa} \tag{3}$$

with the correlation 0,9.

A_c = ø × t was the splitted area.

From all this it seems reasonable to propose

$$\kappa \approx 45 \pm 15 \tag{4}$$

as a basis for predicting the influence from ρ on crack opening.

To define ρ then is a remaining problem. ρ is the content of bonded reinforcement within a sectional area, often assumed to be the maximum area having the same centroid as the reinforcement therein. This, however, is not univocally correct in all situations without having firstly s_s and, after some cracking, s_r known.

The section nearest to the weakest one may have a tensile strength, $\beta\sigma_{ct,ef}$ according to (2) with β in ordinary cases between 1,05 and 1,10, all depending on the scattering of the tensile strength of the concrete from the characteristics of its own and from the way in which it has been cast, worked and cured. In extreme cases β may reach values up to about 1,20. Under indirect restraint even values < 1, e.g. 0,95, are observed.

Then, after some additional strain.

$$\sigma_{ct} = \beta f_{ct,o} (1 + \kappa\rho) = f_{ct,o} [(1 + \kappa\rho) + \Delta\varepsilon E_s \frac{\rho}{f_{ct,o}}] \qquad (5)$$

from which

$$\rho_{min} = \frac{1}{\frac{\alpha}{\beta-1} - \kappa} \qquad (6)$$

where α refers to $E_{c,sec}$ at $\varepsilon_{ct,u}$ with a value hardly below

$$\alpha \approx 30 \qquad (7)$$

In such case, as an example

$$\rho_{min} \approx \frac{1}{\frac{30}{0,075} - 45} = 3\ {}^{o}/oo \qquad (8)$$

Beside problems with an unsufficient knowledge about α, β, and κ it remains to scrutinize the physical model, a reinforcement crossing the opening crack, being strained over a short length, containing the developing crack, to a stress to some extent compensating the loss in the concrete confining the crack, unloaded under vanishing strain.

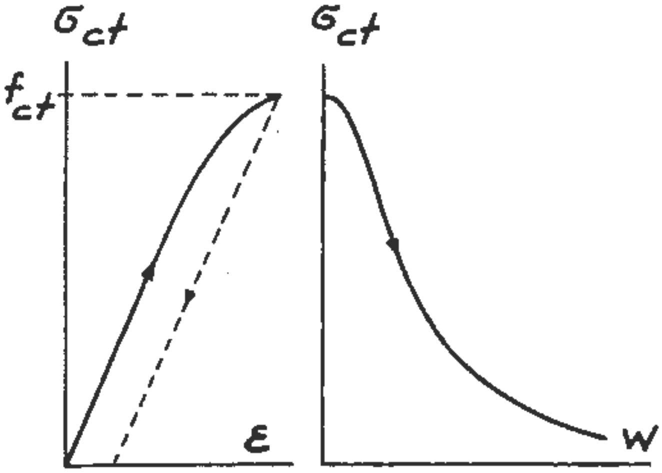

Figure 2 The Hillerborg crack model with crack opening, w, between intact concrete borders under strain, ε.

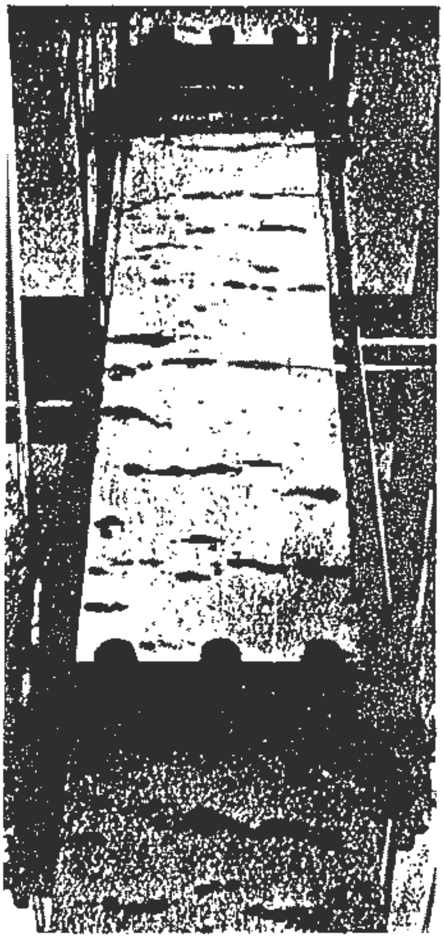

Figure 1 Water penetrating the surface of damp beams under bending, showing the position of cracks to open

The phenomenon with the concrete in the position of a coming crack successively breaking up is known from /Ba 07/, Figure 1. It is described in /Hi 76/, Figure 2, and has been looked upon as a continous course of events, occuring along the reinforcement. A report, /Gr 87/,however, gives a contradictory information.Continuity is a misleading word.

The anthors strained a concrete specimen according to Figure 3 with $\varnothing_s = 4$ mm, c symmetric,and $\rho = 2{,}2$ °/oo to $\varepsilon_m = 0{,}158$ °/oo corresponding to $\sigma_{sm} \approx 32$ MPa and $\sigma_{cm} \approx 3$ MPa and observed strains on opposite sides, nearest the reinforcement. The result from x = 184 mm to x = 204 mm is on the one side varying from + 0,53 °/oo to - 0,44 °/oo, still being quite moderate on the opposite side. See Figure 4.

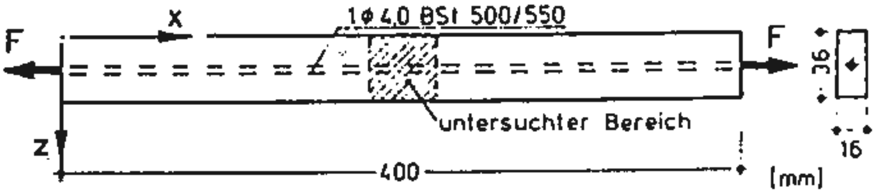

Figure 3 Test specimens according to Gross and von Cramon.

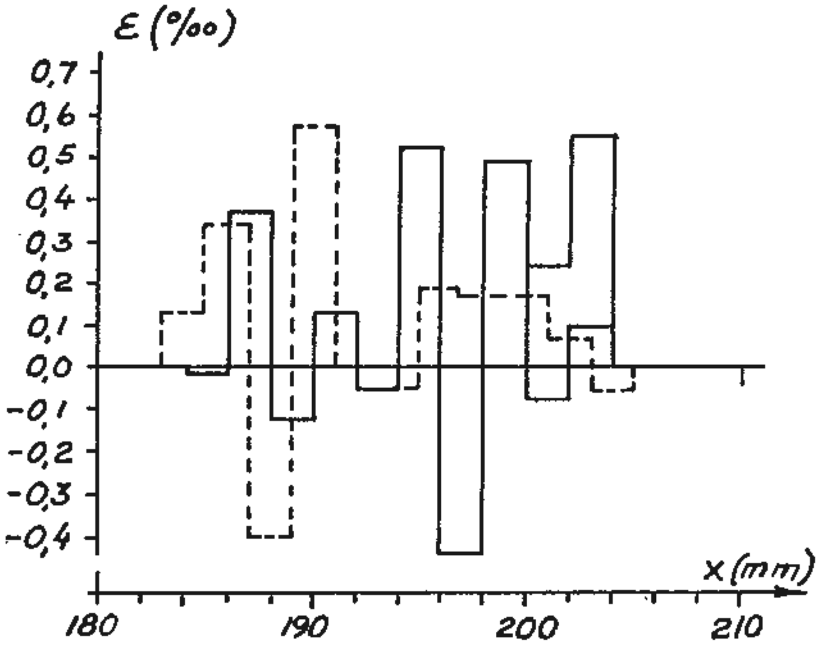

Figure 4 Strains varying on two opposite sides of the test specimen according to Figure 3, full and broken lines.

The situation is obviously that the matrix is strained leaving relatively big aggregate grains nearly unstrained. To fulfil the mean strain the local strain therefore has to vary within wide limits, much wider than are expected for the reinforcement under moderate stresses. See e.g. /Ho 65/. Bond then has to vary along the bar, changing locally from positive to negative, with even varying direction of the bond-slip before cracking.

The physial model mentioned then must be a very rough one at least until $\sigma_{cm} \approx f_{ct}$ is reached. And local values f_{ct} must depend on the distribution of big aggregate grains. So aggregate size could influence minimum reinforcement for crack control. Several problems remain for further studies, amongst them the problem of minimum reinforcement being less neaded as the concrete tensile strength decreases with age. See e.g. /Ja 55/. This, however, ought to be a minor problem as ρ_{min} has only an insignificant connection to f_{ct}. From (6) and (7) follows, beside $\rho_{min} \approx 3\ ^{o}/oo$

$$\frac{\Delta\varepsilon E_s}{f_{ct,o}} = \frac{\sigma_s}{f_{ct,o}} \approx \alpha \gtrapprox 30 \qquad (9)$$

and so

$$\sigma_s \gtrapprox 30\ f_{ct,o} \leqq 30 \times 5 = 150 \approx \frac{400}{3}\ \text{MPa} \qquad (10)$$

High grade reinforcement therefore does not serve effectively as minimum reinforcement for crack control. Grades over 400 are hardly justifiable.

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NOTATIONS

A	Area
A_c	Concrete area
A_s	Steel area
E	Young's modulus
E_c	Modulus for concrete
$E_{c,sec}$	Secant modulus for concrete
E_s	Modulus for steel
F	Force
F_u	Ultimate force
c	Concrete cover
c_h	Horizontal concret coser
d_g	Size of aggregate
f	Strength
f_{ct}	Concrete tensile strength
$f_{ct,o}$	Tensile strength for unreinforced concrete
f_{spl}	Splitting strength
s	Spacing
s_r	Crack spacing
s_s	Steel bar spacing
t	Thickness
w	Crack width
x	Coordinate
α	Modular ratio, E_s/E_c
β	Factor for f_{ct} increasing from crack to crack
ε	Strain
ε_{cs}	Shrinkage
ε_{ct}	Concrete tensile strain
$\varepsilon_{ct,m}$	Concrete mean tensile strain
ε_m	Mean strain
ε_{sm}	Mean steel strain

κ	Factor for f_{ct} increasing with ρ
ρ	Ratio of reinforcement, $A_s/A_c + A_s)$
σ	Stress
$\sigma_{ct,ef}$	Effective concrete tensile stress
σ_s	Steel stress
w/c	Water/Cement ratio
$\Delta\varepsilon$	Increment in ε
$\varnothing$	Diamanter
$\varnothing_s$	Diamanter of steel bar