AN EXPERIMENTAL AND THEORETICAL STUDY OF THE SHEAR CAPACITY OF COMPOSITE BEAMS

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Tests on four full-scale composite beam-ends are reported. The shear capacity for composite beams with a naked steel web as well as for a steel web in composite action with a concrete web has been studied. Theoretical models for both types of composite action in the ultimate limit state are evaluated. For the naked steel web a tension stress field model somewhat underestimates the real shear capacity. In the case of a steel web in composite action with a concrete web a model also based on plasticity showed good agreement between theoretical and experimental shear capacities. For this latter case an approximative formula is given for calculation of a minimum thickness of the concrete web to prevent premature buckling of the concrete web. Formulas for the shear flow to be carried by anchoring elements between steel web and concrete are derived.

Keywords: Beam, composite, shear, plasticity, anchoring bars, full-scale tests.

1. INTRODUCTION AND BACKGROUND

Research on composite structures has for some years been going on at the Division of Concrete Structures, Chalmers University of Technology, Gothenburg. Composite action between steel and concrete as well as between concrete of different strengths and densities is studied. Both the ultimate limit states and the serviceability limit states are objects of our research but the main interest is devoted to the shear capacity of the composite structure as a whole and to the detailing and the strength of the connections between the structural elements. In this paper we will concentrate on some tests with composite steel-concrete beams where the upper flange of the steel beam is substituted by a concrete slab. The connection between the steel web and the concrete slab consisted of bent deformed bars welded to the steel web. By some tests the web itself was designed for composite action by pouring concrete webs on both sides of the steel web. An evaluation of the shear capacity for this kind of composite structure in comparison with a conventional design was supposed to be valuable since it according to our knowledge has not been tested and reported about earlier.

2. TEST SPECIMENS AND MATERIAL QUALITIES

This study comprises four full-scale beam tests with the design of the sections according to Fig. 1a and Fig. 1b.

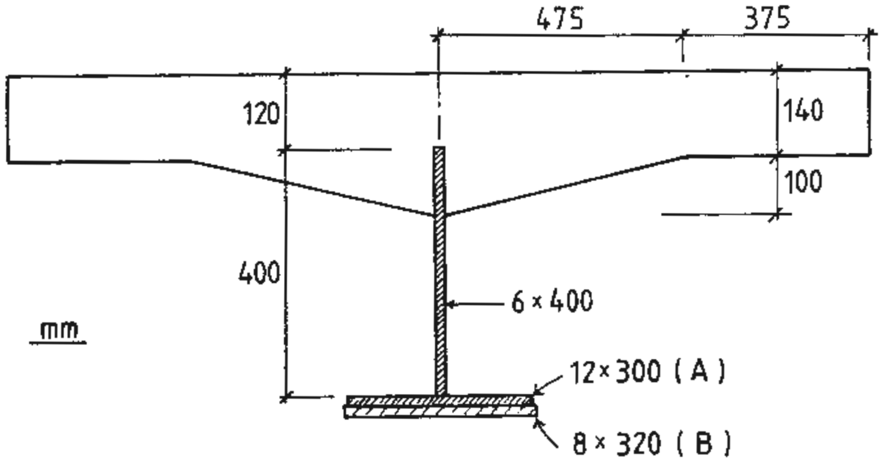

Fig. 1a Sections A and B.

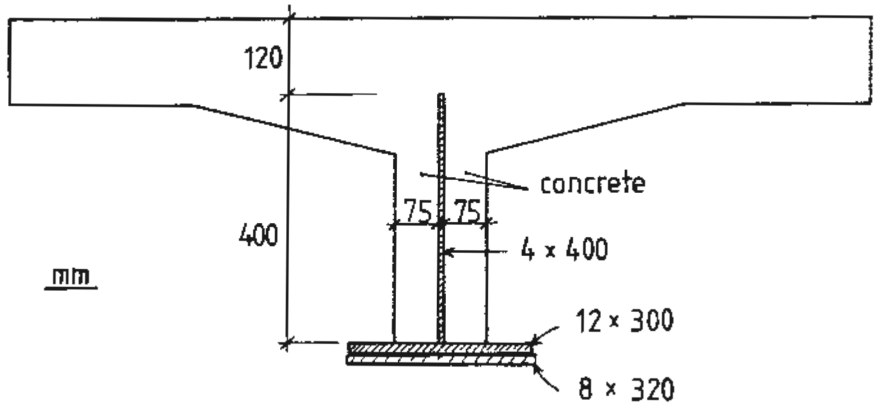

Fig. 1b Sections C and D.

The test specimen A is thus furnished with only one flange plate 12x300 while the others B, C and D were furnished with an extra flange plate 8x320 besides the one used for test A in order to prevent that the shear failure should be affected by yield in the lower flange due to bending. In the shear span the slab reinforcement perpendicular to the beams was in all cases 2ϕ10 s500 in the upper edge and ϕ8 s480 in the lower edge. This reinforcement had a nominal yield strength, f_y = 400 MPa. When all the main tests had been carried out test pieces were taken out from the steel beams and used for tensile tests. The following mean values of the yield strength and the ultimate strength were obtained:

Table 1

Structural detail	Yield strength f_{sy} MPa	Ultimate strength f_{su} MPa
Web plate		
6 mm	330	420
4 mm	330	>420
Flange plate		
12 mm	380	510
8 mm	430	550

The modulus of elasticity for all test pieces was measured and was found to vary slightly around a mean value of E_s = 210 GPa.

The concrete mix was designed to correspond to a concrete strength of 35 MPa (cubes of 150 mm side-length) after 28 days. When the main tests were carried out the actual cube strength was measured on cubes that were stored identically with the main specimens. The age of the concrete at the time of the main tests together with the mean cube strengths f_{ccK} are given below:

Table 2

Test specimen	Age at testing Days	Compressive strength f_{ccK} MPa
A	168	40.5
B	309	37.3
C	308	40.4
D	308	40.4

In the theoretical analysis we have estimated the structural mean strength f_{cc} of the concrete from the expression:

$$f_{cc} = \frac{f_{ccK}}{1.35}$$

which according to the Swedish Standard SS 137207 is used to calculate the relation between the wet cylinder strength and the dry cube strength. The structural strength is thus here assumed to correspond to the wet cylinder strength.

The design of the shear transfer reinforcement between the steel beam and the concrete slab is presented in Fig. 2.

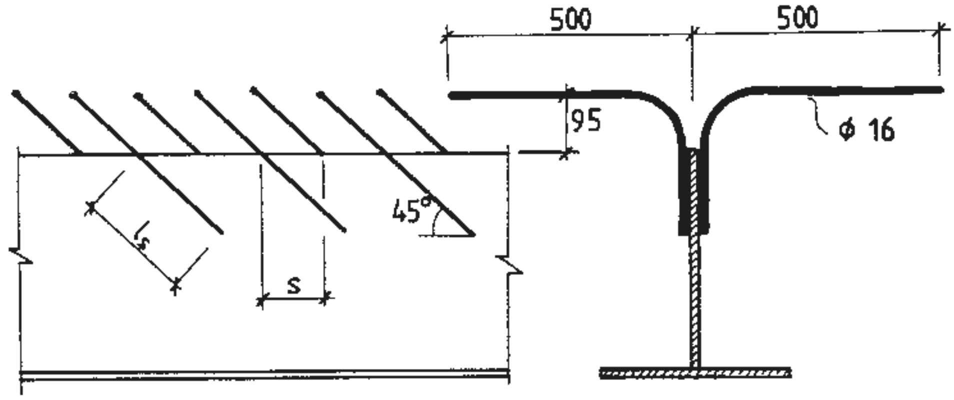

Fig. 2 The design of the shear transfer reinforcement.

Deformed bars of a weldable quality have thus been bent and welded to the beam web with an inclination of 45°. The design of the welding including the welded length ℓ_s has been chosen so that failure is not to be expected in the welded connections. The distance s between the anchoring bars has been chosen different for the different beam tests so that one in some cases could expect an anchorage failure. Tensile tests on the anchorage bars have given the following mean values of the yield strength f_{sy} and of the ultimate strength f_{su} namely: f_{sy} = 670 MPa and f_{su} = 830 MPa. The welded length ℓ_s for the anchoring bars and the distance s between them for the different tests have been put together below.

Table 2

Test specimen	Welded length ℓ_s mm	Distance between bars s mm
A	150	150
B	150	75
C	200	150
D	200	75

The steel webs were fitted with web stiffeners at the supports and the flanges were fitted with indentations in tests C and D in order to secure the shear stress transfer between the concrete compressive struts in the concrete webs and the bottom steel flanges. More information about these details is given in the main report from the research division /1/.

3. TESTING ARRANGEMENTS, TEST RESULTS AND FAILURE MODES

Only a short survey of the testing arrangements will be presented in this paper. The reader is directed to the main report /1/ for a more informative description. The tests were carried out as beam tests with one eccentrically placed point load according to Fig. 3.

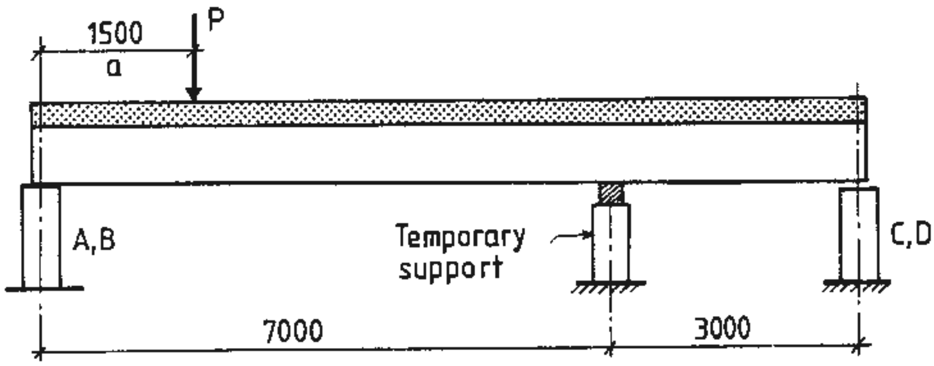

Fig. 3 Loading arrangements at shear tests.

By the testing of for instance the beam end A is thus a temporary support placed according to Fig. 3 in such a way that the beam end C is completely unloaded. By the following test of the beam end C the temporary support was moved to the corresponding position close by the beam end A. The relation between the support reaction and the deflection caused by the point load for the different tests are presented in Fig. 4.

The relative movement between the concrete slab and the steel web was continuously measured during testing and the results are presented in Fig. 5. By means of strain gauges steel strains and concrete strains were measured at positions of interest. Thus the strains in selected anchoring bars and maximal strains in steel and concrete webs were measured. The results from these measurements are commented upon in the discussion of the failure modes below.

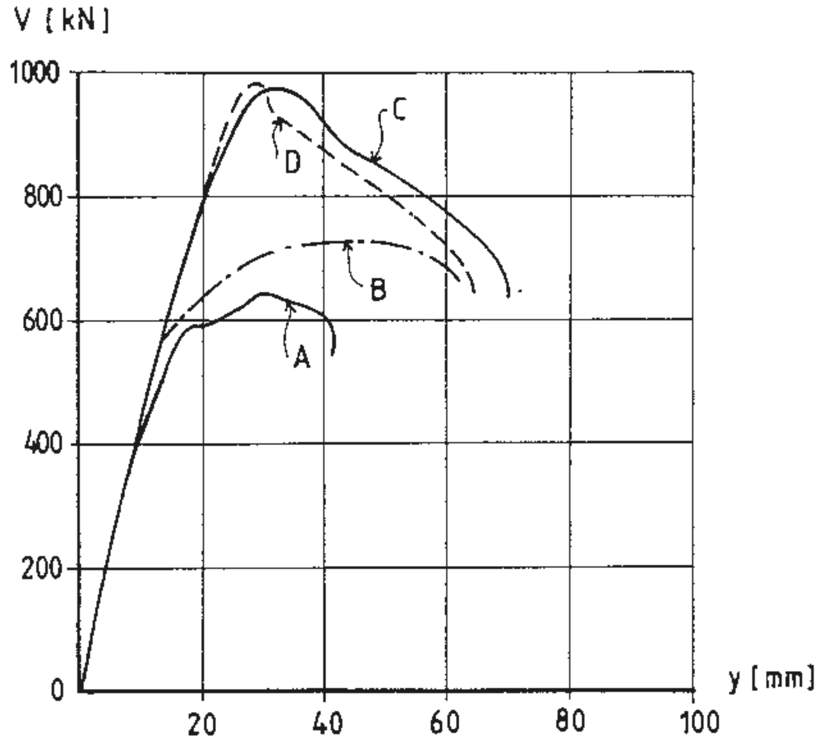

Fig. 4 Load deflection diagrams for the shear tests A, B, C and D.

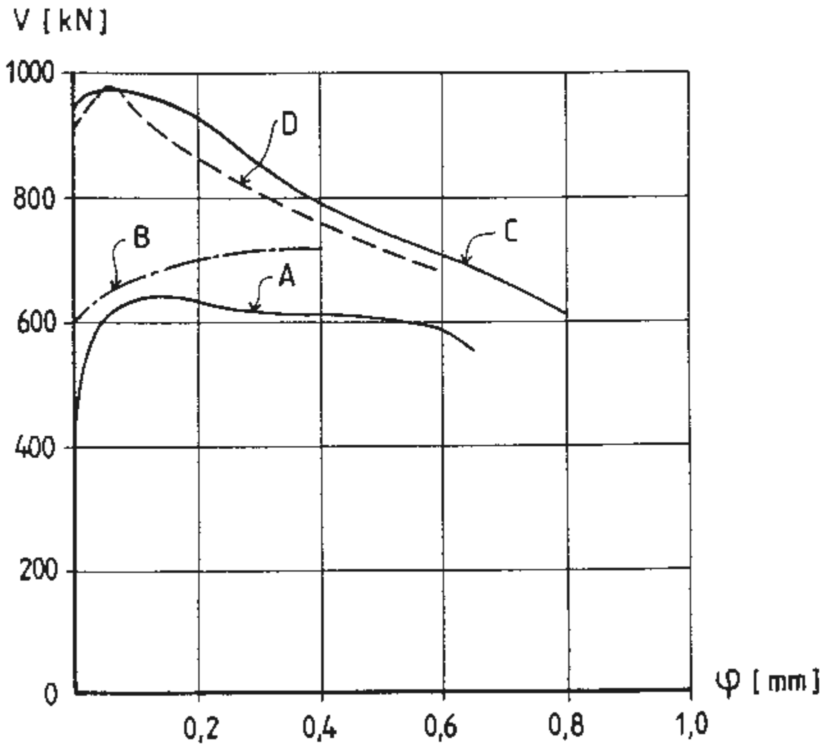

Fig. 5 Load slip relation for the shear tests A, B, C and D.

From Fig. 4 it is obvious that the course of failure is of a ductile character for the beam ends A and B where the naked steel webs are used while the beam ends C and D with composite steel and concrete webs can be characterized as brittle. For the latter beam ends C and D the course of failure and the failure loads are quite identical in spite of the fact that the amount of connection reinforcement is quite different. For these tests it therefore can be concluded that the design of the shear transfer reinforcement was of no primary importance for the behaviour at failure. The measured displacements between the concrete slab and the steel web according to Fig. 5 seem also to be of a secondary or post failure character for tests C and D.

On the contrary it can be seen from Fig. 4 that the deflection for test A deviates from linear behaviour for a value of the shear force equal to V = 400 kN. It is moreover obvious from Fig. 5 that measurable displacements occur for this magnitude of the shear force. Yield in the anchoring bars and more distinct displacements were however not obtained until the maximum load was reached and when a horizontal crack in the beam end appeared at the level of the anchoring bars. At the maximum load the tensile strains in the web were at the order of $10^{\circ}/oo$ and the stresses in the bottom flange had reached the yield level. The final cause of failure for the beam end A can be characterized as an anchorage failure but a combined bending and shear failure of the composite section seemed to be very close.

For the beam end B the stresses in the anchoring bars and in the bottom flange were below the yield stress at maximum load. The strains in the web were however in the main tensile direction of the order of $20^{\circ}/oo$. At maximum load the steel web was heavily buckled with buckles inclined less than 30° ($\theta_s < 30^{\circ}$) giving the impression of a developed tension stress field. Before the maximum load was reached bending cracks occurred in the concrete slab suggesting that the slab transferred a part of the shear force to the support. The failure mode can however be characterized as a web shear failure.

For the beam ends C and D thus rather identical failure courses were obtained but strain measurements suggest that the stresses in the anchoring bars of test specimen C were not far from the yield stress. This was not the case for the test specimen D. At maximum load the steel webs yielded and inclined compression struts separated by cracks had developed in the concrete webs. The primary cause of failure for beam ends C and D seemed to be crushing of the concrete webs close to the supports. The crushing zones however rapidly spread all over the concrete webs and finally buckling of the concrete webs occurred.

Pictures of the beam ends after failure are reproduced in Figs. 6, 7, 8 and 9. The ultimate shear force at the loading tests and the failure modes are summed up in the table below. On account of dead weight and certain preloading cycles the support reactions exceed the values given in the table with about 40 kN.

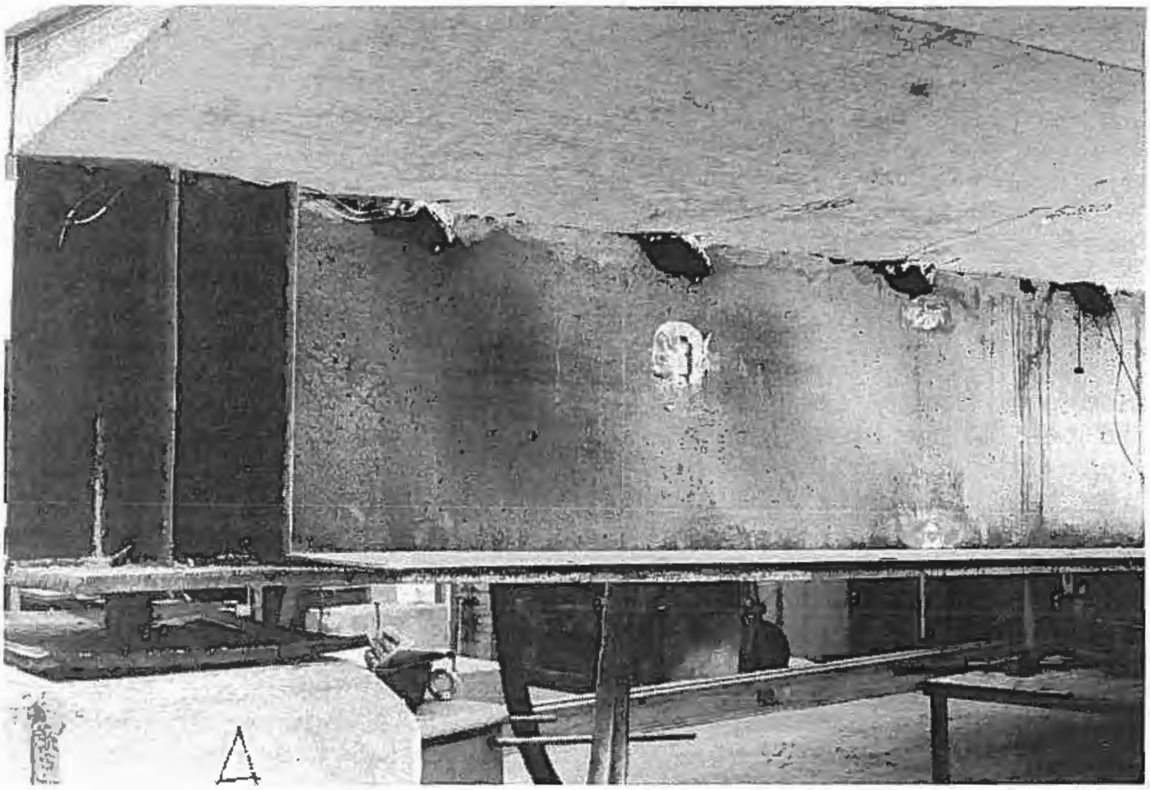

Fig. 6   Beam end A after failure.

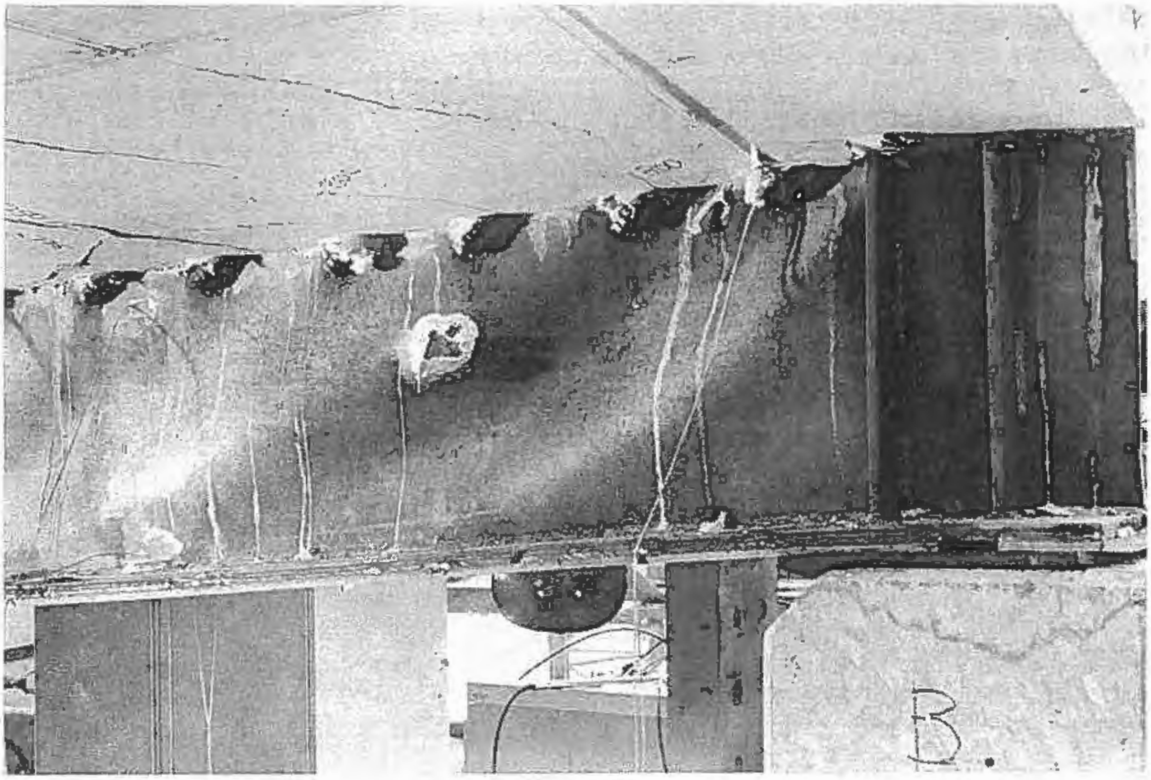

Fig. 7   Beam end B after failure.

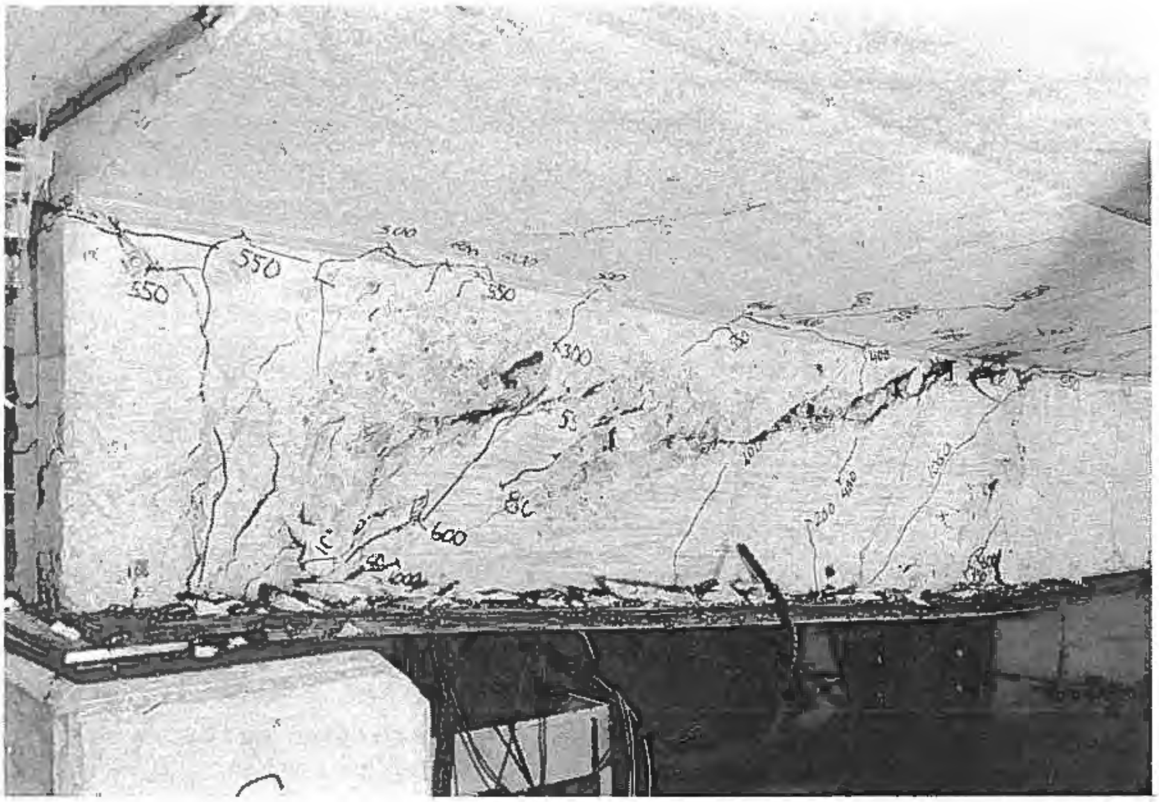

Fig. 8 Beam end C after failure.

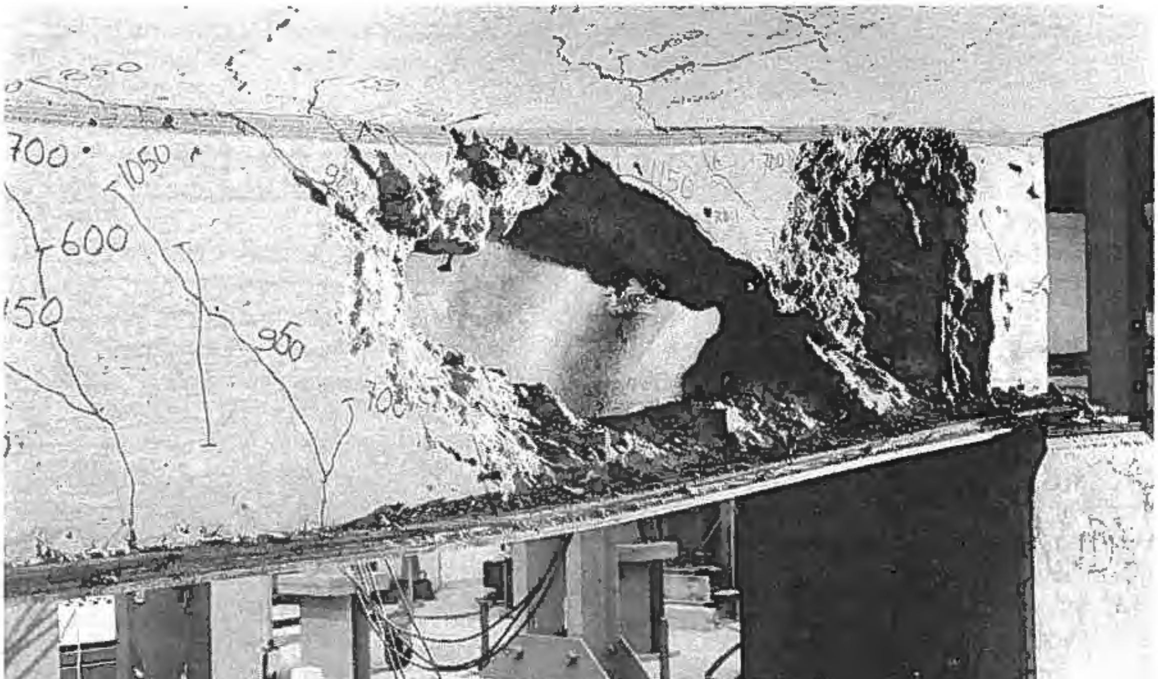

Fig. 9 Beam end D after failure.

Table 4

Test specimen	Ultimate shear kN	Failure mode
A	630	Anchorage failure (Bending-shear)
B	720	Web-shear
C D	970 990	Crushing of concrete struts

4. THEORETICAL ANALYSIS

In this analysis simple models of the observed failure modes are presented and the experimentally obtained ultimate capacities are compared with the theoretically derived. The observed failure modes have been classified as anchorage failure, combined bending shear failure, shear failure and concrete compression failure. Since the anchoring bars are affected differently when the steel webs act alone or when they are in composite action with the concrete webs it is more suitable to present models for the shear failure modes for the different structural designs before the anchorage failure modes are analysed.

For the naked steel web the ultimate shear force capacity V_u when yield occurs both in the web and the bottom flange is assumed to be attained when a tension stress-field has been developed according to Fig. 10. It is obvious that for this stress-field to develop in the steel web also the concrete slab plays an important part. No effort is however presented in this study to derive secondary failure criteria concerning the interaction of the concrete slab. We therefore introduce the suggested model as a first step towards a more refined analysis of the shear failure for composite beams.

For the ultimate limit stage we neglect shear stresses and assume a pure tension stress-field. In the choice of the configuration of the stress-field we have been guided by the experiments. The lower boundary of the tension stress-field is thus assumed to meet the intersection between the support-reaction and the upper flange according to Fig. 10.

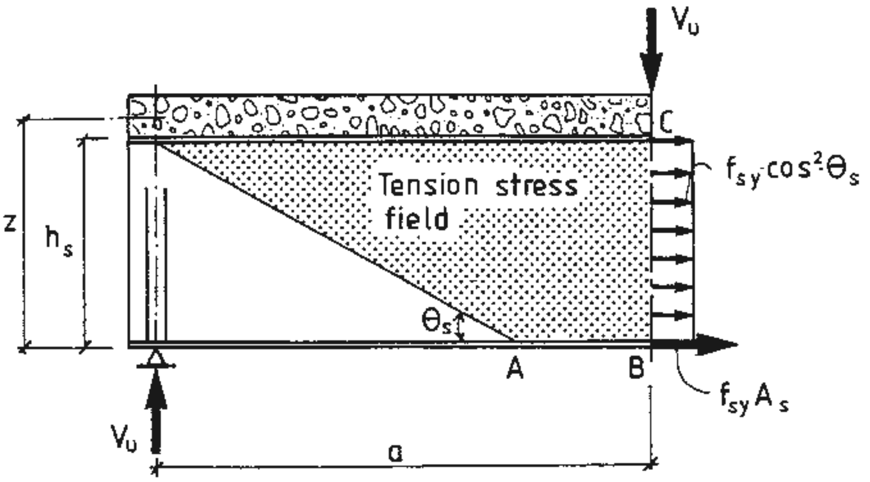

Fig. 10 Model for the ultimate shear capacity of the naked steel web.

The equilibrium conditions for the shear span in Fig. 10 can be formulated as:

$$V_u = f_{sy} \cdot a \cdot t_s \cdot \sin^2\theta_s \tag{1a}$$

and

$$V_u \cdot a = f_{sy} \cdot A_s \cdot z + f_{sy} \cdot t_s \cdot \frac{h_s^2}{2} \cdot \cos^2\theta_s \tag{2a}$$

where the first equation expresses the global equilibrium between interior and outer vertical forces and the second equation corresponds to the bending moment equilibrium. By introducing the reference value

$$V_p = f_{sy} \cdot t_s \cdot h_s \tag{3}$$

and the bending moment M_y:

$$M_y = f_{sy} \cdot A_s \cdot z \tag{4}$$

corresponding to the contribution to the bending moment of tbe bottom flange, the equilibrium conditions (1a) and (2a) can be presented as follows:

$$V_u = V_p \cdot \frac{a}{h_s} \cdot \sin^2\theta_s \tag{1}$$

$$V_u = \frac{M_y}{a} + \frac{1}{2} V_p \cdot \frac{h_s}{a} \cdot \cos^2\theta_s \tag{2}$$

In the transition zone AB the local equilibrium conditions can only be satisfied by additional shear in the bottom flange. A closer study of these secondary effects are beyond the scope of this paper. Elimination of θ_s from Eq. (1) and Eq. (2) results in an expression for the ultimate shear force V_u:

$$V_u = \frac{\frac{M_y}{a} + \frac{1}{2} V_p \cdot \frac{h_s}{a}}{1 + \frac{1}{2}\left(\frac{h_s}{a}\right)^2} \tag{5}$$

If according to Eq. (5) $V_u > V_p$ this is assumed to indicate that yield is not obtained in the bottom flange and modified formulations of Eq. (2) and (5) should be used according to Eq. (6) and Eq. (7) below:

$$V_u = \left(\frac{M_Y}{a} + \frac{1}{2} V_p \cdot \frac{h_s}{a}\right) \cos^2 \theta_s \qquad (6)$$

$$V_u = \frac{\frac{M_Y}{h_s} + \frac{1}{2} V_p}{\frac{M_Y}{V_p \cdot a} + \frac{a}{h_s} + \frac{1}{2}\frac{h_s}{a}} \qquad (7)$$

The application of this model for beam ends A and B results in ultimate shear forces of V_u = 550 kN and V_u = 710 kN. For test specimen A Eq. (5) is decisive and for test specimen B Eq. (7). Yield in the bottom flange is thus obtained according to the theory for the test A and not for the test B in agreement with the experiments. The agreement between theoretical and experimental ultimate shear capacity V_u is not quite satisfactory for test specimen A but observations during the test indicated that the concrete slab itself carried a considerable part of the shear. In order to get a better agreement a more advanced model that pays due regard to this effect must be derived.

For the beam ends A and B with the naked steel webs the whole shear flow Q_u must be transferred by the anchorage bars. In accordance with the model outlined above:

$$q_u = \frac{V_u}{a} \cot\theta_s = \frac{V_u}{a} \sqrt{\frac{V_p \cdot a}{V_u \cdot h_s} - 1} \qquad (8)$$

At maximum load level the following numerical values q_u = 830 kN/m and q_u = 860 kN/m are obtained for beam ends A and B. If the only way for the connection reinforcement to transfer the shear flow is through axial forces the ultimate shear flow q_u can be formulated as

$$q_u = \frac{f_{sy} \cdot A_s}{\sqrt{2} \cdot s} \qquad (9)$$

corresponding to q_u = 650 kN/m and q_u = 1300 kN/m for the two test specimens. This model thus slightly underestimates the real capacity and it is to be supposed that a part of the shear flow is transferred by dowel forces in the bars.

For the beam ends C and D with composite webs of steel and concrete a model for the ultimate capacity is assumed according to Fig. 11. The shear force is thus assumed to be carried by crossing stress fields consisting of one tension field in the steel web and one compression field in the surrounding concrete webs with angles of inclination θ_s and θ_c. The ultimate capacity is assumed to occur when both steel and concrete stresses have reached the plastic limits f_{sy} and νf_{cc} respectively. The factor ν is the so called effectiveness factor introduced by M.P. Nielsen /2/.

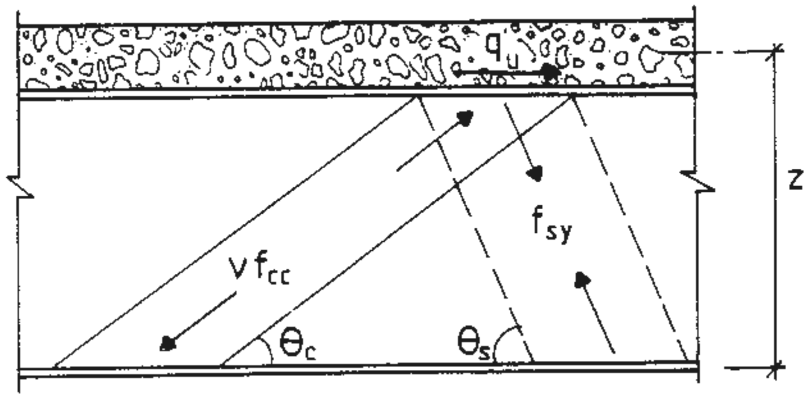

Fig. 11 Model for the shear capacity of the composite web.

The equilibrium conditions for the stresses in the web can be expressed as:

$$\nu f_{cc} \cdot t_c \cdot \sin^2\theta_c = f_{sy} \cdot t_s \cdot \sin^2\theta_s \qquad (10)$$

$$q = \nu f_{cc} \cdot t_c \cdot \sin\theta_c \cdot \cos\theta_c + f_{sy} \cdot t_s \cdot \sin\theta_s \cdot \cos\theta_s \qquad (11)$$

With due consideration of Eq. (10) maximum of Eq. (11) can be derived to be:

$$q_u = \sqrt{\nu f_{cc} \cdot f_{sy} \cdot t_s \cdot t_c} \qquad (12)$$

The ultimate capacity corresponds to the following angles of inclination:

$$\cot\theta_c = \sqrt{\frac{\nu f_{cc} \cdot t_c}{f_{sy} \cdot t_s}} \qquad (13)$$

$$\cot\theta_s = \sqrt{\frac{f_{sy} \cdot t_s}{\nu f_{cc} \cdot t_c}} \qquad (14)$$

The stress fields are thus crossing each other orthogonally. With q_u according to Eq. (12) the ultimate shear can be calculated as:

$$V_u = z \cdot q_u \qquad (15)$$

The thickness of the concrete web must of course be chosen so that premature buckling is avoided. An approximative condition for this can be formulated as:

$$\frac{t_c}{h_s} \geq \alpha\left[1 + \sqrt{1 + \frac{\lambda}{\alpha^2}}\right] \qquad (16)$$

where $$\lambda = 19\,\frac{f_{cc}}{E_c} \qquad (17a)$$

$$\alpha = 9.5\,\frac{f_{cc}^{\,2}}{E_s\,f_{sy}}\cdot\frac{h_s}{t_s} \qquad (17b)$$

For the design according to beam ends C an D this corresponds to

$t_c \geq 70$ mm

to be compared with the actual thickness $t_c = 150$ mm. With the numerical value of the effectiveness factor $\nu = 1.0$ the theoretical ultimate shear capacity amounts to $V_u = 1.08$ MN for beam ends C and D to be compared with the experimentally obtained $V_u = 1.01$ MN and $V_u = 1.03$ MN. The internal lever arm z has in the theoretical application been put equal to the distance between the gravity centres of the concrete slab and the bottom steel flange. It is thus obvious that the model reflects the real behaviour quite satisfactorily. In spite of the brittle behaviour of the concrete it is thus possible to use a model based on plasticity for the composite behaviour. The ductility of the steel seems to be sufficient to compensate for the brittleness of the concrete.

For beam ends C and D it is sufficient that the anchoring bars transfer a part q_s of the shear flow:

$$q_s = f_{sy}\cdot t_s\cdot \sin\theta_s\cdot\cos\theta_s =$$

$$= \frac{f_{sy}\cdot t_s}{\nu f_{cc}\cdot t_c + f_{sy}\cdot t_s}\cdot q_u \qquad (18)$$

At maximum load the shear flow according to Eq. (18) amounted to $q_s = 510$ kN/m and $q_s = 520$ kN/m which is less than the theoretical capacity $q_s \geq 650$ kN/m indicating that anchorage failure was not to be expected. This was also experimentally confirmed.

The shear transfer between the concrete struts and the bottom flange must be designed to resist the shear flow:

$$q_c = q_u - q_s \qquad (19)$$

The indentations used in the tests were sufficient but whether they were too strong or just strong enough could not be found out from the tests. It is also obvious that the web stiffeners have to be designed to resist the reactions of the compressive concrete struts. In this case the two web-stiffeners with concrete poured between them gave a composite action with a quite sufficient resistance.

5. FINAL COMMENTS

This paper deals with some full-scale tests on composite beams and the experimentally obtained ultimate shear capacities are compared with theoretically derived capacities. The shear capacity for composite beams with a single steel web has been studied as well as composite beams with a steel web in composite action with a concrete web. The theoretical models that are applicable for composite beams with a single steel web somewhat underestimates the real shear force capacity. This is according to the authors due to the load carrying capacity of the concrete slab itself which is not explicitly considered in theoretical model.

For the composite beams where the webs were made of steel in composite action with concrete a model based on plasticity resulted in very good agreement between theoretical and experimental shear capacities. It is thus sufficient that one of the components that contributes to the carrying capacity exhibits a ductile behaviour in order to use such a model to calculate the total carrying capacity.

Concerning the connection reinforcement it can be concluded that a model which only takes regard to axial forces in the bars underestimates the shear flow capacity but the experiments are too few to give the possibility to derive a more refined model.

6. ACKNOWLEDGEMENTS

The research presented in this paper has been financially supported by the Swede-Beam System, by the Swedish Foundation for Concrete Research and by the Swedish Council for Building Research. Representatives of these organizations have taken active part in the lay-out of the experimental program.

7. REFERENCES

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NOTATIONS

A_s = area of steel flange, area of anchoring bar

a = length of shear span

E_c = modulus of elasticity of concrete

E_s = modulus of elasticity of steel
f_{cc} = mean structural strength of concrete
f_{ccK} = mean cube strength of concrete
f_{sy} = yield strength of steel
f_{su} = ultimate strength of steel
h_s = height of steel web
ℓ_s = welded length of anchoring bar
s = distance between anchoring bars
t_c = thickness of concrete web
t_s = thickness of steel web
z = internal lever arm
M_y = part of the yield moment carried by the bottom flange
V = shear force
V_p = reference value of shear force
V_u = ultimate value of shear force
q = shear flow
q_c = shear flow corresponding to stresses in the concrete web
q_s = shear flow corresponding to stresses in the steel web
q_u = ultimate shear flow
y = deflection
ϕ = displacement, slip
θ_c = angle of inclination of stress-field in the concrete web
θ_s = angle of inclination of stress-field in the steel web
ν = effectiveness factor