THE SHEAR CAPACITY OF PRESTRESSED CONCRETE BEAMS IN REGIONS OF PLASTIC HINGES

Thomas Pettersson, Lic. Eng., Division of Concrete Structures, Chalmers University of Technology, Göteborg, Sweden.

Earlier investigations at the Division of Concrete Structures, Chalmers University of Technology, have indicated a strong influence on shear capacity of the prestressing force. Lately some douhts about this increasing of the capacity have been put forward, specially in regions of great bending moments or close to plastic hinges.

The paper presents an experimental investigation dealing with this problem and compares the test results with the new Swedish code BBK 79 and the theory of plasticity.

Keywords: Concrete, beams, prestress, shearing, yielding.

1. INTRODUCTION

This paper is a short summary of an investigation reported in /1/.

It has been known for several years that prestress and compression force increase shear capacity. This phenomenon is easy to understand and can be explained by among other things decreasing flexural cracks. In the case where the prestressed bars reach the yield point and the plastic stage prior to shear failure in the same section, the effect of the prestress can be expected to decrease. This case is for instance present in a support section of a continuous beam, especially if ultimate limit design is used. The aim of this test is to study this phenomenon.

2. TEST SPECIMENS

In order to study the problem mentioned above, two main types of prestressed beams were produced. The first type (A) are beams with one end projected as a cantilever and the prestressed bars at the upper side, and designed so that the shear failure would appear on the span side of the support. The second type (B) are simply supported beams with the same span length as the cantilevered ones but with the prestressed bars in the lower side.

All beams were designed according to the new Swedish code BBK 79 but without any safety coefficients. Both types were reinforced with four cold drawn bars $\phi 16$ Ks90[1] in the shear span, two prestressed and two ordinary bars.

The beams were cast in pairs, one of each type and with the same prestress force in both beams. The main types were divided in several subseries e.g. with or without stirrups and different spacing between the stirrups. Fig. 1 and 2 show the dimensions and the reinforcing details for all beams.

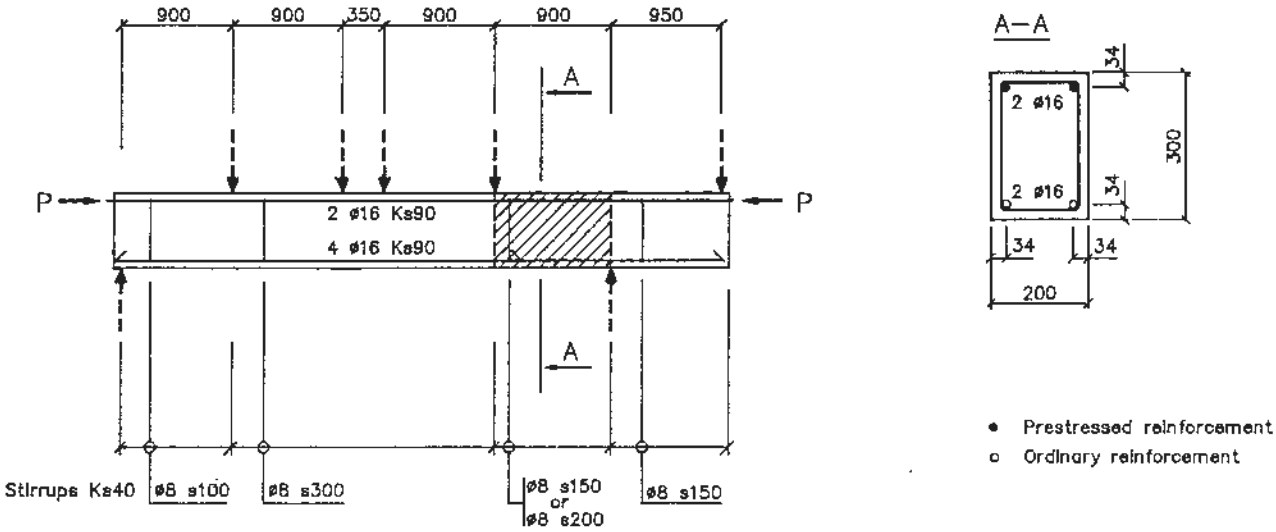

Fig. 1 Test beams - Type (A). The figure shows only beams with shear reinforcement in the test area

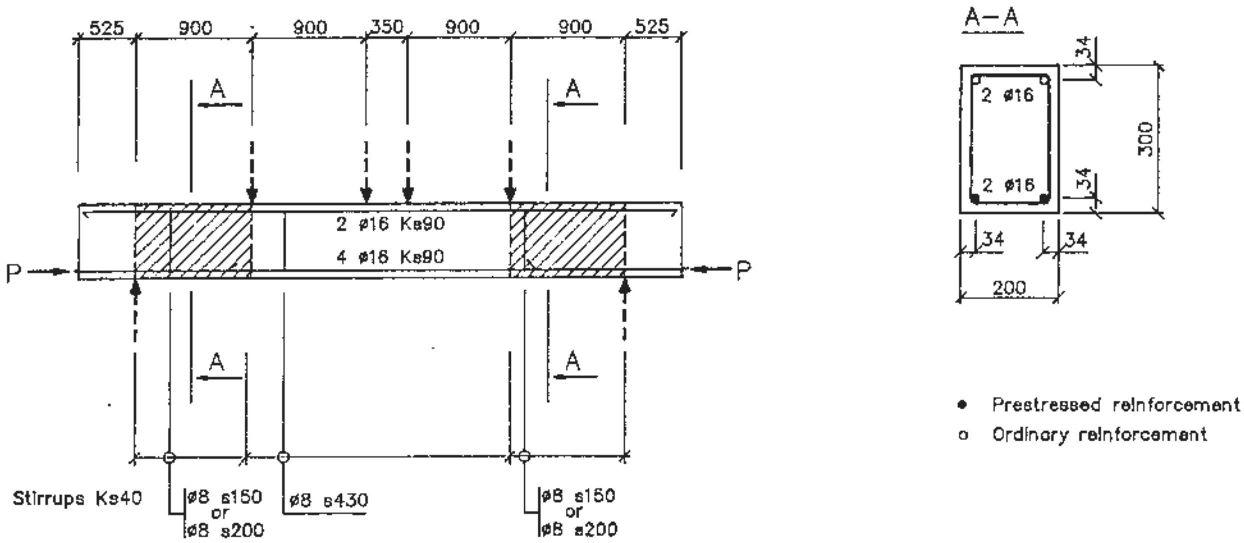

Fig. 2 Test beams - Type (B). The figure shows only beams with shear reinforcement in the test area

[1] Ks90 are deformed bars with the yield strength [f_y] at least 900 MPa

2.3 The test set-up

The beams of type (A) mentioned above were loaded with five loads (see Fig. 3). One was at the end of the cantilever and four in the span. The beams of type (B) were loaded with four loads and they had the supports located 525 mm from the ends of the beam. The reason for this was to get full anchorage between the prestressed bars and the concrete.

The strain in the prestressed bars was measured with two strain gauges, one on each side of the bars. These gauges were also used to measure the prestressing force in the bars before casting and to check when they start yielding above the support of the cantilever during the test.

The loads were measured by load cells, located between the distribution beams and the jacks. In addition we also measured deflections on the beam and observed all cracks when they appeared on the white painted side of the beam.

The loads increased in steps of about 10 kN during the test, until the beam reached its failure load (shear failure).

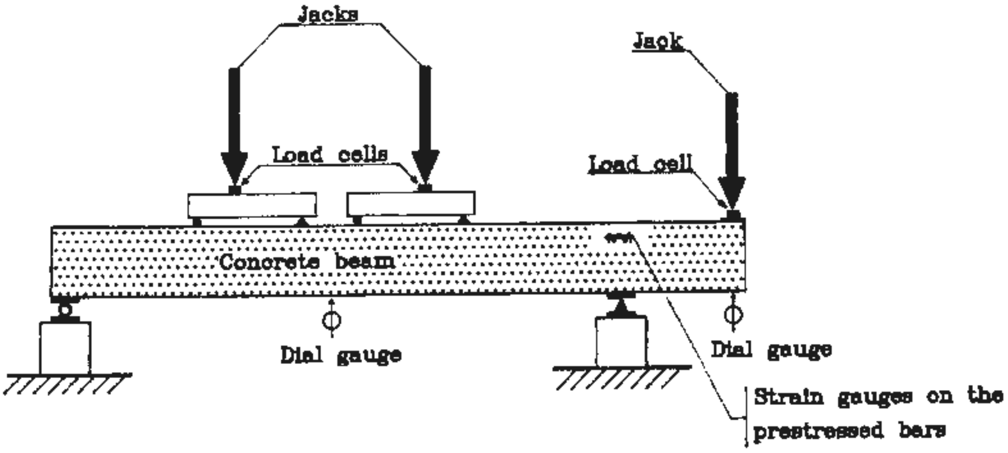

Fig. 3 Test set-up for beams of type (A)

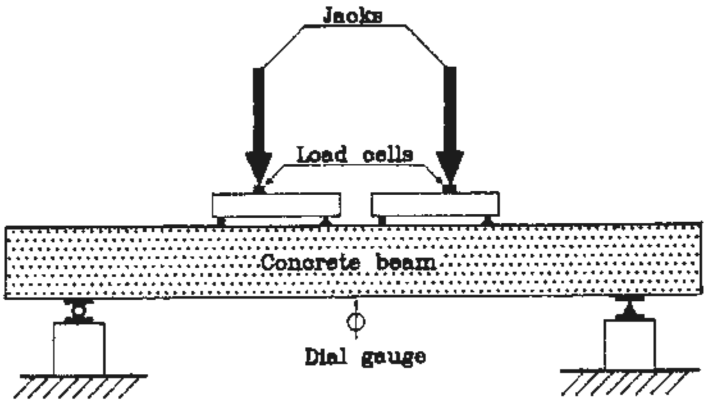

Fig. 4 Test set-up for beams of type (B)

3. TEST RESULT

The concrete strength, measured from cubes (15x15x15) and data for the reinforcement are presented in Table 1 below. The shear force, calculated at the failure load and normalized shear strength are presented in Table 2.

Results from the other measurements are not presented in this paper.

The results in Table 2 are divided in subseries, where test series A1-A3 are beams of type (A) with different numbers of stirrups. B1-B3 are corresponding type (B) beams. Serie A4 is ordinary type (A) beams but tested upside-down. A5 is beams without any stirrups and loaded so that the prestressed bars do not yield.

Table 1 The compression strength for the concrete, measured on 150x150x150 mm cubes. Data for all reinforcement in the beams

Beam No.	f_{cube} MPa	ϕ mm	E_s GPa	f_{02} MPa	f_u MPa	ϕ_s mm	f_{sy} MPa
1	37.5	16	221	999	1024	-	-
2	53.8	16	221	999	1024	-	-
3	41.7	16	221	999	1024	-	-
4	46.0	16	221	999	1024	-	-
5	41.1	16	218	907	916	8	427
6	47.5	16	218	907	916	8	427
7	40.6	16	218	907	916	8	427
8	37.2	16	218	907	916	8	427
9	40.6	16	218	907	916	8	427
10	32.7	16	218	907	916	-	-
11	29.3	16	218	907	916	-	-
12	40.2	16	218	907	916	-	-
13	41.8	16	218	907	916	-	-
14	41.4	16	221	999	1024	-	-
15	41.2	16	221	999	1024	-	-
16	49.1	16	221	999	1024	-	-
17	36.4	16	218	907	916	-	-
18	39.5	16	221	999	1024	-	-
19	39.9	16	218	907	916	-	-
20	30.1	16	218	907	916	-	-
21	32.4	16	218	907	916	-	-
22	34.9	16	218	907	916	-	-
23	38.5	16	218	907	916	-	-
24	51.5	16	218	907	916	8	427
25	38.0	16	218	907	916	8	427
26	31.1	16	218	907	916	8	427
27	47.5	16	218	907	916	8	427
28	32.3	16	218	907	916	8	427
29	34.1	16	218	907	916	8	427

Table 2 Results from tests and normalized shear strength

Test series A1
Without stirrups,
With yielding in the prestressed bars above the support of the cantilever

Beam No.	Prestress MPa	V_u kN	τ/f_{cube}
1	0.0	72.0	0.0407
2	123.0	108.0	0.0425
3	423.0	152.0	0.0772
4	656.0	165.0	0.0760

Test series A2
With stirrups, s = 150 mm.
With yielding in the prestressed bars above the support of the cantilever

Beam No.	Prestress MPa	V_u	τ/f_{cube}
5	101.0	137.6	0.0692
6	392.8	188.6	0.0841

Test series A3
With stirrups, s = 200 mm.
With yielding in the prestressed bars above the support of the cantilever

Beam No.	Prestress MPa	V_u kN	τ/f_{cube}
7	0.0	145.8	0.0761
8	166.8	145.6	0.0829
9	423.1	174.2	0.0909

Test series A4
Without stirrups.
Tested upside-down

Beam No.	Prestress MPa	V_u kN	τ/f_{cube}
10	369.0	69.6	0.0451
11	399.0	69.6	0.0503

Test series A5
Without stirrups.
Without yielding in the prestressed bars

Beam No.	Prestress MPa	V_u kN	τ/f_{cube}
12	0.0	79.0	0.0416
13	0.0	73.0	0.0370
14	417.0	126.0	0.0645
15	408.0	147.0	0.0756

Test series B1
Without stirrups

Beam No.	Pre-stress MPa	V_u kN	τ/f_{cube}
16	0.0	80.0	0.0343
17	0.0	57.0	0.0332
18	159.0	80.0	0.0429
19	172.0	67.0	0.0356
20	443.0	85.0	0.0468
21	425.0	107.0	0.0700
22	426.0	89.0	0.0540
23	698.0	109.0	0.0600

Test series B2
With stirrups, s = 150 mm

Beam No.	Pre-stress MPa	V_u kN	τ/f_{cube}
24	0.0	138.6	0.0570
25	183.1	164.6	0.0918
26	401.9	168.2	0.1121

Test series B3
With stirrups, s = 200 mm

Beam No.	Pre-stress MPa	V_u kN	τ/f_{cube}
27	0.0	163.6	0.0730
28	183.1	166.6	0.1093
29	407.9	166.2	0.1033

4. THE NEW SWEDISH CODE - BBK 79

The method in the Swedish code BBK 79 is almost the same as in the CEB model code /4/. The total shear capacity (V_d) is calculated as a sum of the concrete term (V_c), the shear reinforcement term (V_s) and the prestress term (V_p). Calculated values for each beam are presented in Table 3

$$V_d = V_c + V_s + V_p \qquad \dots 1$$

4.1 The shear capacity for the concrete

The capacity for the concrete, without any shear reinforcement is calculated with Eq. 2, as the nominal shear strength multiplied by the effective area of the beam section

$$V_c = b \cdot d \cdot \xi (1+50\rho) k_v \cdot f_{ct} \qquad \dots 2$$

where b = the width of the beam section

d = the effective depth of the section

ξ = 1.6-d

$$\rho = \frac{A_s}{b \cdot d}$$

f_{ct} = the tensile strength of the concrete

k_v in Eq. 2 is a statistic constant calculated from a large sample of test beams as the 0.05 fractile and is in the code equal to 0.3. We have not used this value because it underestimates the failure load. Instead we have used our ordinary reinforced beams without stirrups to calculate a better value. In that calculation we also replaced the tensile strength f_{ct} with $\sqrt{f_{cube}}$ and received k_v equal to 0.1257

$$V_c = b \cdot d(1.6-d)(1+50\rho)0.1257 \cdot \sqrt{f_{cube}} \qquad \ldots 3$$

4.2 The shear capacity of the shear reinforcement

The shear capacity from the stirrups is calculated as:

$$V_s = \frac{A_{sv} \cdot 0.9 \cdot d \cdot f_{sv}}{s} \qquad \ldots 4$$

where A_{sv} = the steel area of one stirrup

f_{sv} = the design value for the steel tension of the stirrups

s = the spacing between the stirrups

The code uses the design value of steel strength f_{sv}, which almost always underestimates the real shear capacity. We wanted a better value and therefore used the yield strength from the steel test.

4.3 The shear capacity from the prestress force

The increase in shear capacity from prestress is calculated with Eq. 5. M_o is the zero strain moment. The model assumes that the shear capacity increases by the same degree as the flexural moment (M_d) exceeds M_o

$$V_p = \frac{M_o \cdot V_d}{M_d} \qquad \ldots 5$$

where M_o = the zero strain moment for the section

V_d = the shear force

M_d = the flexural moment

Table 3 shows that the shear capacity calculated according to BBK 79 is too low in some beams and too high in others. This is expected as shear tests always give a great spread between the results.

The test results indicate no decrease in shear capacity when the prestressed bars reach the yield points.

Beam nos. 10 and 11 were ordinary type (A) beams but tested upside-down. This means that the prestressed bars were in the compression zone. The code says then that the shear capacity would decrease but the tests still give a small increasing effect from the prestress.

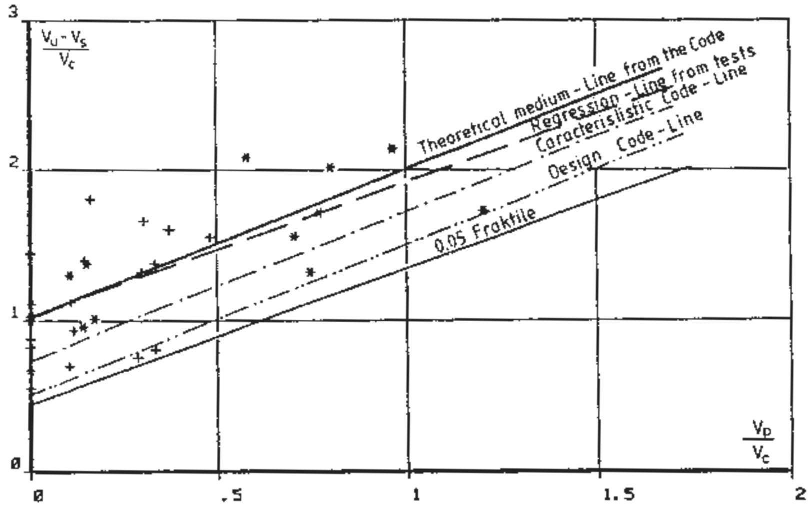

Fig. 5 The test results in comparing with the new Swedish code BBK 79

In Fig. 5 above, the test results are compared with the new Swedish code BBK 79. We can see that the regression line from the tests is almost the same as the theoretical line. One can also see that the design line in the code is very close to the line which corresponds to the 0.05 fractile.

Table 3 Calculated shear capacity in accordance with the new Swedish code BBK 79 compared with the ultimate shear capacity from test $[V_u]$

Beam No.	M_o kNm	M_d kNm	V_c kN	V_s kN	V_p kN	V_d kN	V_u/V_d
1	0.0	104.0	70.0	0.0	0.0	70.0	1.03
2	8.4	104.0	83.0	0.0	8.7	91.7	1.18
3	28.9	105.0	73.0	0.0	42.0	115.0	1.32
4	45.0	101.0	77.0	0.0	74.0	151.0	1.09
5	6.6	86.3	73.2	67.2	10.5	150.9	0.91
6	25.9	89.7	77.8	67.2	54.4	199.4	0.94
7	0.0	114.0	71.9	50.7	0.0	122.6	-
8	6.8	89.9	68.8	50.7	10.6	145.6	1.12
9	27.6	88.2	71.9	50.7	55.1	174.2	0.98
10	-9.5	60.8	64.5	0.0			
11	-10.2	54.1	61.1	0.0			
12	0.0	39.0	71.4	0.0	0.0	71.4	1.11
13	0.0	45.0	72.8	0.0	0.0	72.8	1.00
14	28.5	41.0	72.8	0.0	87.0	126.0	0.79
15	27.9	70.0	72.8	0.0	58.0	147.0	1.12
16	0.0	110.0	79.8	0.0	0.0	79.8	1.00
17	0.0	64.0	68.6	0.0	0.0	68.6	0.83
18	8.6	89.0	71.4	0.0	7.7	79.1	1.01
19	9.3	75.0	71.4	0.0	8.3	79.7	0.84
20	23.9	98.0	61.6	0.0	20.7	82.3	1.03
21	22.9	124.0	64.4	0.0	19.8	84.2	1.27
22	22.9	101.0	67.2	0.0	20.2	87.4	1.02
23	37.6	122.0	70.0	0.0	33.6	103.6	1.05
24	0.0	159.5	80.9	67.2	0.0	148.1	0.93
25	12.0	188.6	69.5	67.2	10.4	147.1	1.12
26	26.9	192.4	62.9	67.2	23.5	153.6	1.09
27	0.0	188.3	77.8	50.7	0.0	128.5	1.27
28	12.1	190.9	64.1	50.7	10.6	125.4	1.33
29	26.9	189.3	65.9	50.7	23.6	140.2	-

5. THE THEORY OF PLASTICITY

The theory of plasticity /5/, /6/ and /7/ has been used with success during the last decades to get upper and lower bound for the ultimate capacity. The aim has been both to calculate the shear capacity and to explain the phenomenon of shearing.

The model takes neither consideration to the tension strength of the concrete nor to the prestressing effects in any other way than through an effectiveness factor ν. This factor is calculated from beam tests and becomes in the tests reported in /5/, /6/ and /7/ about 0.6. In our tests we received much lower values as one can see in Table 5.

5.1 Beams without shear reinforcement

M P Nielsen presents in /7/ how to calculate the ultimate shear capacity in beams without stirrups. He uses the failure mechanism presented in Fig. 6 which resulted in Eq. 6.

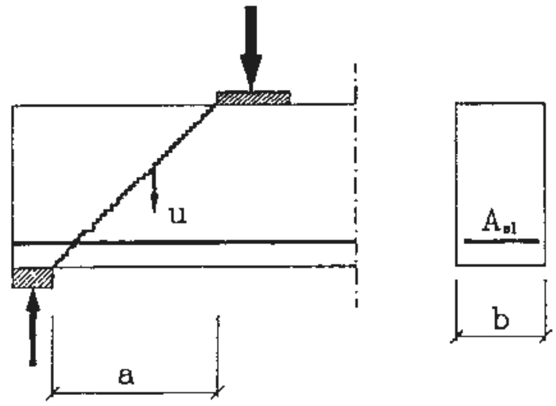

Fig. 6 Failure mechanism for beams without shear reinforcement

$$\frac{\tau}{f_c} = \frac{1}{2}\left(\sqrt{4\phi(1-\phi) + \left(\frac{a}{h}\right)^2} - \frac{a}{h}\right) \qquad \ldots 6$$

In Table 4 the shear strength, calculated from Eq. 6, is compared with test results. We can see that the calculated shear strength seems to be acceptable in beams without prestress but much too low in the others. The calculations are carried out with the effectiveness factor ν equal to 1.0 and a/h = 3.8 for all of the beams.

$$\frac{\tau}{f_c} = \frac{V_u}{b \cdot h \cdot f_c} \qquad \ldots 7$$

$$\phi = \frac{A_{s\ell} \cdot f_{02}}{b \cdot h\ f_c}$$

where
- V_u = ultimate shear force from the tests
- b = beam width
- h = the distance between the tension and the compression reinforcement
- f_c = the compression strength in the concrete, measured on cylinders ($f_c = f_{cube}/1.4$)
- f_{02} = the steel strength
- $A_{s\ell}$ = the steel area

Table 4 The shear strength from tests and corresponding strength, calculated from Eq. 6

Beam No.	$(\tau/f_c)_{test}$	ϕ	$(\tau/f_c)_{calc}$ from Eq. 6
1	0.0582	0.3245	0.0568
2	0.0608	0.2262	0.0405
3	0.1105	0.2918	0.0536
4	0.1087	0.2646	0.0505
12	0.0569	0.2748	0.0517
13	0.0529	0.2643	0.0505
14	0.0922	0.2940	0.0539
15	0.1081	0.2954	0.0540
16	0.0494	0.2479	0.0484
17	0.0475	0.3035	0.0548
18	0.0614	0.3081	0.0553
19	0.0509	0.2769	0.0520
20	0.0856	0.3671	0.0602
21	0.1001	0.3410	0.0582
22	0.0773	0.3166	0.0561
23	0.0858	0.2870	0.0531

5.2 Beams with shear reinforcement

The shear strength, calculated in accordance with formulas in /6/ and with ν such that the strength should be the same as from the tests, is presented in Table 5. We can see that the effectiveness factor ν had to be very small for all beams except those with the highest grade of prestress.

Table 5 Calculated shear strength with ν such that the strengths should be the same as from the tests

Beam No.	$(\tau/f_c)_{calc}$	ν
5	0.0981	0.25
6	0.1211	0.39
8	0.1188	0.39
9	0.1309	0.50
24	0.0816	0.21
25	0.1313	0.38
26	0.1636	0.38
27	0.1049	0.38
28	0.1560	0.57

6. CONCLUSIONS

The tests show that the prestress has an increasing effect on the shear capacity, independently of any yielding in the prestressed bars. This can probably be explained by the fact that the steel area which is yielding is too small to affect the shear capacity.

Today it is very difficult to use the theory of plasticity in beams with prestress and stirrups because of the difficulties to choose an appropriate effectiveness factor. More research is needed in this field.

7. REFERENCES

/1/ Pettersson T: Tvärkraftskapacitet hos förspända balkar i snitt med stort moment (Shear capacity in prestressed concrete beams in sections with high flexural moment), Rapport 86:1, Betongbyggnad, Chalmers tekniska högskola, Göteborg 1986.

/2/ BBK 79: Bestämmelser för betongkonstruktioner (Regulations for concrete structures), Statens Betongkommitté, Stockholm 1979.

/3/ Losberg A, Hedman O: Dimensionering av betongkonstruktioner med hänsyn till tvärkraft (Structural design of concrete construction based on shear failure), Nordisk Betong 5/1975, s 19-29.

/4/ CEB-FIP Model Code for Concrete Structures, 1978.

/5/ Nielsen M P, Braestrup M W, Jensen B C, Bach F: Concrete Plasticity Beams, Shear-Shear in Joints, Punching Shear, Technical University of Denmark, Lyngby Structural Research Laboratory, Copenhagen, October 1978.

/6/ Braestrup M W: Effect of main steel strength on the shear capacity of reinforced concrete beams with stirrups, R 110, Technical University of Denmark, Lyngby Structural Research Laboratory, May 1979.

/7/ Nielsen M P: Limit analysis and concrete plasticity, Prentice-Hall Series in Civil Engineering and Engineering Mechanics, 1984.