THE CALCULATION METHODS OF CONCRETE BEAMS UNDER THERMAL GRADIENT AND MECHANICAL LOADING

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ABSTRACT

The paper deals with calculation methods of concrete structures subjected to combined thermal and mechanical loading. Calculations can be carried out either by means of elastic theory or by non-linear methods based on the elastic theory which take cracking and creep of concrete as well as yielding of reinforcing steel bars into consideration. Use can also be made of the plasticity theory.

Key words: concrete beams, temperature gradient, calculation of forces

1 INTRODUCTION

Calculation methods of concrete structures under combined thermal and mechanical loads are examined below. Thermal load is understood as a load caused by a temperature difference between the surfaces of a structure. Such loads as these develop e.g. in oil drilling and production platforms and in structures of heating and nuclear power plants.

As the action effects due to thermal loads are proportional to the stiffness of a structure, the value conforming to the real stiffness of the structure must be used in the calculation of these action effects caused by thermal loads. In this case the effect of thermal loads on these actions diminishes or almost disappears due to cracking and yielding. Correspondingly, the effect of mechanical loads acting upon a structure will change when action effects due to changes in stiffness are redistributed.

The calculation of action effects can be carried out, depending on the degree of stress, either by the classical elastic theory taking into account changes in the stiffness of a structure in a cracked state or by a non-linear method. By means of non-linear methods based on the elastic theory not only cracking of concrete but also the non-linearity of moment curvature relation as well as the effect of creep of concrete and that of yield hardening of reinforcing bars can be taken into account. In addition to the above methods use can be made of methods based on the plasticity theory.

2 STIFFNESS OF CONCRETE STRUCTURES

The determination of the actual stiffness value of a concrete structure is of primary importance in order that thermal load could be determined, since the magnitude of action effects is directly proportional to the stiffness of the structure.

The factors which have an effect on the stiffness of the structure can be divided into three groups. These may be dependent on geometry of the structure, the physical properties of a building material or on the distribution of action effects.

A noticeable decrease in the stiffness of concrete structures usually takes place when the structure cracks. The amount of tension bars greatly influences the extent to which stiffness decreases. The smaller the amount of tension bars is, the lower the stiffness ratio of the cracked state to the uncracked state. The stiffness of the structure further depends on the loading level, in which case the stress-strain diagram with increasing loading begins to bend and the tension bars may reach the yielding stage.

If the tension bars, however, reach the so-called yield hardening stage, the structure may stiffen again. Furthermore, as creep of concrete brings a reduction in stiffness about, stiffness is therefore smaller at sustained loads. After release of sustained loads there remains deformation in the structure, which is the so-called irreversible proportion of creep. Thus the stiffness of the structure is also dependent on the loading history of the structure. Since stiffness and the intensity of action effects are interdependent, one ends up in iterative methods which are usually employed for solutions. The magnitude of a normal force should be mentioned as a factor which has a most important effect on the stiffness of concrete beams. This force can result either from prestressing or it can also be an external load. Stiffness of the structure increases by the action of compressive normal force and decreases due to tensile normal force /3, 6/.

3 CALCULATION OF ACTION EFFECTS

3.1 Linear method based on elastic theory

The most simplified calculation methods of concrete structures are based on the linear elastic theory. In these calculations stiffness is then used which conforms either to the cracked state or the uncracked state. The validity of the result greatly depends on how stiffness is chosen particularly when calculating the action effects caused by a thermal load. When the amount of the upper and lower reinforcements of beams differs from each other non-continuous stiffness distribution must be used in calculations. The change point of stiffness can then be assumed to be the zero point of the bending moment and, for improving the accuracy of calculations at least two calculation rounds can be carried out, by means of which the location of the zero point of the moment is iterated /6, 16, 20).

3.2 Non-linear method based on elastic theory

In order to obtain an exact result when analysing concrete structures non-linear methods based on the elastic theory can be used. These methods take cracking and creep of concrete into consideration together with redistribution of action effects due to yield hardening of reinforcing bars.

In the following, the calculation method based on the elastic theory is presented for calculation of action effects caused by mechanical load and thermal gradient in bent structures. The method takes into account the loading history of a structure, cracking and creep of concrete as well as yield hardening of reinforcing bars, i.e. a real stiffness of the structure is used in this method. This method as a manual process requires plenty of work in the case of multi-span beams and frames but is efficient when a computer is used which gives as accurate action effect distribution as possible.

The idea of this method is to calculate, as presented in Fig. 1, for each cross section an increase in curvature caused by non-linearity.

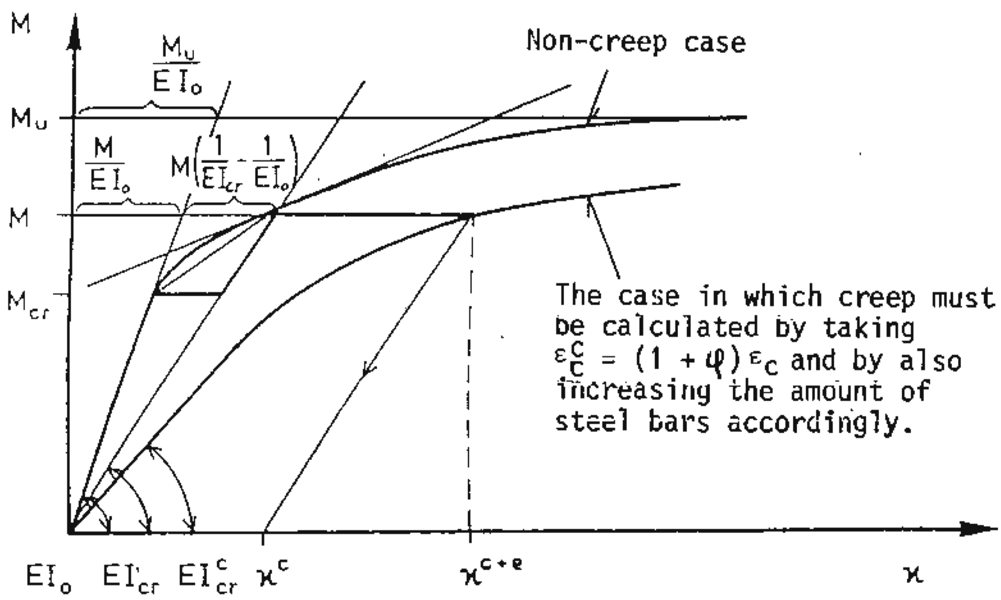


Fig. 1. Calculation phases.

1. Using the elastic theory the bending moment surface M_o of a structure is solved by assuming that the structure is in an uncracked state (stiffness EI_o conforming to the uncracked state is used). Loading is composed of mechanical and thermal loads.

2. The cracked areas of the structure are traced down. The cross section is interpreted as cracked when at the boundaries of the cross section the tensile stress exceeds the characteristic tensile strength.

3. For cracked portions of a structure the stiffness EI_{cr} conforming to the cracked state is calculated.

4. An increase in cracking or, if the steel reinforcing bars are at the yield point, an increase in curvature due to yielding of reinforcing bars in cracked areas is calculated

$$\Delta\kappa(x) = M_o(x)\left(\frac{1}{EI_{cr}(x)} - \frac{1}{EI_o(x)}\right) \quad \text{when} \quad \sigma_s(x) < f_y \text{ and } \sigma_c(x) < 0.6\, f_c$$

$$\Delta\kappa(x) = \frac{\varepsilon_s(x) - \varepsilon_c(x)}{d(x)} - \frac{M_o(x)}{EI_o(x)} \quad \text{when} \quad \sigma_s(x) = f_y \text{ or } \sigma_c(x) \geq 0.6\, f_c$$

5. By means of curvatures $\Delta\kappa(x)$ the rotations, caused by cracking or yielding of bars, are calculated by integrating a curvature change over the deformed portion of the structure and by moving it to the cornes to act

$$\Theta_i = \int \frac{L - x}{L} \Delta\kappa(x)\ dx$$

$$\Theta_j = \int \frac{x}{L} \Delta\kappa(x)\ dx$$

6. The bending moment surfaces caused by unit rotations are calculated by placing in turn the rotation $\Theta_i = -1$ to the corners between the bars which have undergone deformation. Thus the moment surfaces M_i^1 are obtained.

7. The final moment surface is obtained by superposing the effect of cracking into the moments conforming to the uncracked state:

$$M(x) = M_o(x) + \Sigma M_i^1\ \Theta_i$$

8. The checking of the accuracy in calculations is carried out by calculating $\Delta\kappa(x)$ again at the value of $M(x)$ in conformity with Item 4 and, if the necessary accuracy is achieved, calculations can be terminated. If the required accuracy is not attained, calculations can be continued by iteration in conformity with Items 5 to 8. Finally, the bending moment distribution conforming to the non-creep case is obtained, in which the effect of cracking and the real stress-strain relation of reinforcing bars is taken into consideration.

9. The effect of creep can be taken into account by calculating the curvature κ^{c+e} of the crept beam corresponding to the bending moment, in which case an increase in curvature in the cracked and crept areas is

$$\Delta\kappa(x) = \kappa^{c+e} - \frac{M_o(x)}{EI_o}$$

The crept curvature is calculated by multiplying deformation calculated in comformity with the short-term elastic modulus of concrete by $(1 + \varphi)$ and by increasing steel strain accordingly.

The effect of temperature on creep can be taken into consideration by increasing $(1 + \varphi)$ which corresponds to the temperature.

If loading is altered during the course of loading and calculation, the new situation can be calculated in conformity with Items 1 to 9 by storing the earlier crept curvatures κ^c of the structure and by considering the effect of these curvatures in Item 9 by substituting the value $\kappa^{c+e} + \kappa^c$ for κ^{c+e}.

If mechanical loading is removed altogether, creep caused by this loading can be taken into account with the relaxation of action effects due to the thermal gradient.

10. The displacements of a structure can be calculated by means of corner-rotations ϕ and curvatures κ^{c+e}.

In manual calculations, the above-mentioned method can be simplified in such a way that the plasticized area is replaced by a hinge. The hinges are placed at the maximum values of the moment. The moment distribution is assumed to be in agreement with the elastic theory, The moment-rotation relation as presented in Fig. 2 is used for each hinge in the method. Using the calculation method true to the method the moment of the order of M_u in the limit state can be allowed for each hinge providing the structure has an adequate rotation capacity at the hinge points.

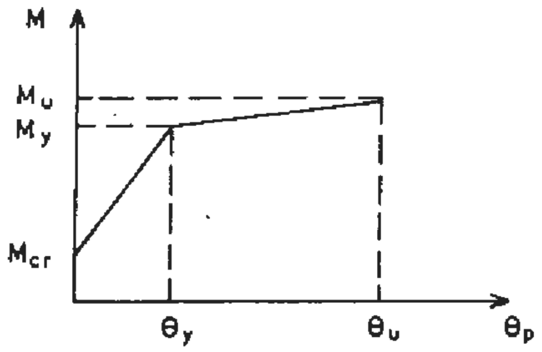


Fig. 2. Simplified method.

3.2 Plasticity theory

In analysing concrete structures the plasticity theory can also be used. In this case the structure is considered as a mechanism shown in Fig. 3. It must be checked, however, that a sufficient amount of hinges can develop in the structure without exceeding the rotation capacity in any hinge.

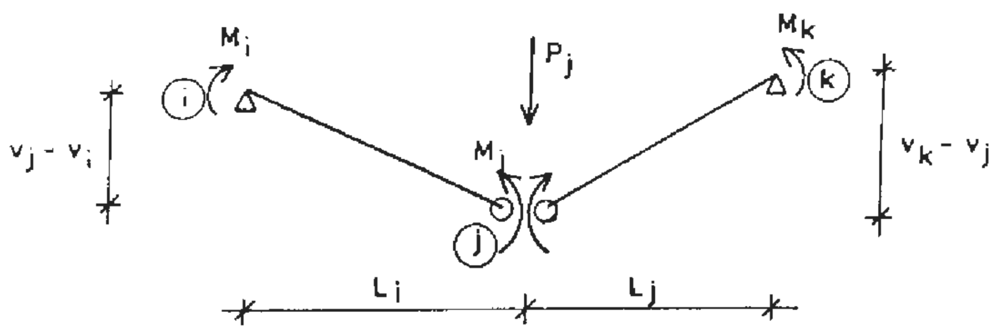


Fig. 3. Mechanism composed of three hinges.

For a material either the general moment-curvature relation of a non-linear material or the bi-linear moment-curvature relation can be used.

In solutions usually iterative methods have to be used since an analytical solution attainable in a closed form is only rarely found.

In the solution conforming to the plasticity theory we arrive at the set of equations

$$[A_{ij}]\ \{v_j\} + [B_{ij}]\ \{M_j\} = 0 \qquad \text{(n compalibility equations)}$$

$$[C_{mn}]\ \{M_n\} = \{P_m\} \qquad \text{((n - r) equilibrium equations)}$$

It is assumed above that there are n hinges in the structure (n unknown moments), r restraints and n - r unknowns in the equations. The dependence is non-linear when the matrix $[B_{ij}]$ is a function of moments M_i.

In order to solve the simultaneous equations the latter equation can be changed to the form

$$[C_{mr})]\ \{M_r\} + [C_{ms}]\ \{M_s\} = \{P_u\}$$

where there are r of M_r moments and (n - r) of M_s moments. By calculating $[M_r]$ from the equation and by placing it into n compatibility equations the set of equations

$$[A_{ij}]\ \{v_j\} + [B_{ir}]\ [(C_{mr})]^{-1}\ \{P_m\} - [B_{ir}]\ [C_{rs}]\ \{M_s\} + [B_{is}]\ \{M_s\} = 0$$

is obtained /1/.

The equation can be solved step by step by a suitable algorithm (see e.g. "Zienkiewics: The finite element method", Mc Graw Hill, 1977).

4 CALCULATION OF ROTATION CAPACITY

4.1 General

When using plasticity theory in calculation of the ultimate capacity of the structure the rotation capacity must be checked. There are many factors which affect the magnitude of the rotation capacity. The factors can be divided into main groups such as material factors, geometrical factors and loading factors. The most important material factors are the strength and strain factors of concrete and reinforcement. The most important geometrical factor is the area of tension reinforcement and of the loading factors a tensile or compressive normal force /5, 8, 9/.

4.2 Calculation

The rotation capacity of the concrete structure is usually calculated with formulas based on research results. Methods based on the energy principle can also be used as well as methods based on calculation of crack width between uncracked concrete sections.

In the following, a new method is presented. In this method the rotation capacity is calculated by the formula

$$\Theta = 0.5 \cdot k_M \cdot L_{cr} \cdot \kappa_u$$

where k_M is a factor depending on the moment distribution

k_M 0.5 when the moment distribution is not constant

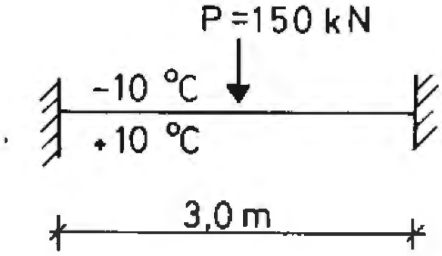
k_M = 1 when the moment distribution is constant

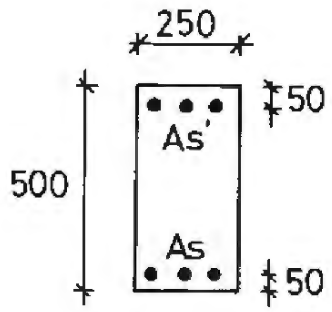
L_{cr} is the cracked section of the beam

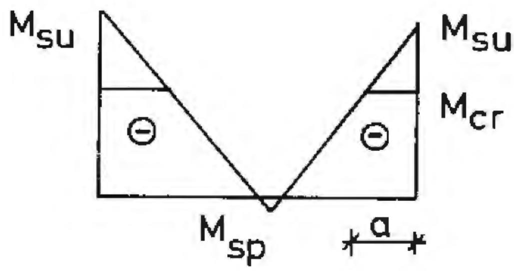
κ_u is the ultimate curvature of the beam.

<u>Example beam</u>

- $b \times h = 250 \times 500$
- concrete strength K 40
- $A_s = A_s' = 24.12\ cm^2$ ($3\ \phi\ 32$)
- $f_{ct} = 3.0$ MPa
- $E_c = 31622\ MN/m^2$
- $E_s = 210\ 000\ MN/m^2$
- $n = E_s/E_c = 6.6$
- $\alpha_{\Delta T} = 10 \cdot 10^{-6}/^oC$
- $\rho = \frac{A_s}{bd} \cdot 0.021 = \rho'$
- $I_o = 0.00368\ m^4$
- $I_{cr} = 0.00184\ m^4$







$$M_{su} = M_{su}^{\Delta T} + M_{su}^{P}$$

$$= -\frac{\alpha \cdot \Delta T \cdot E_c \cdot I}{h} - \frac{P \cdot L}{8}$$

$$= -0.102\ MNm$$

$$M_{sp} = -\frac{\alpha \cdot \Delta T \cdot E_c \cdot I}{h} + \frac{PL}{8}$$

$$= +0.0102\ MNm$$

Stresses in the non-cracked state

$\sigma_{su} = 6.9$ MPa $> f_{ct}$ (cracks!)

$\sigma_{sp} = 0.7$ MPa $< f_{ct}$

Increase in curvature due to cracking:

$$\Delta\kappa(x) = M \left(\frac{1}{EI_{cr}} - \frac{1}{EI_o}\right)$$

Rotation due to increase in curvature

$$\phi = \int_{o}^{a_{cr}} \Delta\kappa(x)\, dx$$

Change in support-moment

$$\Delta M_{su} = \frac{2\, E_c I_o}{L}\, \phi$$

New moments

$$M_{su}^{\;i} = M_{su}^{\;i-1} + \frac{2\, E_c I_o}{L}\, (\phi_i - \phi_{i-1})$$

$$M_{sp}^{\;i} = M_{sp}^{\;i-1} - \frac{2\, E_c I_o}{L}\, (\phi_i - \phi_{i-1})$$

Moments in iteration cycles

i	M_{su}	M_{sp}
0	-0.1020	0.0102
1	-0.0750	0.0360
2	-0.0907	0.0203
3	- 0.0835	0.0275
4	-0.0877	0.0232
5	-0.0849	0.0260
6	-0.0865	0.0243
7	-0.0859	0.0249
8	-0.0865	0.0242

The support moment decreases ∿ 15 % and span moment increases ∿ 135 % due to cracking.

The condition for complete relaxation of thermal moment is the rotation

$$\phi = \frac{\alpha \cdot L \cdot \Delta T \cdot E_c \cdot I_o}{2\, h} = 0.0698$$

The rotation capacities are:

Baker's method	$\Theta_p = 0.0043$
Corley's "	$\Theta_p = 0.0066$
Mattock's "	$\Theta_p = 0.0066$
Huovinen's "	$\Theta_p = 0.0025$

So at ultimate state the thermal moment decreases only about 4...9 %.

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NOTATIONS

A_s, A_s'	Reinforcement areas
E_c	Modulus of elasticity of concrete
E_s	" " " reinforcement
I_o	Non-cracked moment of inertia
I_{cr}	Cracked " "
K	Nominal concrete strength
L	Span
L_{cr}	Cracked section of the span
M_o	Non-cracked moment
M_{cr}	Cracking moment
M_y	Yielding moment
M_u	Ultimate moment
M_{su}	Support moment
M_{sp}	Span moment
ΔT	Temperature gradient
d	Effective depth of cross-section
h	Heigh of cross-section
k_M	Rotation capacity factor
v	Reflection
x	Coordinate
ε	Relative strain
κ	Curvature
σ	Stress
Θ	Rotation