A FRACTURE MECHANICS AND EXPERIMENTAL APPROACH ON ANCHORAGE SPLITTING

Per-Ake Olsson, MSc, Division of Concrete Structures, Chalmers University of Technology, Göteborg, Sweden.

An extension of the fictitious crack model (FCM) considering unloading of stable cracks is presented. The model is used to analyse the splitting of the concrete cover in the anchorage zone of a beam, reinforced with deformed bars.

A testing method for experimental verification is presented.

Keywords: bond, anchorage, splitting, fracture mechanics, constitutive model

1. INTRODUCTION

1.1 General

Anchorage of deformed reinforcing bars in concrete is a very complex problem and therefore very complicated to model. To the knowledge of the author, no model has yet been published that tries to take all known parameters into consideration. However, a lot of researchers all over the world have been, and are working with bond and anchorage problems stipulating some kind of restrictions and simplifications in order to make the problem possible to grasp.

A lot of work has been done concerning the transverse cracking around a deformed bar embedded in concrete, and the effect on the bond slip behaviour. Many theoretical models, both analytical and numerical, have been proposed, based on a local bond relation. This local bond relation is normally based on a pull out test where a bar with a short embedment length is pulled out from the centre of a concrete cube. If we permit ourselves to make a distinction between bond failure and splitting failure then the pull out test will represent a bond failure.

In most real cases, with deformed bars used in a structure and with normal concrete cover the anchorage failure will probably be a splitting failure. This lack of agreement between the theoretical models and a real construction would be partly overcome if there was a way of modelling an effective local bond relation taking into consideration the splitting of the concrete cover and the presence of transverse reinforcement.

The major aim with the work, partly described in this paper, is to model such a relation using a combined fracture mechanics and experimental approach.

1.2 Splitting failure theory

When a deformed bar is anchored in concrete, inclined compressive forces are acting between the concrete and the bar. These forces are concentrated to the bar ribs and they cause an internally cracked zone around the bar. This cracking has been experimentally verified by Goto et al, /1/, see Fig. 1.2.

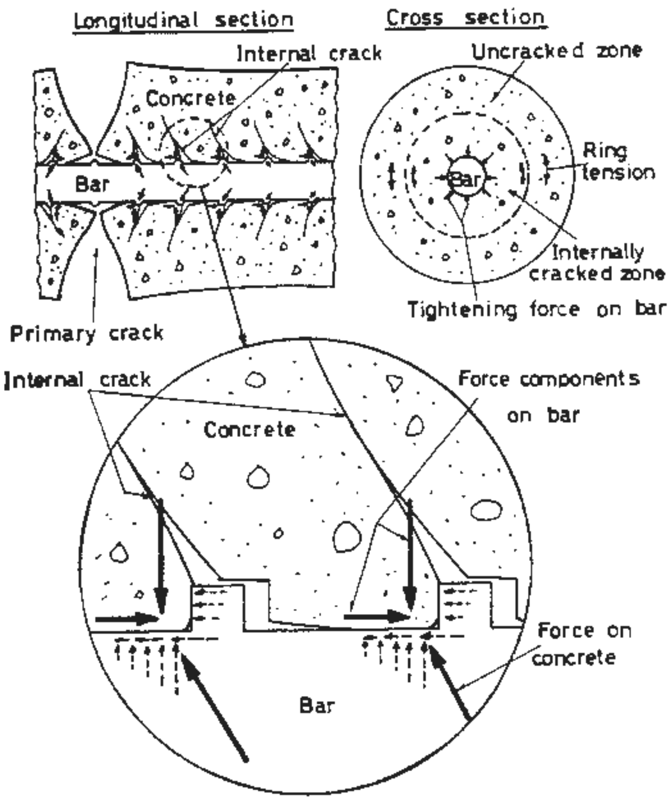


Fig. 1.1 Forces and internal cracking around a deformed bar, fig. from /1/

The radial force components can be looked upon as an internal pressure acting on the surrounding concrete. This leads to tensile stresses in the concrete, see Fig. 1.2. If these stresses in any point reach the concrete tensile strength a crack starts to propagate. Depending on the thickness of the concrete cover, influence of transverse reinforcement, interaction between several bars etc this can lead to an anchorage failure due to splitting of the concrete cover.

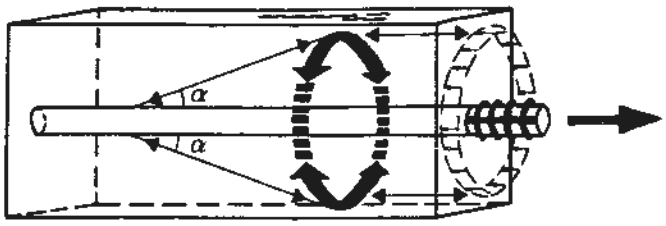


Fig. 1.2 "Schematic representation of how the radial components of the bond forces are balanced against tensile stress rings in the concrete in an anchorage zone", from /2/

Depending on the geometry of the cross section, different crack patterns are possible, see Fig. 1.3.

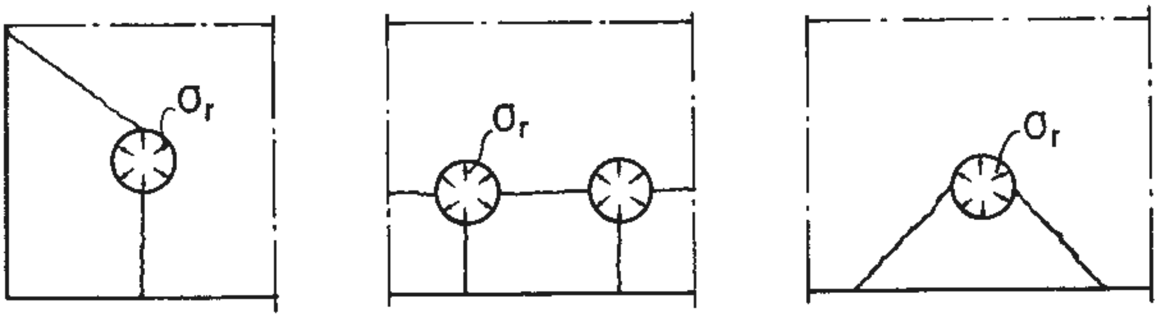


Fig. 1.3 Examples of different possible crack patterns at anchorage failure due to splitting (splitting failure)

If an assumption is made about the tensile stress distribution in the cracks at splitting failure, then $\sigma_{r,max}$ can be calculated from an equilibrium condition. Such a simple stress distribution, based on plasticity in the concrete, is proposed by Tepfers, /3/. He assumes that the tensile stress equals the concrete tensile strength f_{ct} all along the crack, see Fig. 1.4.

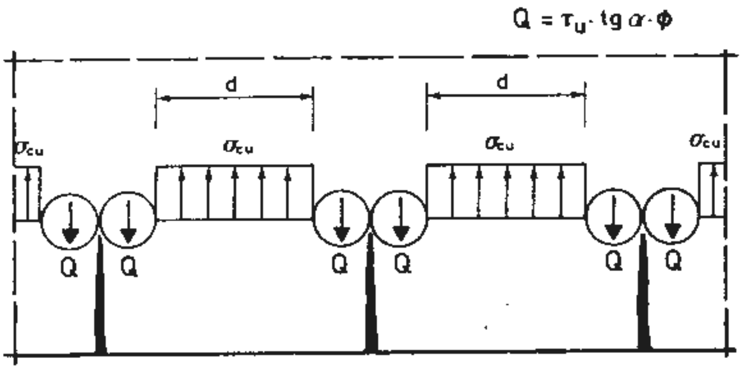


Fig. 1.4 Splitting failure assuming perfect plasticity in the concrete. Fig. from /3/

This model has been adopted in the Swedish concrete code, BBK 79.

2. A NON-LINEAR FRACTURE MECHANICS APPROACH FOR ANCHORAGE SPLITTING

2.1 General

Based on the splitting failure theory described in part 1.2 a cross section of a beam within the anchorage zone of a deformed bar is studied. The bar is modelled as a hole with an internal pressure $p = \sigma_r$. If the pressure increases a splitting failure will occur when $p = f_r$. The splitting failure is a result of crack propagation through the concrete cover, see Fig. 2.1.

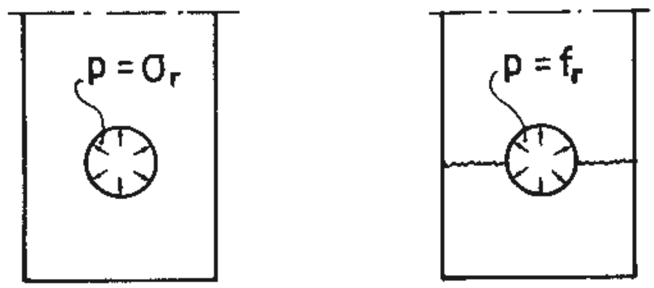


Fig. 2.1 Principle for splitting failure in a concrete cross section loaded with an internal pressure

Since the anchorage failure due to splitting is a crack growth problem it seems very reasonable to adopt a fracture mechanics approach.

2.2 The fictitious crack model (FCM)

The fictitious crack model, /4/-/6/ is a fracture mechanics model developed especially for concrete. It takes into account the relatively big damage zone in concrete and it also takes into account the decreasing of the stress when the crack width increases within the zone. However, the most important feature of the model when studying anchorage splitting is that FCM does not need an initial crack.

The features of FCM are:

- In front of the real, open crack there is a fictitious crack that is able to transfer stresses. This part of the crack is called the fracture zone or the damage zone, Fig. 2.2
- The fracture zone starts developing when the first principal stress is equal to the tensile strength.
- The fracture zone develops perpendicular to the first principal stress.

- The stress transfer in the ficitious crack is depending on the crack width, w, in the stress direction according to a σ-w curve. The σ-w curve is based on direct tensile tests, see Fig. 2.3.
- The width of the fracture zone is zero when it starts to develop.
- The material outside the fracture zone is described by a σ-ε curve. In practical use the material is often supposed to be linear elastic up to $\sigma_t = f_t$.

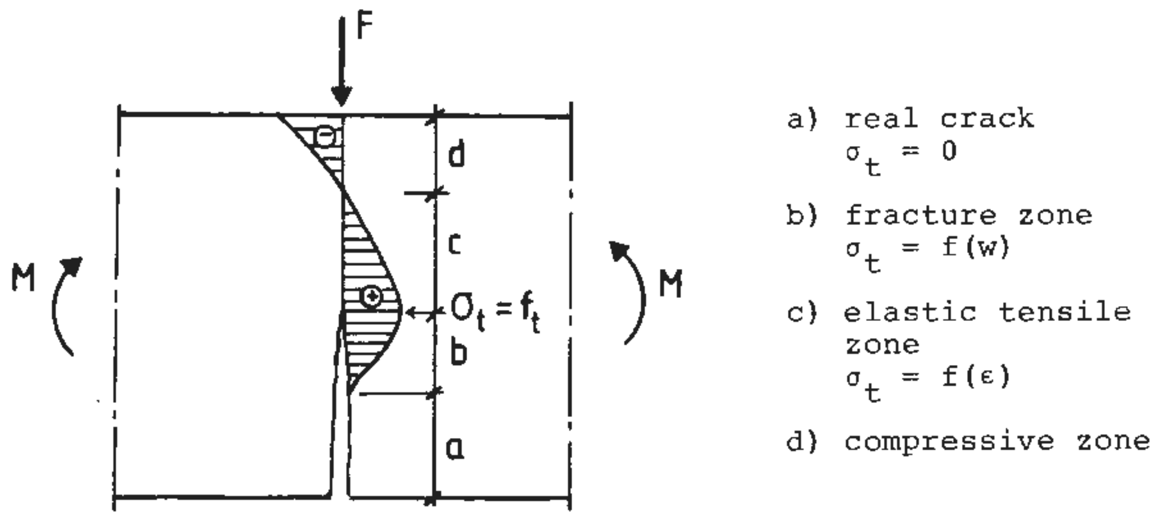


Fig. 2.2 Stress distribution at crack propagation according to FCM

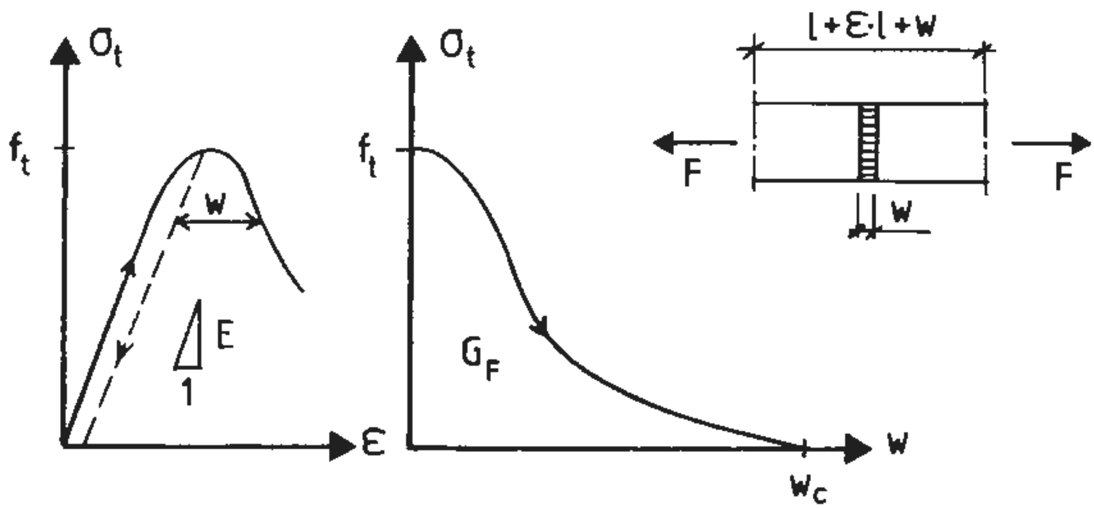


Fig. 2.3 Description of the constitutive modelling according to FCM

FCM has three major advantages compared with other possible models:

- FCM is based upon simple assumptions and criterias which makes the model easy to understand and to use together with the finite element method (FCM).
- Using FCM it is possible and relatively easy to follow all parts in the failure process. From the initiation of the crack growth to the crack propagation through the entire tensile region.
- The crack propagation must not necessarily start at a stress concentration in front of a notch tip or in front of an already existing crack. This means that FCM is not a pure fracture mechanics model, it can also be used to analyse traditional strength problems.

Note that the third advantage in the list above is necessary to be able to analyse geometries where the crack propagation starts from a hole.

There are four "parameters" that must be known in order to perform a FCM analysis. These are the tensile strength f_t, Young´s modulus of elasticity E, the fracture energy G_F and the shape of the σ-w curve. The fracture energy is the energy absorbed per unit area of the fracture surface (projected area). G_F is also equal to the area below the σ-w curve. In numerical analyses the shape of the σ-w curve is modelled with straigth lines. For plain concrete, a linear curve according to Fig. 2.4a can be used. However, a bilinear curve according to Fig. 2.4b is a much better approximation, see Fig. 2.5.

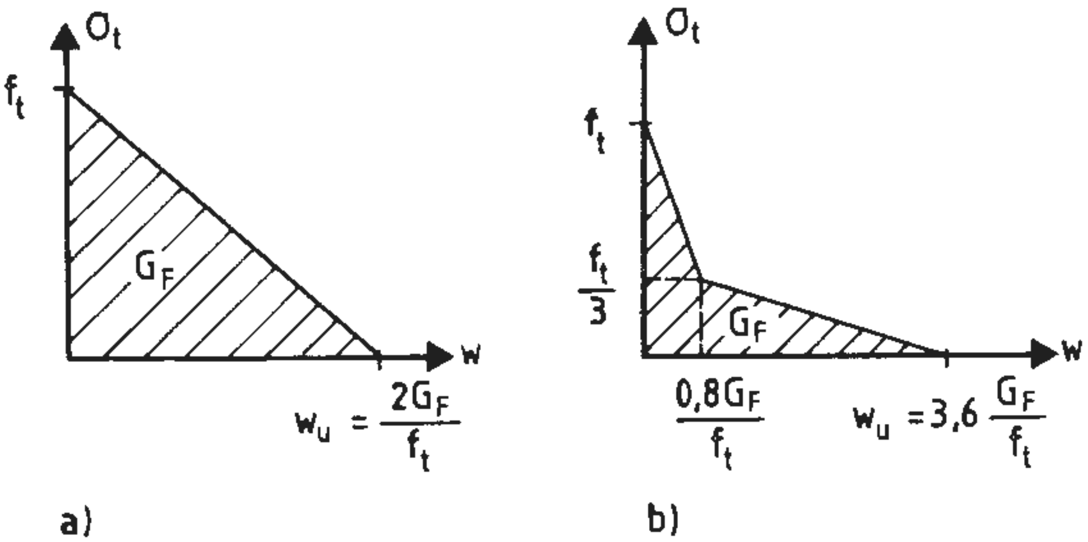


Fig. 2.4 Approximations of the σ-w curve for plain concrete. b) is proposed by Pettersson, /5/ based on direct tensile tests

Values of the fracture energy, G_F can be determined from a three point bend test on a notched beam, see /6/ and /7/. In the literature, values of G_F can be found in, for example, /5/, /8/ and /9/.

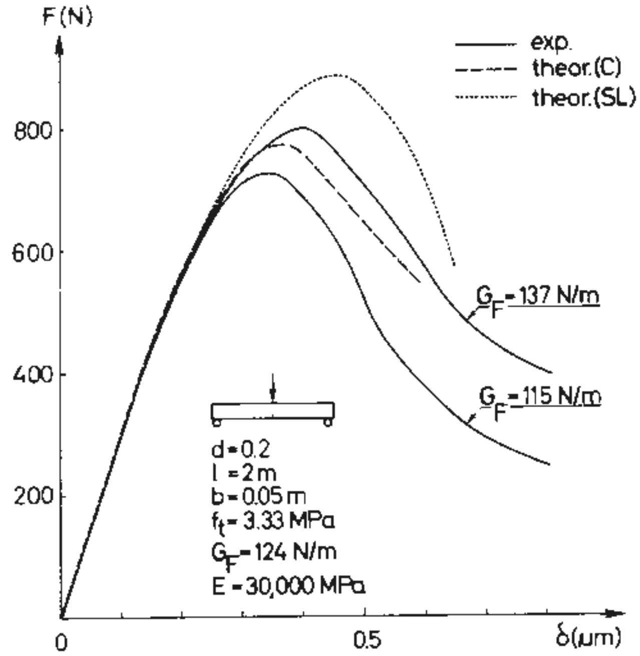


Fig. 2.5 Experimental and theoretical load-deflection curves for three-point bend tests on notched beams. (SL) is the straight line model. (C) is the bilinear model /5/

The unit on the x-axis should be (mm)!

2.3 An unloading model for stable fictitious cracks

2.3.1 General

The fictitious crack model, as presented by Hillerborg et al, /4/, does not include unloading and reloading. Gylltoft, /9/ used FCM as a base in a fatigue model and proposed an extension considering crack-closing and crack-opening when changing the sign of the stress.

In section 2.3.2 in this paper a constitutive unloading model based on FCM is presented. The model considers crack-closing in a different way than the model by Gylltoft. It also considers compressive stresses. Now, the following question might arise. Why the need of an unloading model?

The load is monotonically increasing. However, when analysing the cover splitting, more than one crack must be modelled at the same time. At maximum load only one of them become instable. When this happens the load has reached its maximum. The stresses outside the fracture zone of the instable crack are therefore also decreasing. This means that the stable cracks in the concrete cover will be unloaded i.e. both the stresses and the crack widths are decreasing.

2.3.2 An extension of the fictitious crack model

Consider the fracture zone as a lot of fibres, slipping against each other, see Fig. 2.6.

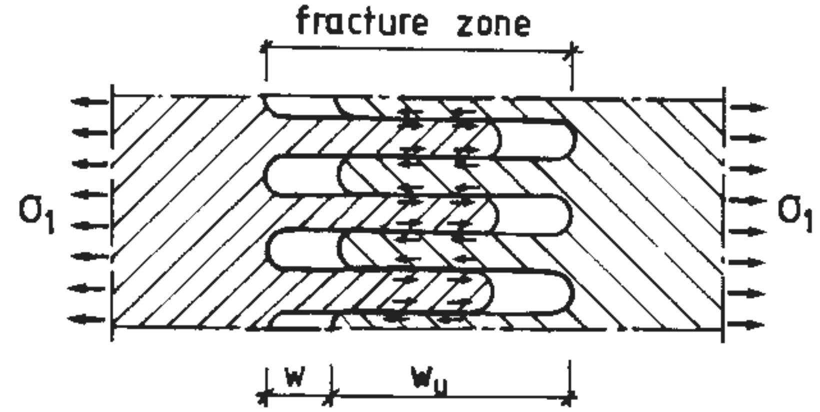


Fig. 2.6 Phenomenological, "slip friction" hypothesis to explain the strain softening in the fracture zone

Due to the friction between the fibres it is possible to transfer stresses even after that the tensile strength has been reached. Let σ increase from 0 to σ_N and then decrease to $-2\sigma_N$ using the straight line monotonic σ-ε and σ-w curves. Adopting the "slip friction" hypothesis a constitutive model according to Fig. 2.7 is formulated.

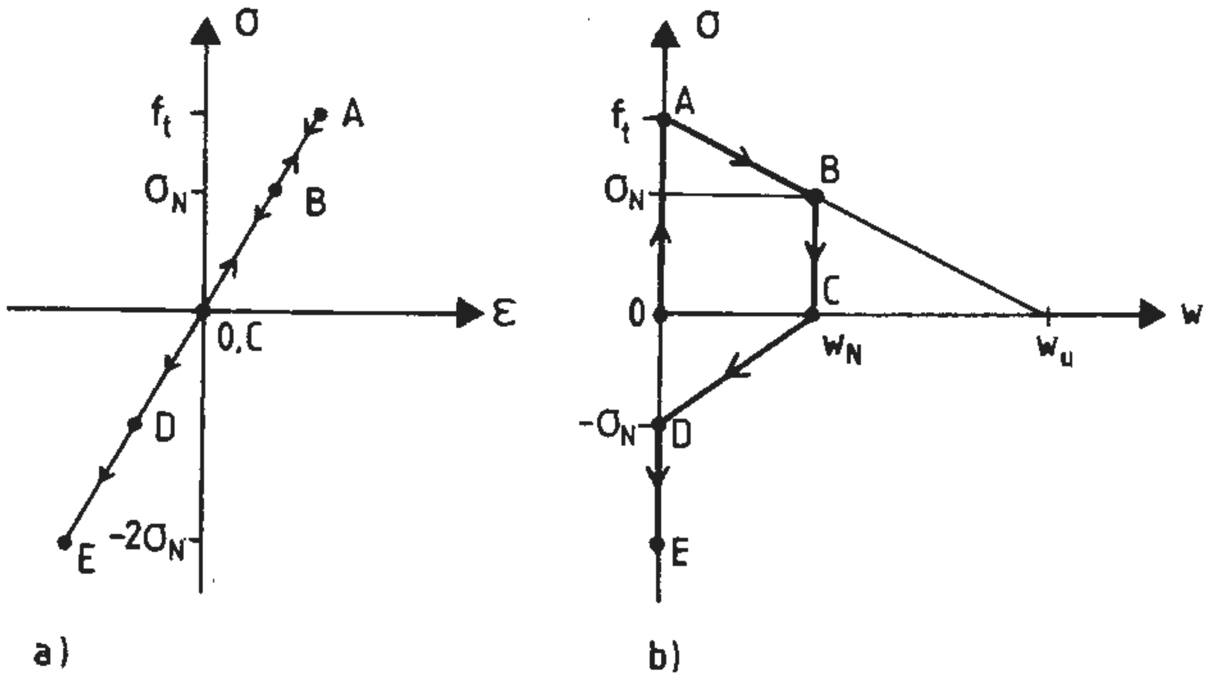


Fig. 2.7 Constitutive model including unloading.
a) outside the fracture zone
b) across the fracture zone

The model can be described:

O-A: Elastic stage; $d\sigma/dt > 0$, $w = 0$

A-B: Fictitious crack stage; $d\sigma/dt < 0$, $dw/dt > 0$
B-C: Unloading stage; $d\sigma/dt < 0$, $dw/dt = 0$
C-D: Closing stage; $d\sigma/dt < 0$, $dw/dt < 0$
D-E: Compressive stage; $d\sigma/dt < 0$, $w = 0$

The elastic and fictitious crack stages follow FCM completely. During the unloading stage $dw/dt = 0$ since the elastic deformations are assumed to be zero within the fracture zone. During the closing stage $d\sigma/dt \neq 0$. This is a principle difference from the model proposed by Gylltoft, /9/. Because of the "friction between the fibres" energy must be absorbed during the closing of the fictitious crack and that implies $\sigma \neq 0$. It seems reasonably to model that the closing starts at $\sigma = 0$ and is completed at $\sigma = -\sigma_N$ with a straight line in between. In this way the closing stiffness decreases with decreasing σ_N, i.e. if the unloading starts late, when the crack is almost a real crack then the closing stiffness is very low.

3. NUMERICAL APPROACH

3.1 General

For analysis of the fracture behaviour of a structure, a numerical technique is needed. In this work the finite element method (FEM) has been chosen. A detailed description using FEM together with FCM is given in /5/.

In the present work a slightly different technique has to be used to be able to follow the simultaneous growth of two or more cracks.

3.2 Crack element for the FEM modelling

To model the fracture zones a special crack element has been developed. The conditions for the features of the element are mainly:

- it should model the crack continiously
- it should be easy to implement in the FEM model in any direction
- it should be able to handle booth normal stresses and shear stresses in a FCM analysis
- it should give stresses and deformations as output.

A four node element with two degrees of freedom in every node was chosen, Fig. 3.1.

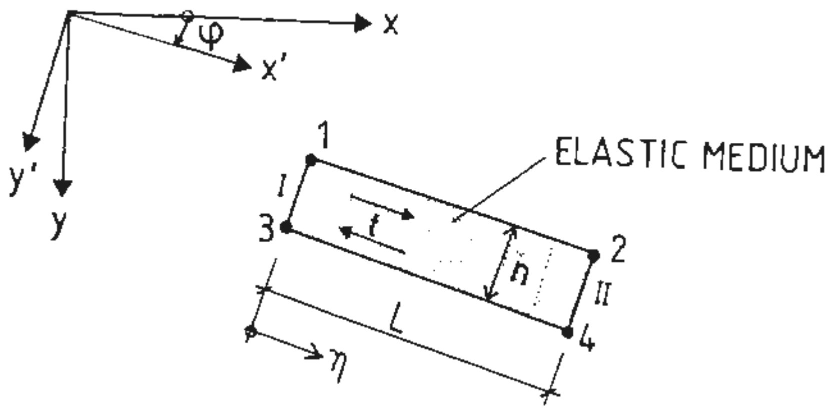


Fig. 3.1 Four node crack element

The two interesting forces N (perpendicular to the crack direction, η) and T (along the crack direction, η) are related to the relative displacements over the elastic medium through the constitutive equations

$$\begin{bmatrix} T \\ N \end{bmatrix} = \begin{bmatrix} E_1 & 0 \\ 0 & E_2 \end{bmatrix} \begin{bmatrix} t \\ n \end{bmatrix}$$

3.3 Numerical solution procedure

During the finite element modelling, crack elements are inserted where cracks are expected to appear. The element stiffness is initially given a very high value to simulate uncracked concrete. Theoretically this method makes it possible to study an unlimited number of cracks in the same analysis. Practically however, the amount of input and output data to handle will put some limits.

The analysis of a structure is performed stepwise where each step consists of one FEM-part and one FCM-part. The principle procedure for such a step is briefly described below.

Calculation step No. N

a) FEM-calculation with the internal pressure equal to 1

b) FCM-calculation based on the output from a). In this part, the stresses from all crack elements in the structure are studied together with the corresponding, accumulated values from earlier steps. Considering all the limits given by the constitutive model in section 2.3.2, the smallest load increment causing a change of stage (according to Fig. 2.7) to any crack element are calculated.

Assuming linear elasticity within each step, all the output values from a) are now being multiplied with the value of the calculated load increment. Then the accumulated values after load step No. N are calulated.

c) The finite element model is now updated with the new stiffness value for the "critical" crack element found in b).

d) Continue with step No. N+1.

The described procedure is performed completely automatically by interaction between a microcomputer and a mainframe, see section 3.3.

3.3 Computer programs

The calculation process described in section 3.2 is separated into two different parts. The FEM-calculations are made in a mainframe with the general program GENFEM, /10/, and with the developed crack element implemented. The FCM-calculations are made in a microcomputer (MC) with a program developed by the author.

Beside doing the FCM-calculations, the MC also controls the whole process. It calculates input data for the mainframe. It sends the data via a serial interface to the mainframe and execute the GENFEM program. When GENFEM is ready, the MC reads the output from GENFEM via the serial interface and then starts the FCM calculations.

The accumulated results after each step are stored on a diskette and the result from each step is being printed together with the actual status of every crack element (uncracked, fictitious crack, open crack, closing crack etc). This makes it possible to, in detail, follow what is happening in the different cracks during the failure process.

4. EXPERIMENTAL APPROACH

4.1 General

In order to verify the theoretical model and to close the gap to the local bond relations described in section 1.1, experimental tests are going to be performed. At the moment two kinds of tests are planned.

4.2 Pressure splitting test

This test is aimed to verify the theoretical model. The specimen is thought to act like a piece of a beam, between two transverse cracks, see Fig. 4.1. Instead of longitudinal reinforcement there are holes.

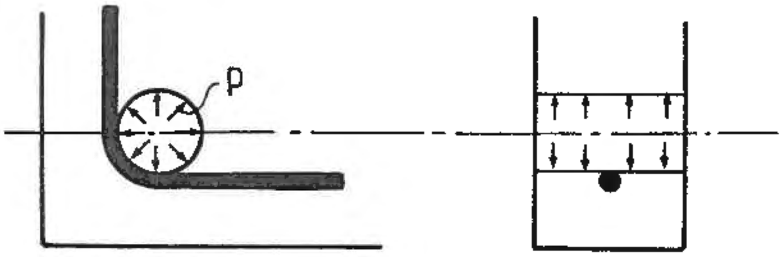


Fig. 4.1 Test specimen to study anchorage splitting

With a special hydraulic tool an internal pressure is applied at the concrete surface, inside the hole, see Fig. 4.2. During the test the pressure is increased until failure. Only pilot tests without measuring are performed up to now. It might be difficult to follow the decending part of the curve but an attempt to measure the pressure and the crack opening during the failure shall be done.

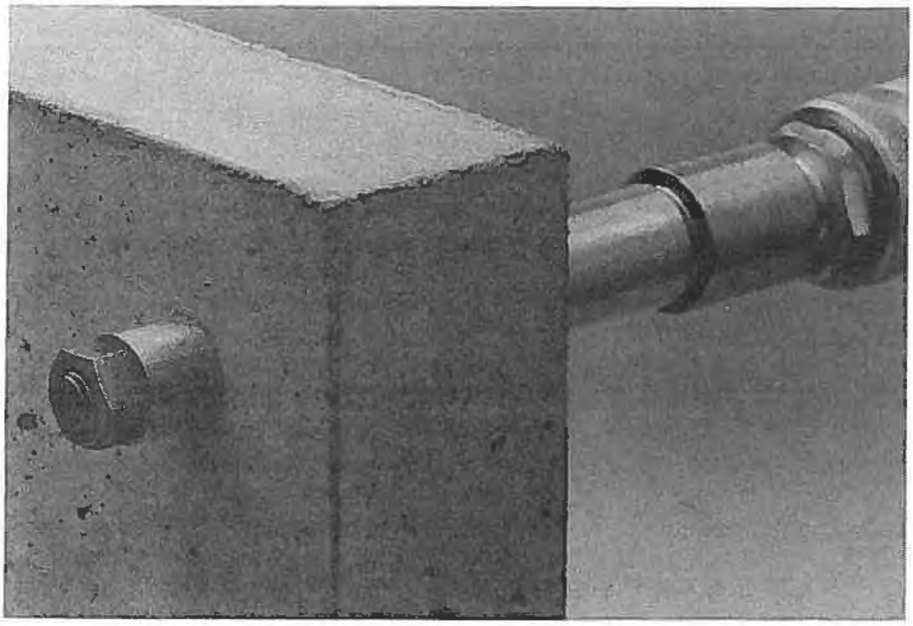
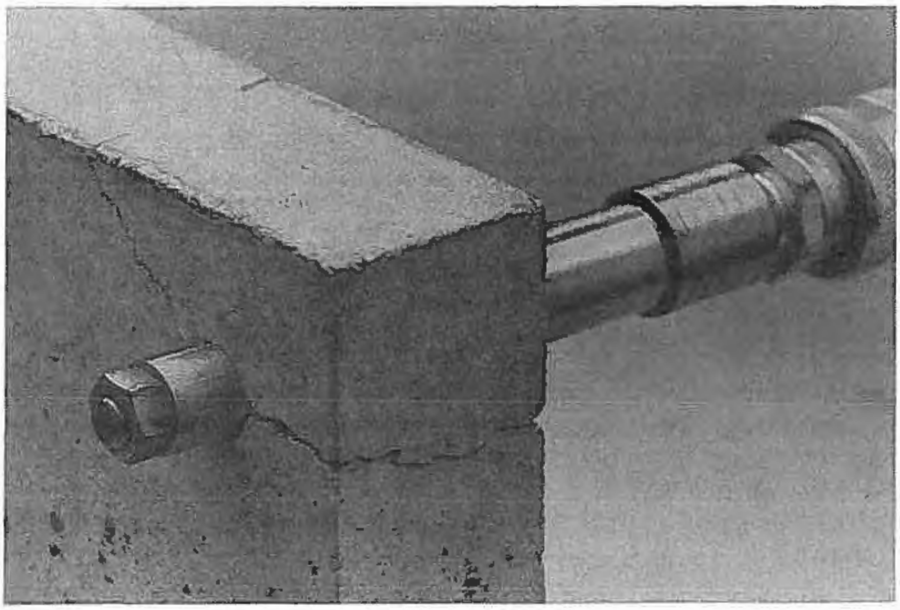


Fig. 4.2 A testing method to simulate anchorage splitting
caused by a deformed bar



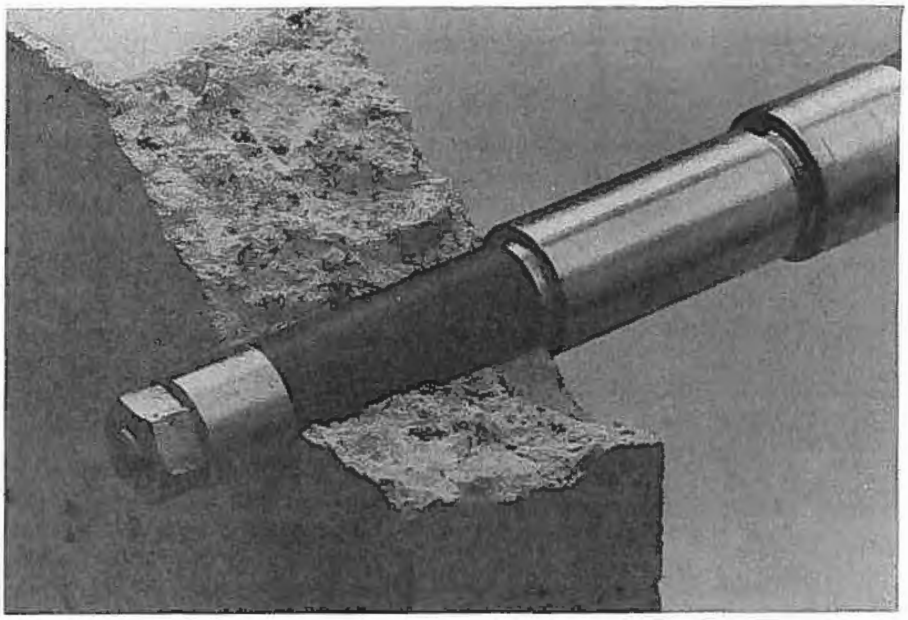


Fig. 4.2 A testing method to simulate anchorage splitting caused by a deformed bar

Test series with and without transverse reinforcement and with different geometry are planned.

4.3 Pull out splitting test

In order to connect the studied "splitting stiffness" with the bond slip behaviour an experimental study is planned. This can of course also be modelled and studied using FEM but that is not planned within this work.

The pull out splitting tests are not planned in detail but they will be based on the philosophy described in the RING TEST in /11/.

5. NUMERICAL RESULTS

5.1 General

Here, the results from two numerical analyses are presented and discussed. The σ-w curve is chosen to be a straight line according to Fig. 2.4a). Other parameters in the analysis have been:

$$G_F = 60 \text{ N/m}$$

$$f_t = 3.0 \text{ MPa}$$

$$E = 30.0 \text{ GPa}$$

The values are chosen based on results presented in /5/. The low value of G_F is justified by the assumption that the maximum particle size is smaller below the reinforcing bars than in other parts of the beam and thus a value corresponding to mortar was chosen.

In the FEM-calculations the length of all crack elements is 2 mm. Compared with the length of the fracture zone, the chosen element length is sufficient. On the other hand the number of elements along the crack are possibly too few in the analysis with the small bar spacing.

In the calculations, plain stress conditions were used. However, when concrete is far from yielding it is of a little importance if plain strain or plain stress conditions are used.

In this paper, the analyses of two different geometries are presented. They both deal with anchorage zones in slabs, i.e. the anchored bars are assumed to be so far from the edge of the concrete cross section that the disturbance can be neglected. In both cases, all parameters are the same except from the distance between the bars, see Fig. 5.1.

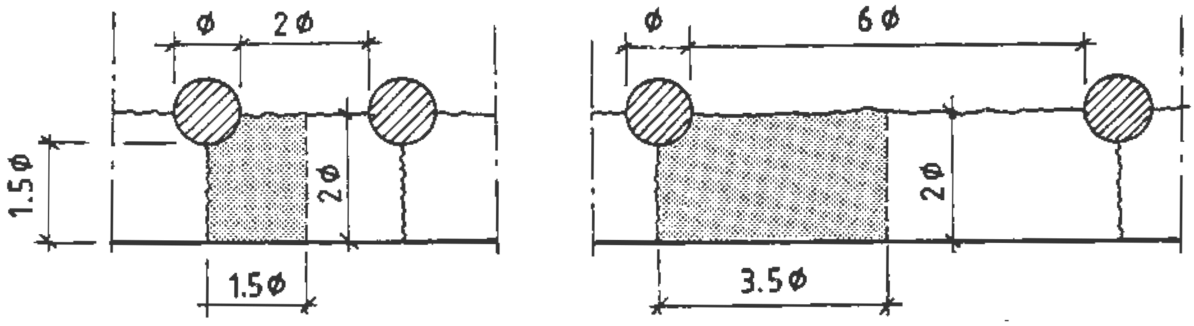


Fig. 5.1 The two geometries being analysed. ϕ = 16 mm. Due to symmetry, only the dotted part of the cover is modelled

As can be seen from Fig. 5.1 two cracks might appear during the splitting failure. The horizontal crack is the primary one which finally causes the failure. The vertical cracks are predicted by Tepfers, /3/. In the present analyses, both cracks are modelled.

In order to change the element size without using triangular elements, GENFEM uses a subsidiary conditions technique where the displacement of "the node in the middle" is related to the displacements of the two "surrounding nodes". In the present analyses the three nodes are specified to be on a straight line.

The internal pressure is modelled with nodal loads acting on each node inside the hole.

5.2 Diagrams

The finite element models used in these analyses are shown in Fig. 5.2.

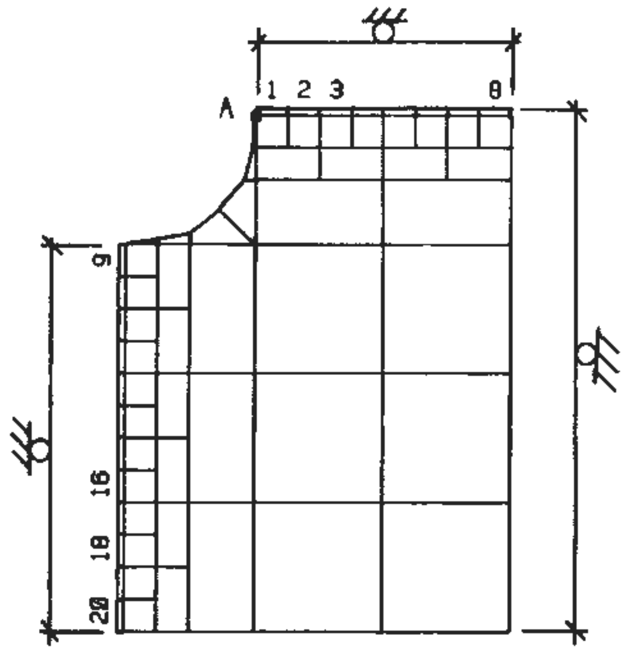


Fig. 5.2a Small bar spacing

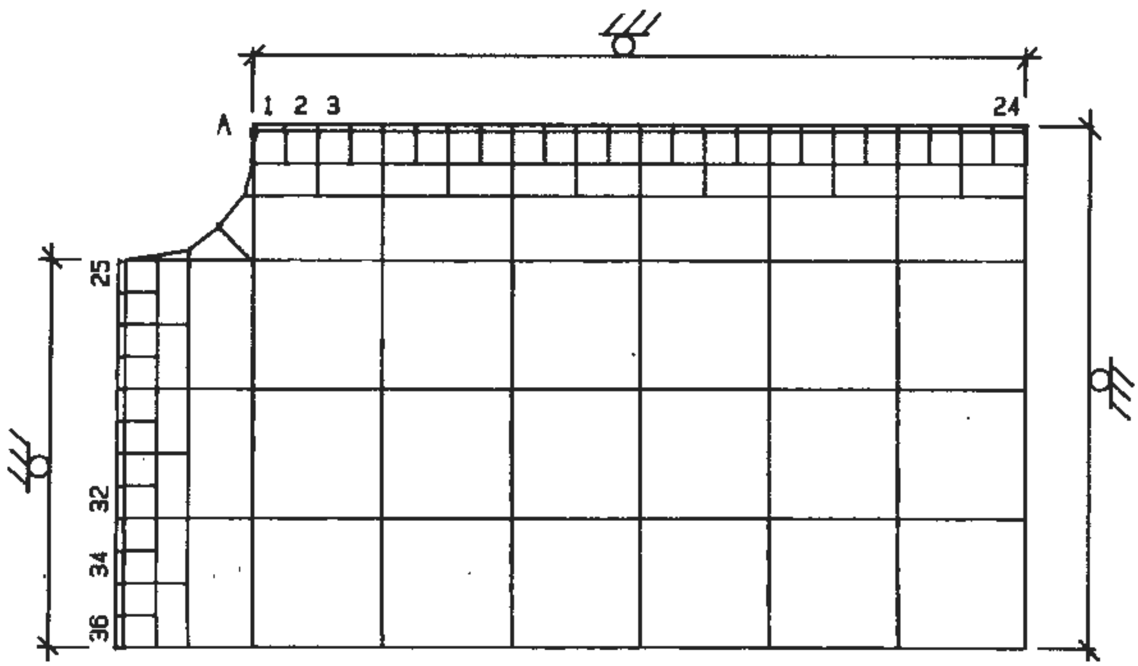


Fig. 5.2b Large bar spacing

Fig. 5.2 Finite element models

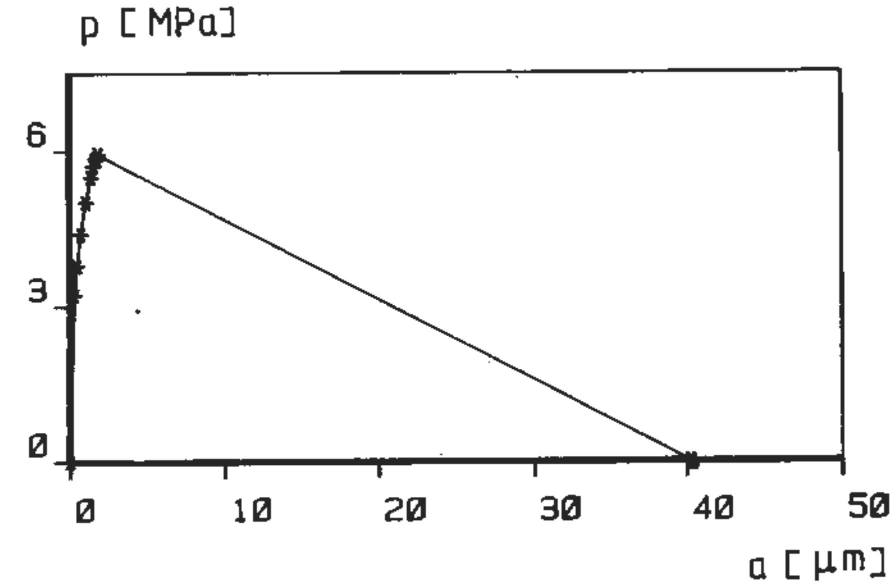


Fig. 5.3a Small bar spacing

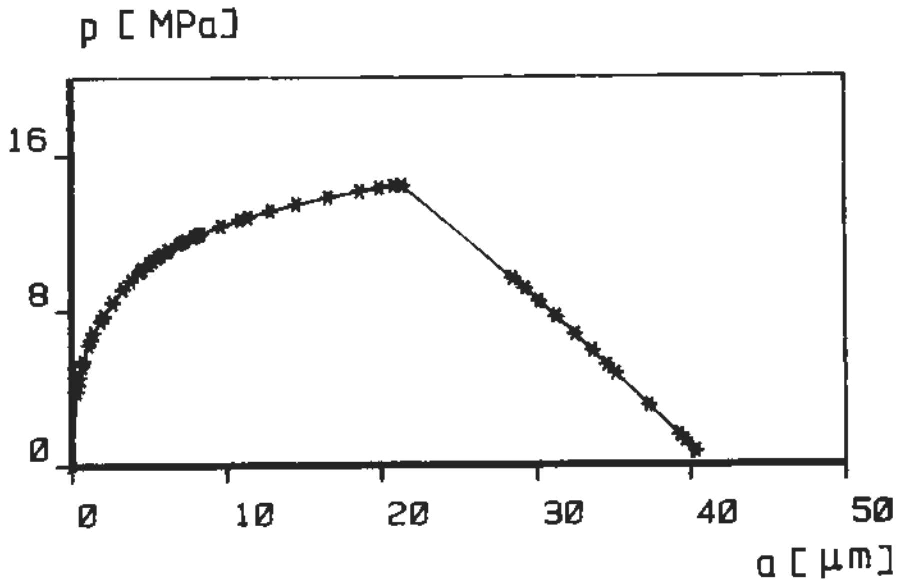


Fig. 5.3b Large bar spacing

Fig. 5.3 Internal pressure, p, in the hole plotted versus vertical displacement, a, in point A. Point A is defined in Fig. 5.2

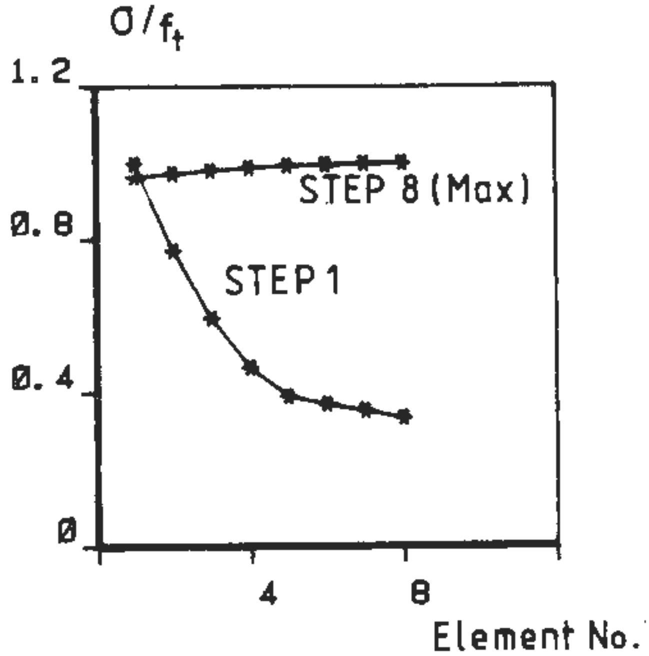


Fig. 5.4a Small bar spacing

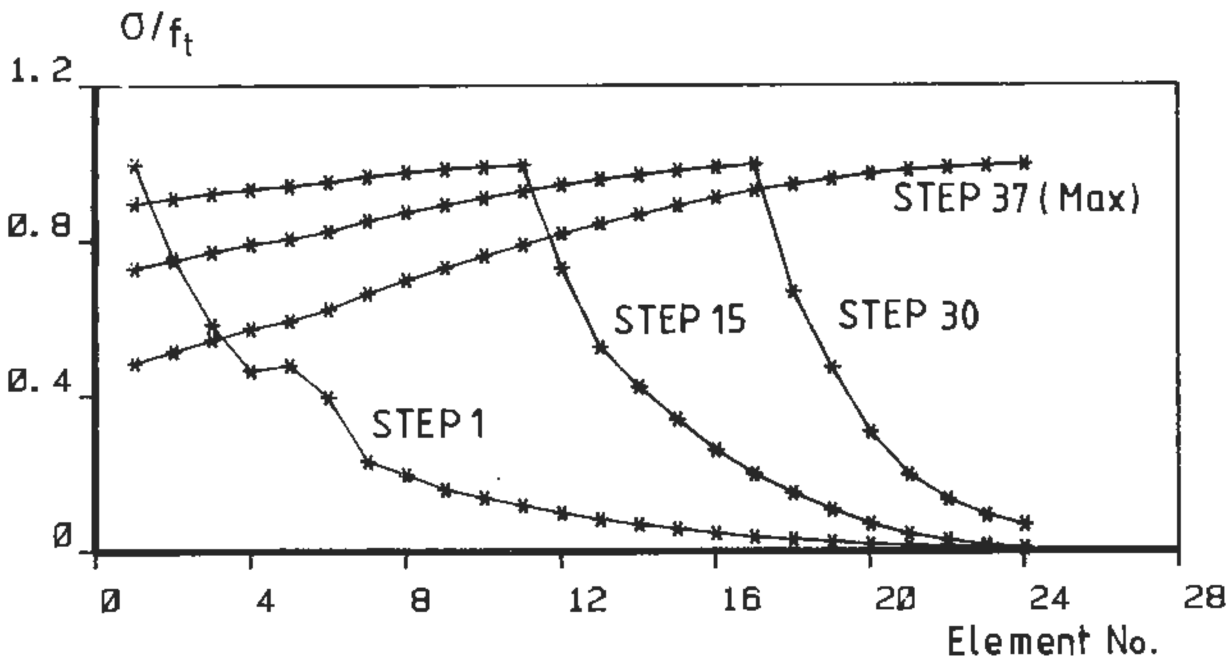


Fig. 5.4b Large bar spacing

Fig. 5.4 Relative stress distribution in horizontal crack line. Element No. refers to crack elements, see Fig. 5.2

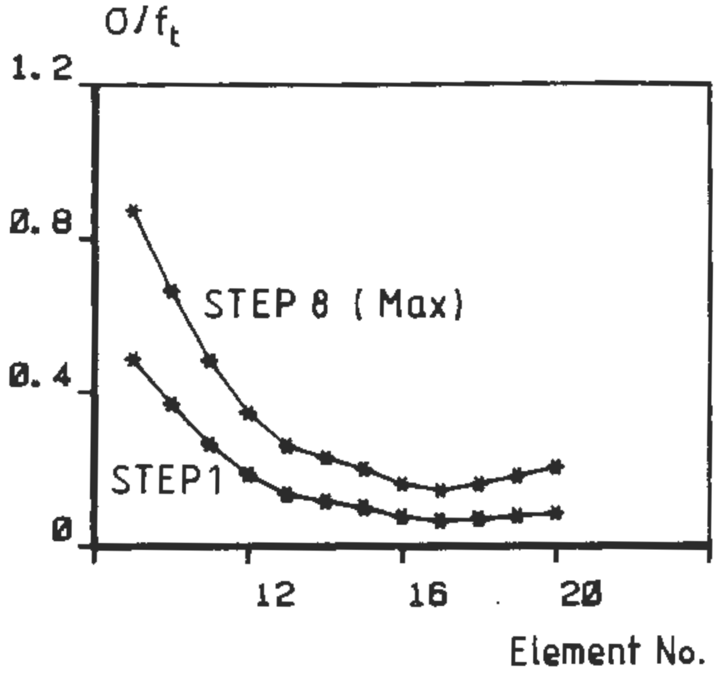


Fig. 5.5a Small bar spacing

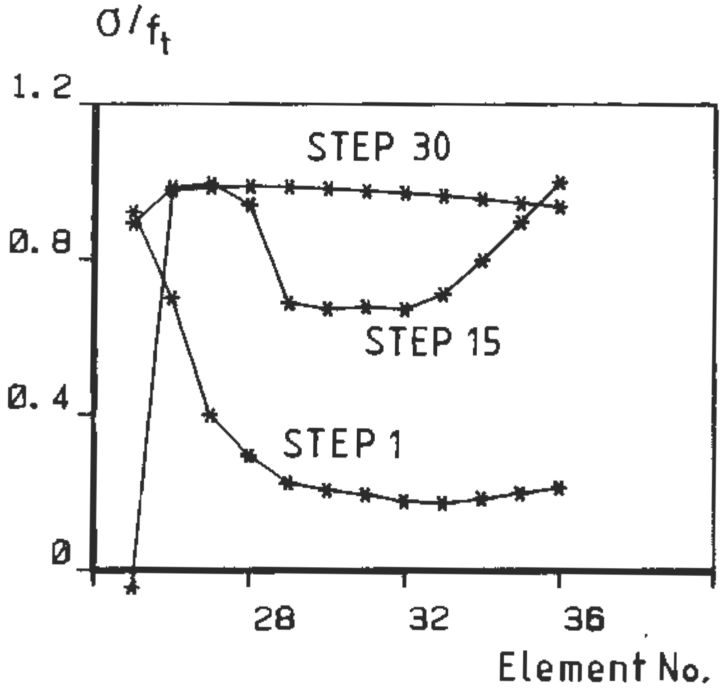


Fig. 5.5b Large bar spacing

Fig. 5.5 Relative stress distribution in the vertical crack line. Element No. refers to crack elements, see Fig. 5.2

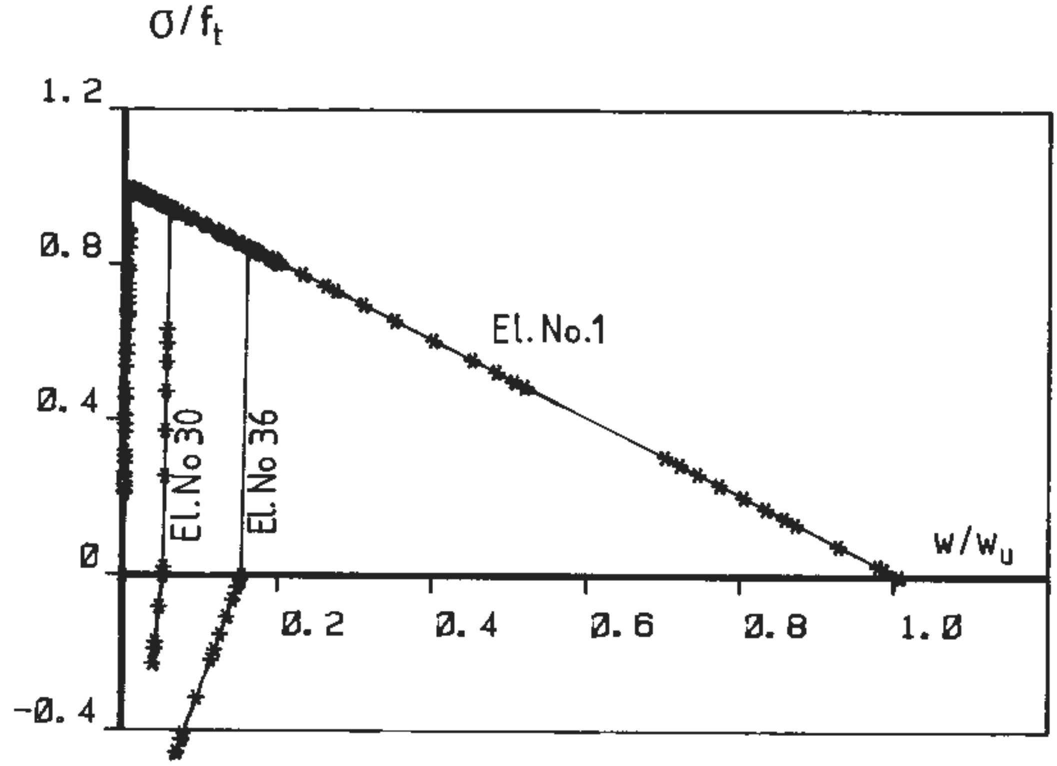


Fig. 5.6 Relative σ-w curve from three different crack elements in the "large bar spacing" geometry

5.3 Comments

As expected the large spacing gives a softer failure and a higher failure load then the small spacing does. Assuming perfect plasticity the failure loads will be 6.0 MPa and 18.0 MPa respectively, compare section 1.2.

Fig. 5.4 shows a difference between the two geometries. The large spacing cause the stress to decrease to about $0.5f_t$ at maximum load. In the small spacing geometry however, perfect plasticity with $\sigma = f_t$ is a good assumption.

In the vertical crack of the small bar spacing geometry the stress is highest near the hole, but the concrete is still uncracked. This is not the case in the large spacing geometry. Here, the fictitious crack starts to grow in the zone close to the hole. However, at approximately step No. 15 the bending of the cover causes a fictitious cracking to start at the surface of the concrete (el No. 36). This crack propagates towards the hole and in the end there are compressive stresses close to the hole.

From Fig. 5.6 the constitutive model according to Fig. 2.7 can be recognized.

The analyse technique presented in this paper has shown to be very easy to handle and it also gives a lot of interesting information about the splitting failure. In the closest future the method will be compared with test results. The sensitivity ot different input parameters such as the initial crack element stiffness etc will also be studied. After this the method will be used to study a lot of different geometries, including transverse reinforcement.

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NOTATIONS

E	modulus of elasticity
E_1	shear stiffness in crack element
E_2	normal stiffness in crack element
F	force
G_F	fracture energy
M	bending moment
N	normal force in crack element
T	shear force in crack element
f	strength
f_{ct}	concrete tensile strength
f_r	radial pressure at splitting failure
f_t	tensile strength
a	displacement
ℓ	length
n	normal displacement in crack element
p	internal pressure
t	time, shear displacement in crack element
w	crack width
w_N	crack width at stress σ_N
w_u	crack width when a real crack starts
$x,y,$	global co-ordinates
x', y'	local co-ordinates
ε	strain
η	local crack element co-ordinate
σ	stress
σ_N	stress in fictitious crack when unloading starts
σ_r	radial compressive stress
σ_t	tensile stress
σ_1	first principal stress

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