DUCTILITY OF CONCRETE AND TENSILE BEHAVIOR

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By combinding theories based on fracture mechanics and continuous damage mechanics a method is presented which appears to give a good basis for describing ductility and tensile behavior of concrete. This opens up the possibility of obtaining an improved input on material characteristics to advanced computer programs for structural design based on finite element analysis.

Keywords: Concrete, ductility, tensile behavior

INTRODUCTION

All cracks which occur in concrete are caused by tensile stresses, and the load-bearing capacity of the material primarily depends on the extent of cracking developing. The tensile behavior of concrete is, therefore, of basic importance for estimating the behavior of concrete under mechanical loading more in general.

By combinding theories based on fracture mechanics [1] and continous damage mechanics [2] a basis has been obtained as shown in the following for describing the ductility and tensile behavior of concrete. Hence, a more fundamental basis for estimating the behavior of concrete under mechanical loading is also obtained.

In the present work efforts have been made to develop a procedure for testing which is sufficiently simple to be used on a routine basis. In the following the theoretical background for such a procedure and some preliminary experiences are briefly presented.

FRACTURE MECHANCIS

Of the more recent works carried out in the field of fracture mechanics applied to concrete, the Fictitious Crack model (FCm) as introduced by Hillerborg, Modéer and Petersson[1] appears to be a most interesting approach. The concept which is based on the models previously developed by Dugdale[6] and Barenblatt[7], describes the microcracked zone ahead of a crack tip as a fictitious crack with a certain ability to transfer stresses. In Fig. 1 the load-bearing capacity of this fictitious crack is related to the crack opening (w), and the area under the σ-w-cruve represents the fracture energy (G_F) of the material:

$$G_F = \int_0^{w_1} \sigma(w)\,dw \qquad (1)$$

where G expresses the energy dissipating per unit of new fracture area forming in the fracture zone, and σ is the tensile stress. A simplified linear estimate of the σ-w-relationship shown in Fig.1 gives:

$$G_F = \frac{1}{2} f_t \cdot w_1 \qquad (2)$$

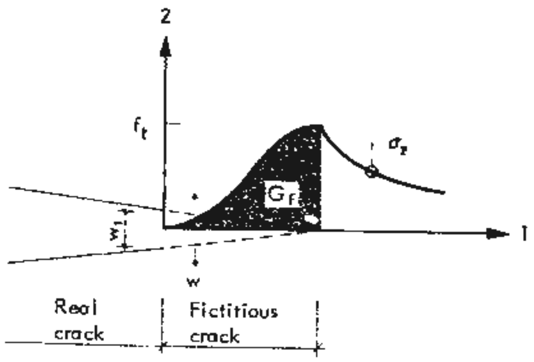

Fig. 1 Fictitious Crack model (FCm) with indication of fracture energy, G_F. w denotes the fictitious crack opening.

The experimental determination of G_F according to FCm can for example be based on a three-point loaded, notched beam as shown in Fig. 2.

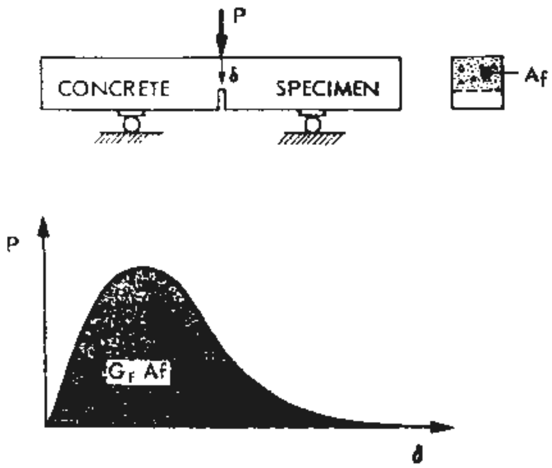

Fig. 2 Experimental determination of fracture energy, G_F, according to FCm[3].

If the depth of the notch is sufficiently big relative to the height of the beam, and the width of the notch is adjusted to the maximum aggregate size of the concrete, most of the fracture energy will be locally concentrated to the notched cross-section. If also the length of the beam is sufficiently big, and the stiffness of the testing machine sufficiently high, then it is possible to obtain a stable determination of the load-deflection curve for calculation of the fracture energy.

CONTINUOUS DAMAGE MECHANICS

Parallel to the concept of fracture mechanics another interesting concept has also been developed over resent years in the form of continuous damage mechanics[4,5]. This concept considers that a material may contain a number of defects in the form of voids and microcracks, and that it is only the mechanically intact material in between which is able to transfer stresses.

Since concrete is a very inhomogeneous material containing numerous voids and cracks, a continuous damage model (CDm)[2] was developed on the basis of continuous damage mechanics. The model estimates the load response (e.g. stress-strain relationship) of concrete.

In this model the damage (ω) includes all kinds of defects in the concrete such as voids and cracks. The damage is further defined as the relative portion of the nominal cross-sectional area which is not mechanically intact:

$$\omega = \frac{A - A_n}{A} \qquad (3)$$

where A = total cross section

A_n = net load-bearing cross section

The relationship between the tensile net stress (s) - which is transferred through the net cross-sectional area - and strain is defined as shown in Fig. 3. In the same figure the assumed relationship between damage and strain is also demonstrated. The initial value of damage (ω_i) includes all original defects such as voids and possible shrinkage cracks. If the concrete has been preloaded, ω_i may also include some permanent damage due to this loading. All damage is irreversible.

For a continuously increasing deformation of a concrete specimen more and more inhomogenities and defects will expand. Hence, the amount of damage will increase exponentially (Fig. 3). At a certain strain level, the strain capacity (ε_c) of the material is exceeded in such a way that a fracture zone occur. Hence, the strain capacity also reflects the strain level where the nominal stress (σ) reaches its maximum value, the uniaxial tensile strength (f_t). Fig. 3 shows some of the most important principles on which CDm is based.

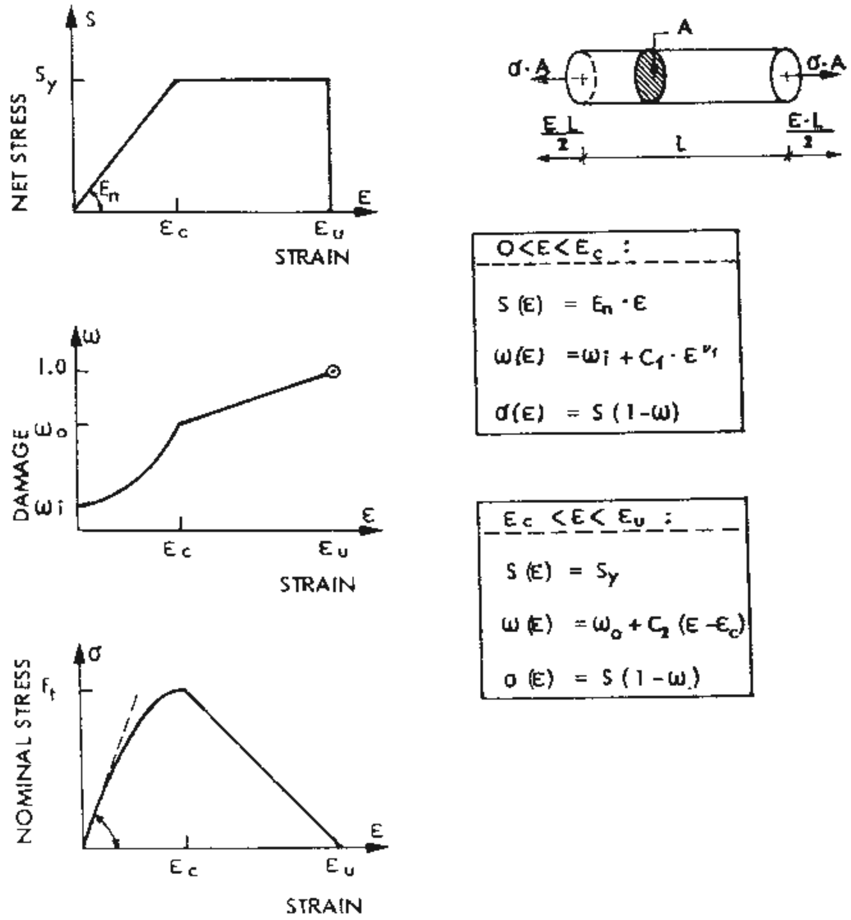

Fig. 3 The most important principles on which the Continuous Damage model (CDm) is based.

In a concrete specimen being exposed to a uniaxial tensile stress the growth of damage in the strain range $0 < \varepsilon \leq \varepsilon_c$ will be evenly distributed throughout the volume of the specimen. When the strain capacity (ε_c) is exceeded, however, the further growth of damage will only take place locally in the fracture zone. Hence, the deformation of this zone will be the total amount of both the external deformation and the contraction taking place in the rest of the material which becomes unloaded. This is demonstrated in Fig. 4 where the stress-increasing parts of the various stress-strain diagrams are identical.

The initial modulus of elasticity (E_i) can be expressed as:

$$E_i = E_n(1 - \omega_i) \qquad (4)$$

where E_n is a fictitious net modulus, namely the modulus corresponding to a damage $\omega = 0$. The value of E_i is easily determined from the stress-strain curve.

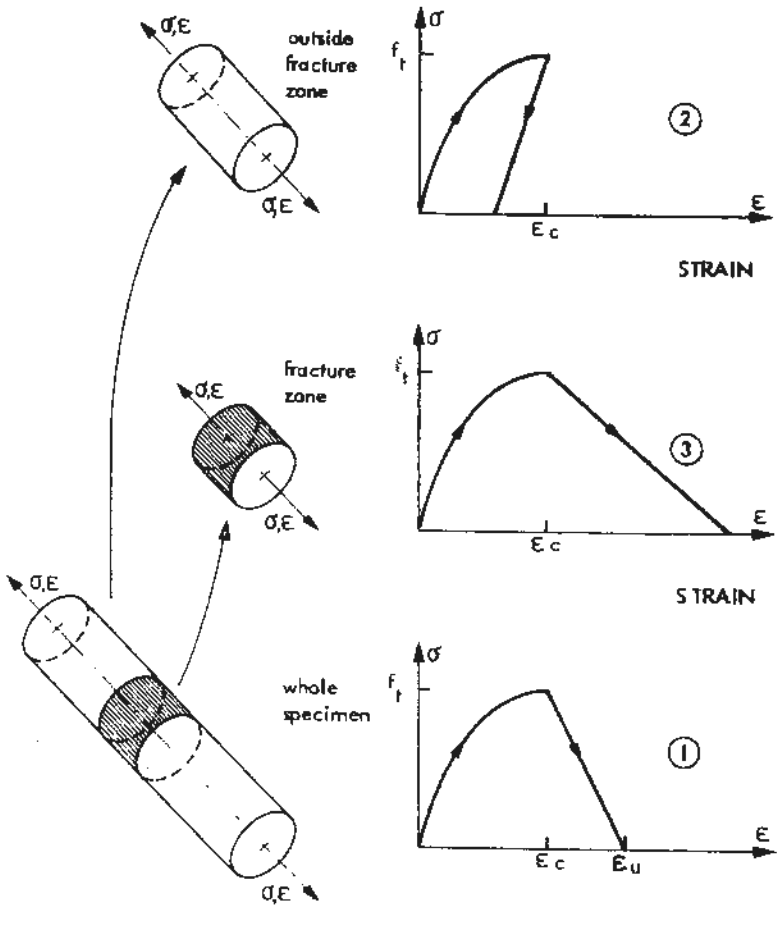

Fig. 4 Stress-strain diagrams for the various parts of a specimen subjected to tensile loading.

For the material outside the fracture zone (zone (2) in Fig. 4) the slope of the unloading part of the curve (E_i') can approximately be considered equal to $-E_i$. This means that the deviation of the stress-strain curve from a linear behavior further can be considered as plastic deformation. Another possibility would be to consider the material as pure elastic. In that case the slope of the unloading part would be:

$$E_i' = -E_n(1-\omega_0) = -E_i\left\{\frac{1-\omega_0}{1-\omega_i}\right\} \qquad (5)$$

Preliminary results indicate that the best estimation of the unloading curve - estimated by a straight line - is obtained if the slope of this line is somewhere in the range $-E_1$ to $-E_1'$.

The ultimate deformation (ε_u) of the total spesimen (case (1) in Fig. 4) is composed of the maximum deformation in the fracture zone and the plastic deformation of the material outside the fracture zone.

COMBINATION OF FCm AND CDm

For tensile loading as shown in Fig. 5 the deformation of the fracture zone corresponding to a growing damage from $\omega = \omega_0$ to $\omega = 1,0$ can be expressed as:

$$\Delta L_1 = \frac{2G_F(1-\omega_0)}{f_t} \quad (6)$$

This corresponds to the following strain value:

$$\varepsilon_1 = \frac{\Delta L_1}{L} = \frac{2G_F(1-\omega_0)}{f_t\,L} \quad (7)$$

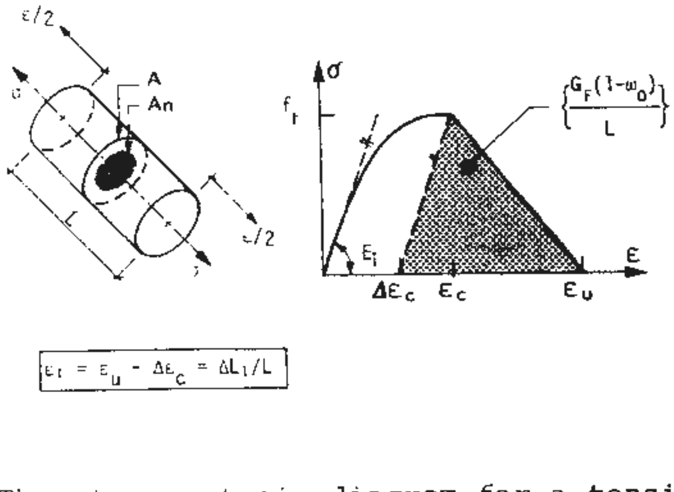

Fig. 5 The stress-strain diagram for a tensile-loaded specimen related to the fracture energy, G_F.

As previously discussed the ultimate deformation (ε_u) of the total specimen is composed of the maximum deformation in the fracture zone (ε_1) and the plastic deformation of the material outside the fracture zone. According to Fig. 5, ε_u can therefore be expressed as:

$$\varepsilon_u = \frac{2G_F(1-\omega_0)}{f_t\ L} + \varepsilon_c - \frac{f_t}{E_i} \qquad (8)$$

Hence, the information necessary for calculating ε_u is based on the fracture energy (G_F) and the increasing part of the stress-strain diagram in uniaxial tension. G_F can be determined as previously described in Fig. 2, while f_t, E_i and ε_c can be determined on each of the two halves of the same specimen tested as shown in Fig. 6. In this testing the load-deformation diagram is obtained by use of inductive displacement transducers.

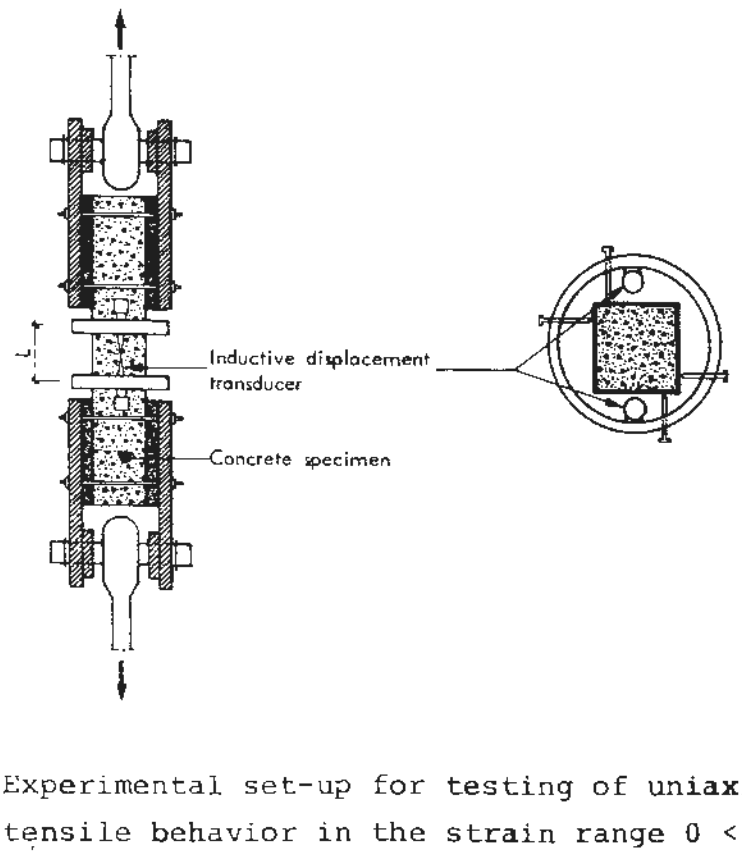

Fig. 6 Experimental set-up for testing of uniaxial tensile behavior in the strain range $0 < \varepsilon < \varepsilon_c$.

Preliminary experiences indicate that determination of ε_u by use of eq. (8) gives basic information on the ductility of the material. This equation also gives sufficient information for the CDm to describe the stress-strain relationship in the strain range $\varepsilon_c < \varepsilon < \varepsilon_u$. Hence, a basis for describing the complete tensile behavior of concrete has been obtained, which can be used as an input to advanced computer programs for structural design based on finite element analysis.

It should be noted that the ductility of concrete as described by eq. (8) depends on the dimensional sizes of the concrete volume. Therefore, the ductility, ε_u, must also be defined for a certain length of specimen. In the following a length, L, of 100 mm has been chosen and the corresponding ductility has been denoted $\varepsilon_{u_{100}}$.

RESULTS AND DISCUSSION

The results from a test series including nine different concrete mixes are shown in Figs. 7-10. The composition of the various mixes is shown in Table 1. All testing was carried

Table 1 Data on Mix Design and Tensile Strength.

Mix No.	Mix Proportions by Weight				Tensile Strength (MPa)
	Cement	Sand	Gravel	Water	
1	1	4.68	3.12	0.89	2.2
2	1	4.68	3.12	0.80	2.4
3	1	4.68	3.12	0.77	2.0
4	1	4.36	3.09	0.77	2.6
5	1	2.70	1.80	0.52	3.4
6	1	2.70	1.80	0.39	4.2
7	1	2.04	1.36	0.43	3.9
8	1	2.04	1.36	0.39	4.4
9	1	2.04	1.36	0.37	4.4

out on 10x10x150 cm specimens after three months of wet curing at 20°C. During testing all specimens were also kept wet in order to avoid any effect of drying shrinkage. The testing included determination of

- Compressive strength
- Fracture energy (Fig. 2)
- Tensile behavior in the strain range $0 < \varepsilon < \varepsilon_c$ (Fig. 6)

As demonstrated in Fig. 7 the relationship between fracture energy (G_F) and tensile strength (f_t) is not very clear. The data indicate, however, a certain trend towards increasing fracture energy for increasing strength, but the scatter is very big. It may be difficult, therefore, to use G_F for describing the ductility of the material. If G_F is used as an input to eq. (8), however, the well known relationship between tensile strength and ductility appears as demonstrated in Fig. 8.

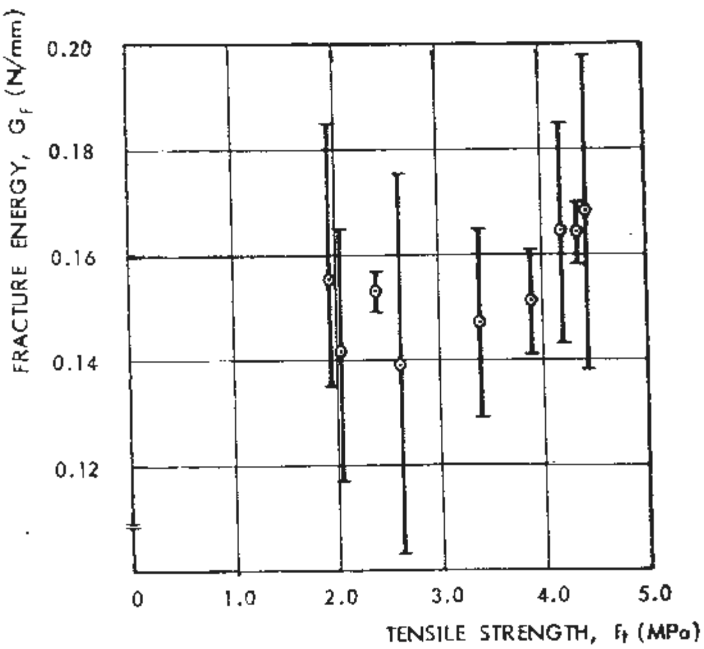

Fig. 7 Relationship between fracture energy, G_F, and tensile strength, f_t. 90% range of confidence is indicated.

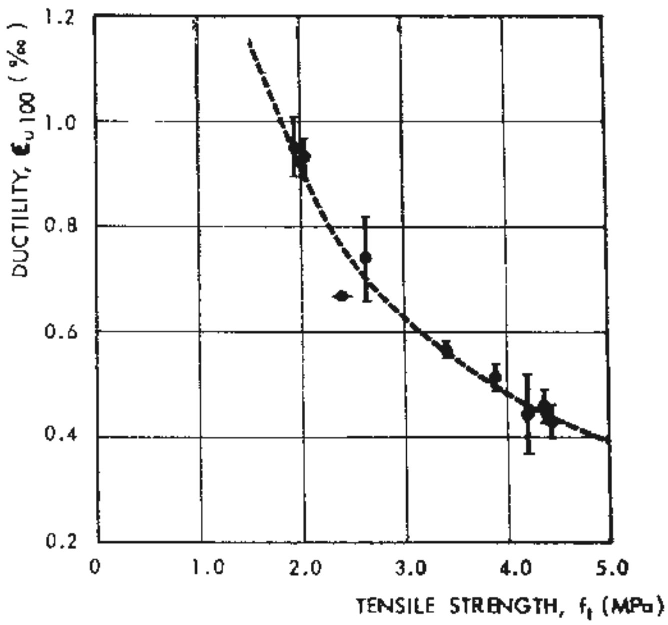

Fig. 8 Relationship between ductility, ε_u, and tensile strength, f_t. 90% range of confidence and exponential regression curve (r^2 = 0,96) is given.

As previously discussed a combination of eq. (8) and CDm can also be used for describing the stress-strain diagram of the material. As can be seen from Figs. 9 and 10 a good agreement between the experimental and the theoretically determined increasing part of the σ-ε-curves is obtained. Hence, it appears that CDm provides a good basis for describing concrete behavior subjected to uniaxial tensile loading.

In Figs. 9 and 10 the importance of the dimensional sizes on the descending part of the σ-ε-diagram is clearly demonstrated. Based on a certain length it is possible, however, to predict the descending part for other lengths of the specimen. This can be done provided that the simplified linear description is used (Figs. 9 and 10).

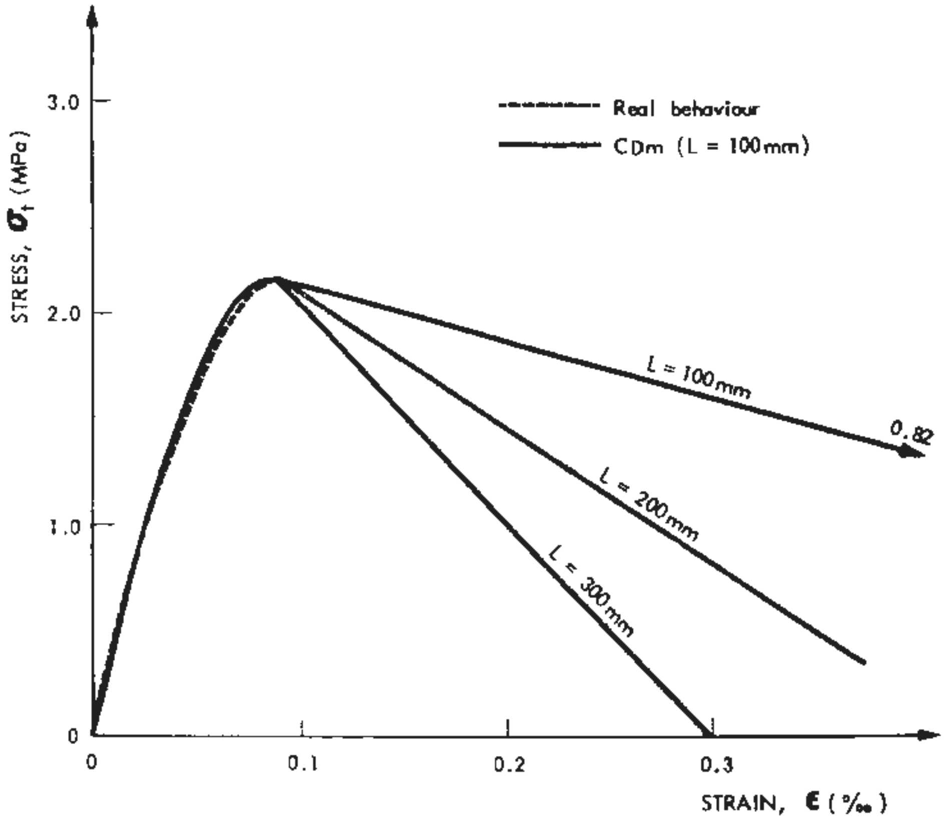

Fig. 9 Experimental and theoretically determined relationship between stress and strain for concrete mix no. 1.

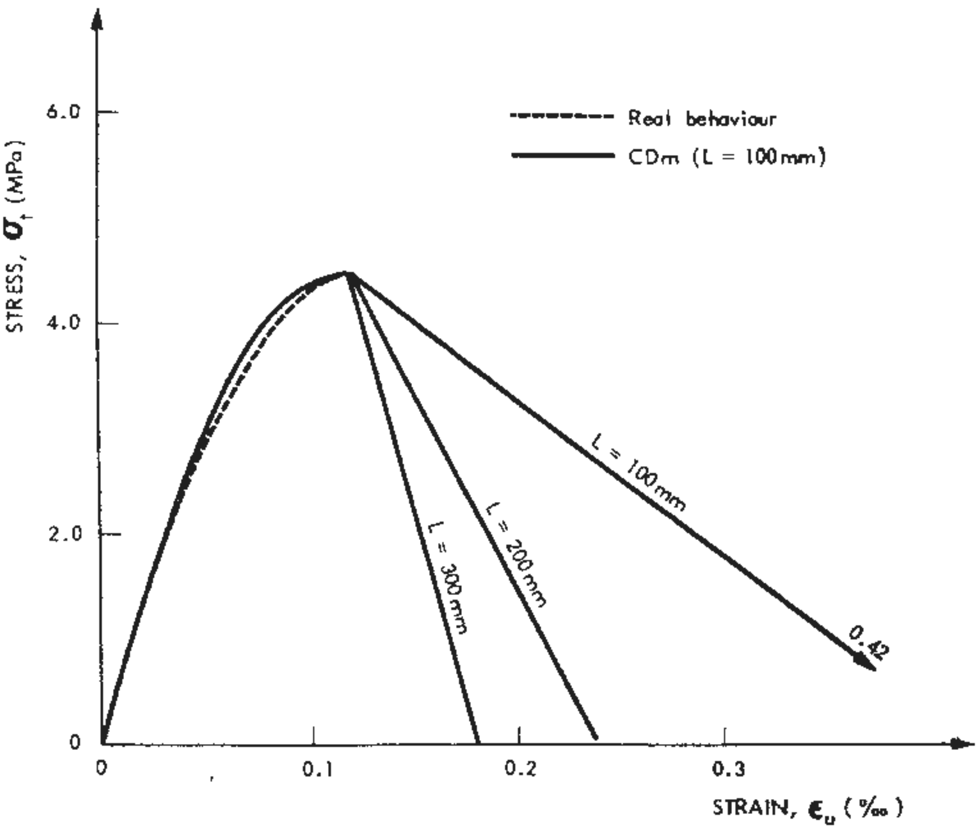

Fig. 10 Experimental and theoretically determined relationship between stress and strain for concrete mix no. 8.

CONCLUSIONS

The proposed method of combinding fracture mechanics and continuous damage mechanics appears to give a good basis for describing ductility and tensile behavior of concrete. This opens up the possibility of obtaining an improved input on material characteristics to advanced computer programs for structural design based on finite element analysis.

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