EXPERIMENTAL STUDY CONCERNING REINFORCED CONCRETE BEAMS UNDER THERMAL AND MECHANICAL LOADS

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ABSTRACT

In order to verify the results of theoretical research, an experimental study on reinforced concrete beams, 13 in all, was carried out. A bending restraint force (resultant) or combined restraint and load forces (resultant) were applied to nine beams and also a restraint normal force, in addition to bending, to four beams. The restraint forces were produced by heating the upper or lower sides of the beams. The test beams were reinforced and designed in different ways. Loading arrangements varied with different beams. The most important measurements in a service state were connected with cracking and in the ultimate state with the determination of rotation capacity.

1. INTRODUCTION

In an article published in the Nordic Concrete Research Publication No. 3, 1984/1/ the behaviour and design of reinforced concrete structures under thermal gradients were discussed. When a pure normal thermal stress is acting, there is then always in design a question of crack limitation. If an external mechanical load is also acting simultaneously, the ultimate limit state design must also be carried out. Further, the effect of a pure thermal force shows that the propagation phase of cracks mostly corresponds to the formation of the first cracks. In this case the crack spacing is indefinite, and the width of cracks cannot be estimated by traditional means, which call for the stabilised cracking phase, i.e. the crack spacing is constant and in average the smallest possible. In the case where an external load acts simultaneously the crack propagation phase, depending on the size of the external load, may be anything between the initiation of the first cracking phase and the stabilised cracking phase. In publication /2/, the cracking phases are defined as follows:

- the first cracking phase, in which case $s_r \geq 2\ l_b$
- the crack propagation phase, in which case $s_r < 2\ l_b$
- the stabilised cracking phase, in which case $s_r = s_{rm} =$ constant,

where s_r is average crack spacing in crack propagation phase

s_{rm} average crack spacing in stabilised cracking phase

l_b anchorage length.

The development of cracks in the first cracking phase and in the crack propagation phase takes place stochastically due to the nature of the tensile strength of concrete. On certain assumptions concerning the tensile strength, the crack spacing before stabilisation can be expressed as a value varying with stress /2/. Thus the crack width may also be expressed as a function of stress, in which case the crack limitation can be made to correspond to the actual state of stress and the actual cracking phase.

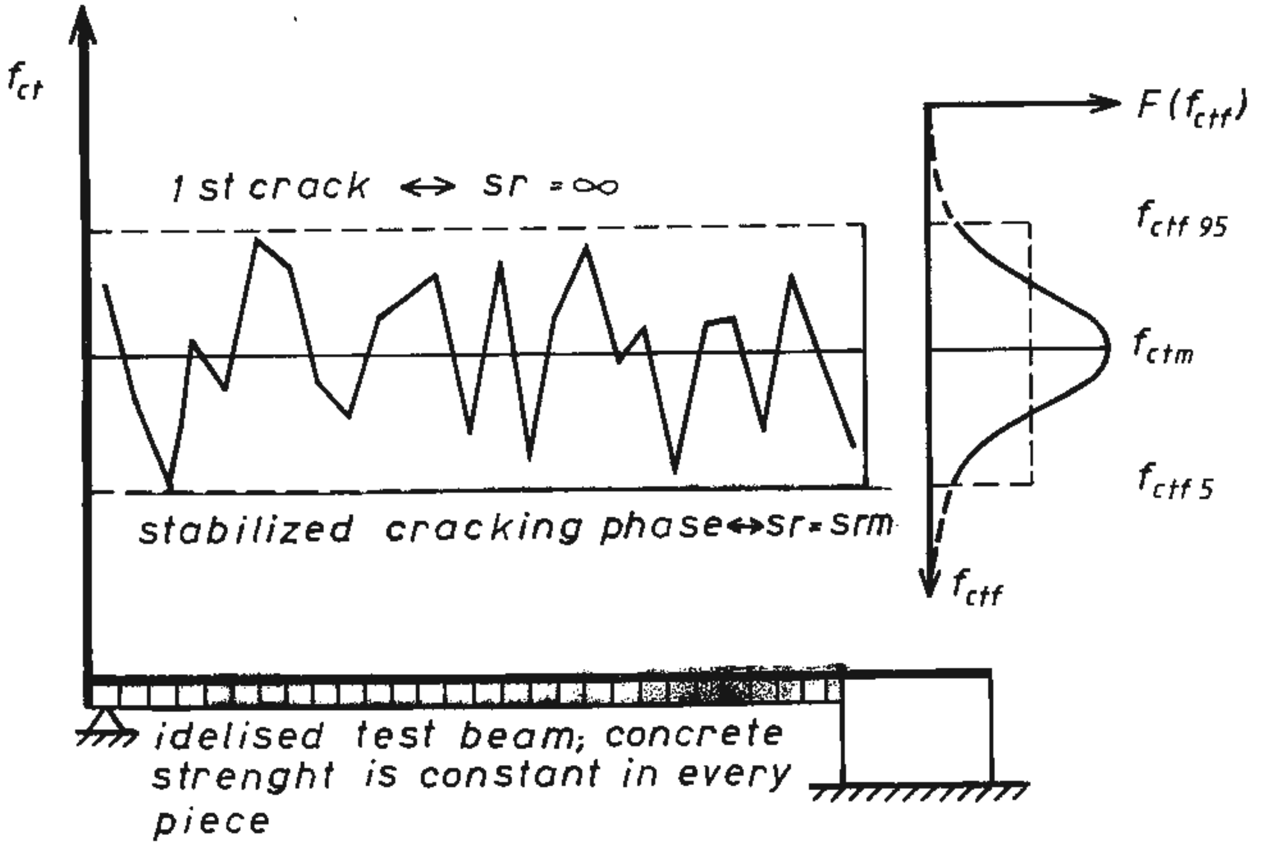


Fig. 1. Stochastic nature of concrete tensile strength and the model of crack propagation phases.

The crack widths corresponding to the above-mentioned cracking phases are determined correspondingly as follows:

- the first cracking phase $w_m = 2\int_{y=o}^{y=l_b} (\varepsilon_{sy} - \varepsilon_{cy})dy$

- the crack propagation phase $w_m = 2\int_{y=o}^{s_{r/2}} (\varepsilon_{sy} - \varepsilon_{cy})dy$
- the stabilised cracking phase $w_m = s_{rm} \cdot \varepsilon_{sm}$

where w_m is mean crack width
ε_{sy} strain in steel
ε_{cy} strain in concrete
ε_{sm} mean steel strain between two cracks.

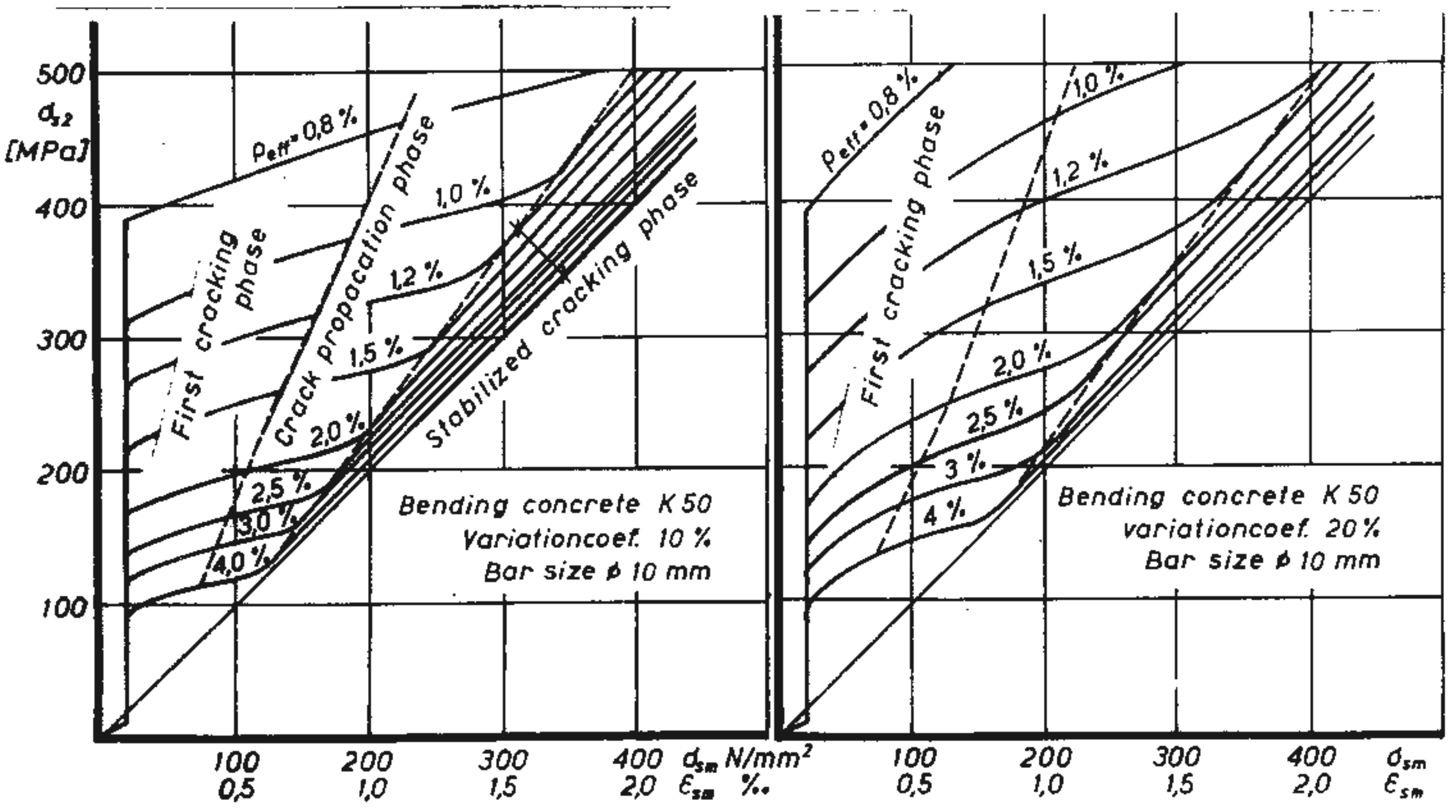


Fig. 2. Calculated values of the mean steel stresses between two cracks and steel stresses at the crack at various stress level.

In the formula for a crack width in the stabilised cracking phase ε_{sm} is an mean strain in steel along the full cracked length of a structure and s_{rm} is the mean crack spacing corresponding to the stabilised phase. Thus in using this formula the crack width in the first cracking phase and crack propagation phase is under-estimated, since the average strain within the anchorage length is greater than ε_{sm} and since the integration is carried out at a shorter distance s_{rm}, instead of 2 l_b. On the other hand, in light reinforced, under restrained forces, in prestressed structures and under normal and flexural force the cracking pattern in service state is not yet stabilised.

As an example of pure bending, it is shown in Fig. 2 that at the level of service stresses (200 - 250 N/mm^2) the stabilised cracking phase exists not before the effective steel area is at least 1.8...2.2 % (variation coefficient of the concrete tensile strength is 10 %) or 2.5...3.0 % (variation coefficient 20 %).

2. PURPOSE OF TESTS

The purpose of the load tests was to study the behaviour of statically-indeterminate reinforced concrete structures of various types upon which the thermal gradient acts alone and the external mechanical load and the thermal gradient in combination. By means of parallel tests it was possible to demonstrate the effects of loading history, i.e. the order of various loading types.

The effect of a normal force was examined by comparing the behaviour of beams subjected to bending stress only to that of beams acted upon by normal force and bending moment.

The effect of a design method was examined in such a way that some beams were designed according to the conventional force resultant distribution of the elastic theory and some beams by using the simple nonlinear method, which takes into consideration the effect of cracking on a structure.

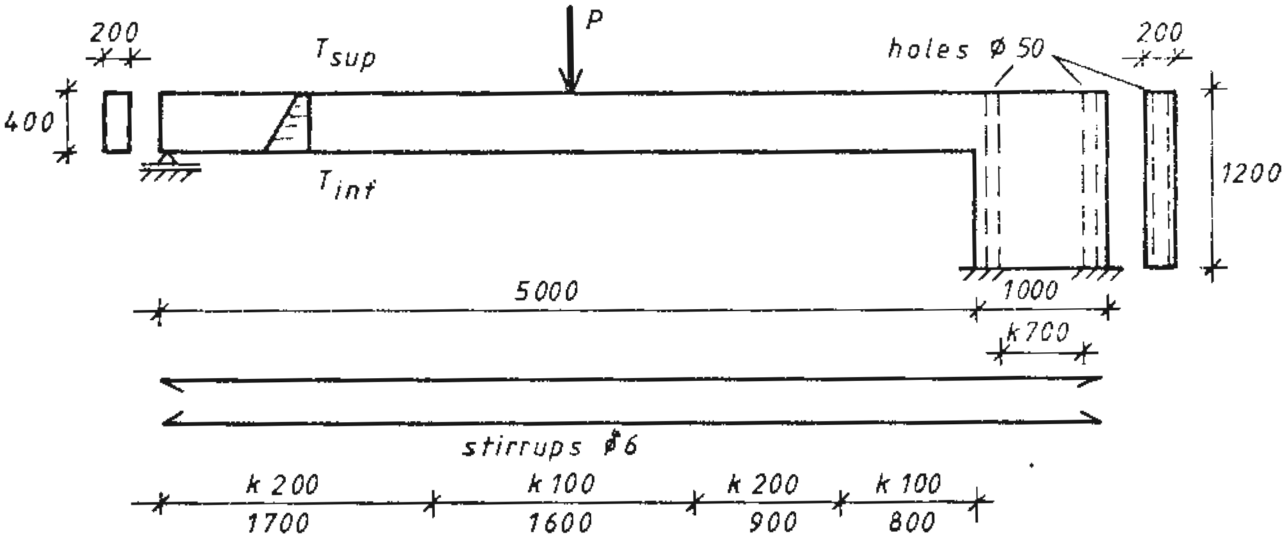
On the other hand, the aim was to verify the results developed in conformity with the theories of crack design. The results based on the assessment methods of stiffness were also compared with the test results. The effect of restraint force resultants on the redistribution of force resultants and on the relaxation of restraint force resultants was investigated.

In the tests, the time of action of the thermal gradient was of relatively short duration, although some relaxation of restraint forces had already clearly occurred as in the tests reported in /3/.

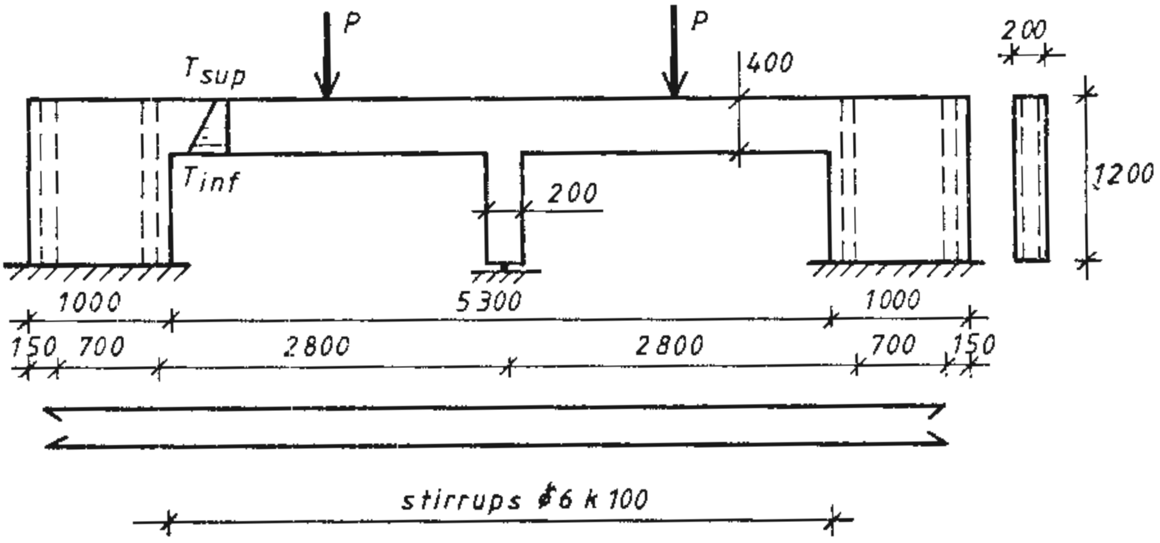
Long-term tests on four beams were started in order to examine the effect of temperature on the relaxation and creep rate as well as to investigate the relaxation of restraint force resultants due to the displacement of supports.

3. TEST SPECIMENS

In the tests two different types of beam in regard to their static system were used; the dimensions and reinforcement of the beams are set out in Fig. 3. The thermal gradient produced in the beams and the position of the external load are also shown in the figure.



BEAMS TYPE 1



BEAMS TYPE 2

Fig. 3. Types of test specimen.

The type of reinforcement and test specimens as well as the design method are given in Table 1.

Table 1. Type of reinforcement, steel areas and the design method of the test beams.

Test specimen	Number	Bottom ρ (%)	Top ρ' (%)	Reinforcement 1)	Design method
1a	3	0.43	0.43	A400H	nonlinear
1b	1	0.43	1.72	A400H	elastic (elastic theory)
1c	3	1.29	1.29	A400H	nonlinear
1d	1	1.29	2.69	A400H	elastic (elastic theory)
1e	1	0.43	0.43	B500K	nonlinear
2a	2	0.43	0.43	A400H	nonlinear
2b	2	1.29	1.29	A400H	nonlinear

1) A400H is hot rolled deformed bar and B500K cold worked deformed bar with corresponding yield strengths of 400 MPa and 500 MPa.

The characteristic strength of the concrete used for making the test specimens was K 50.

4. LOADING ARRANGEMENTS AND TESTS

During loading the direction of heating and the loading order varied as shown in Table 2.

Table 2. Order of loads acting upon the test beams.

Test beam	Load	Loading order and load values						
		1	2	3	4	5	6	7
1a/1 and 1c/1	P	$\to P_k$	$\to 0$	0	$\to P_k$	$\to 0$	0	$\to P_u$
	ΔT	0	0	$\to 80$ °C	80 °C	80 °C	$\to 0$	0
1a/2,1a/3 1b,1c/2,1c/3, 1d,1e,2a/1, 2a/2,2b/1 and 2b/2	P	0	0	$\to P_k$	$\to 0$	0	$\to P_u$	-
	ΔT	$\to 80$ °C	$\to 0$	0	0	$\to 80$ °C	80 °C	-

In the case of beams 1a/3 and 1c/3 the upper sides of the beams were heated. The heating of other beams was carried out from the lower sides of the beams. ΔT shown in Table 2 is the temperature difference between the upper and lower sides of beam. P refers to the external load and its value P_k refers to the service load of the beam, which corresponded to the steel stress of 200 N/mm^2 with the beams in question. P_u refers to the ultimate load of the beam.

During the tests the following measurements were made:

- widths of cracks
- crack spacings
- displacements
  - deflections
  - rotations
- deformations on the upper and lower sides of the beam
  - strain in steel
  - compressive strain and strain in concrete
- support reactions and
- temperature in different parts of the beam.

5. RESULTS

5.1 Materials

The values given in Table 3 proved to be the average material strengths of the test beams on the basis of samples taken.

Table 3. Material strengths of the test beams.

Beam	Reinforcement N/mm^2 field (bottom) f_y	Reinforcement N/mm^2 field (bottom) f_u	Reinforcement N/mm^2 support (top) f_y	Reinforcement N/mm^2 support (top) f_u	Concrete cube strength MN/m^2
1a	458	690	458	690	52
1b	458	690	434	630	53
1c	434	630	434	630	51
1d	434	630	505	735	50
1e	660	725	660	725	54
2a	458	690	458	690	52
2b	434	630	434	630	51

5.2 Service state

Fig. 4 shows the temperature change of the beam loaded with a thermal gradient at different heights of the beam during the time the thermal gradient acts as well as the development of deflection in the centre of the beam and that of shear force at the free end.

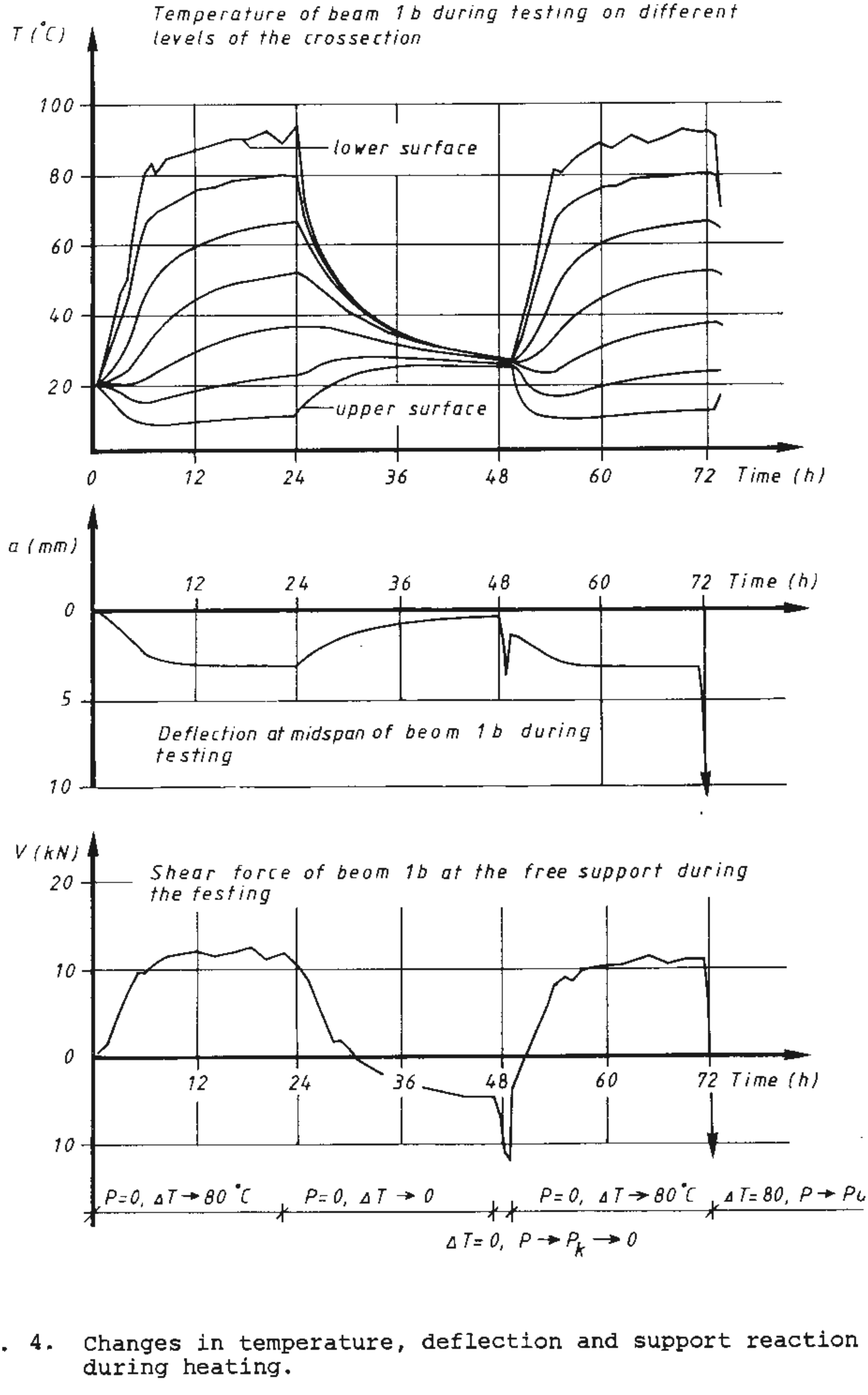


Fig. 4. Changes in temperature, deflection and support reaction during heating.

Measurement results obtained in the service state of the test beams together with calculated values are shown in Table 4.

Table 4. Results calculated and measured in the service state.

Beam No.	Bar size	Calculated values					Measured values		
	mm	ΔT, corresponds to the initiation of 1st crack 1) °C	ΔT, corre-ponds to the initiation of 1st crack 2) °C	width of 1st crack 1) w_m mm	crack width when ΔT = 80 °C 2) w_m mm	crack width when ΔT = 80 °C 3) w_m mm	ΔT, corresponds to the initiation of 1st crack °C	width of 1st crack mm	crack width when ΔT = 80 °C mm
1a/1 4)	10	33.6	15.3	0.09	0.17	0.09	38	0.04	0.12
1a/2	10	33.6	15.3	0.09	0.17	0.09	23	0.02	0.12
1a/3 5)	10	33.6	15.3	0.09	0.17	0.12	39	0.01	0.09
1b	20	30.0	16.5	0.02	0.09	0.03	19	0.02	0.04
1c/1 4)	20	30.0	15.7	0.03	0.12	0.05	42	0.01	0.02
1c/2	20	30.0	15.7	0.03	0.12	0.12	25	0.02	0.06
1c/3 5)	20	30.0	15.7	0.03	0.12	0.06	41	0.01	0.07
1d	25	29.7	21.0	0.01	0.08	0.03	22	0.02	0.06
1e	10	34.9	15.3	0.09	0.17	0.10	18	0.02	0.08
variation		30... 34	15.3... 21.0	0.01... 0.09	0.08... 0.17	0.03... 0.12	18... 42	0.01... 0.04	0.02... 0.12

1) based on real bond properties and first cracking phase /2/
2) based on real bond properties, actual cracking phase, stochastical nature of concrete tensile strength (variation coefficient 20 %) and destroying of bond near the crack /2/
3) calculations made in compliance with CEB Model Code
4) external load acted before thermal load, actions at support to the same direction
5) actions at support in different direction.

The effect of the thermal gradient and that of combined external and thermal loads on the stiffness of beam 1a/2 is shown in Fig. 5. The stiffness distribution of a corresponding beam in the ultimate limit state i also shown in the figure.

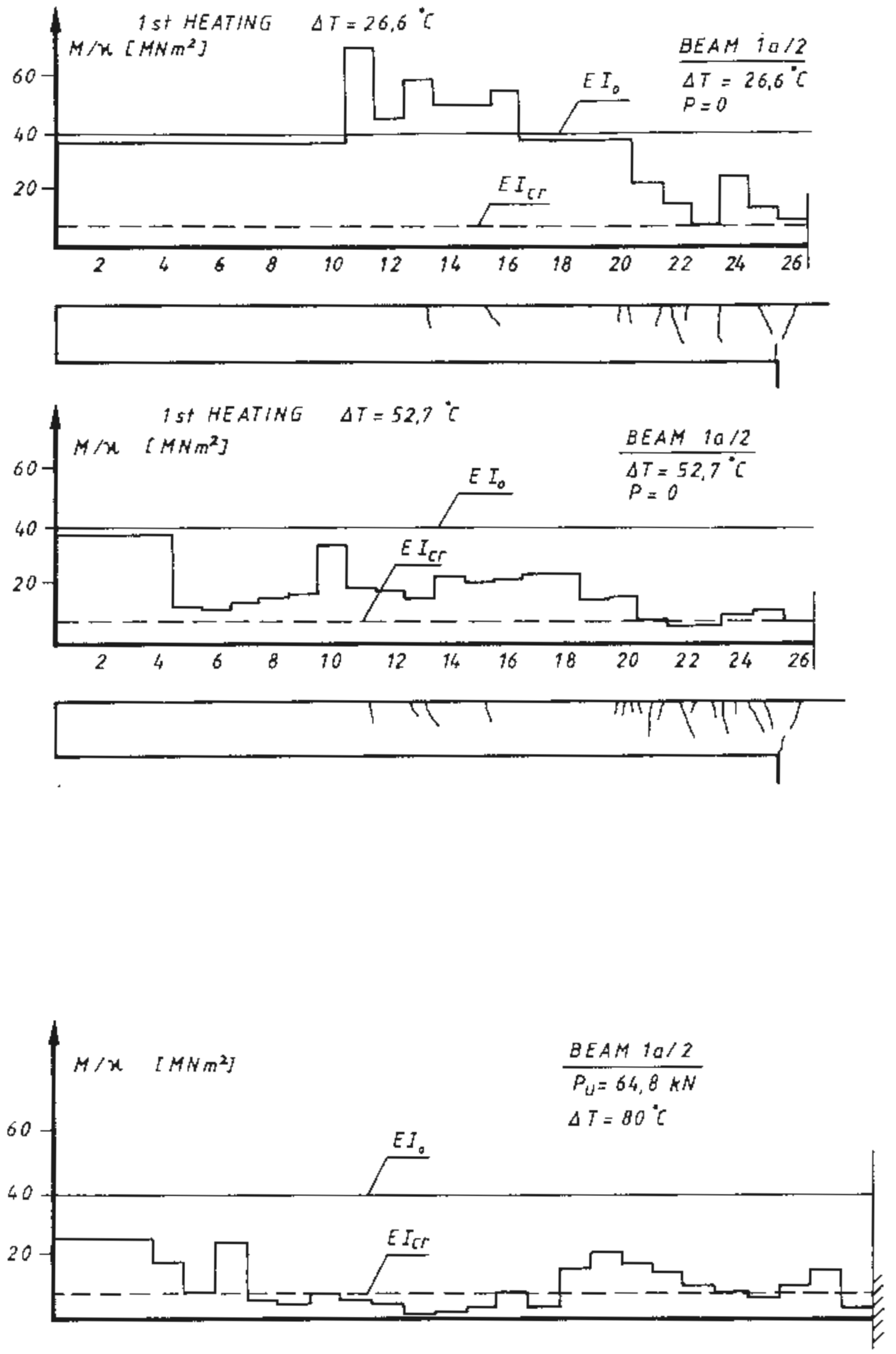


Fig. 5. Stiffness distribution of the beam 1a/2 at service and ultimate state.

5.3 Ultimate limit state

Fig. 6 shows the curvature distributions of beams 1a/1 and 1a/2 in the ultimate limit state. The corresponding results obtained from beam 2a/1 and beam 2a/2 are presented in Fig. 7.

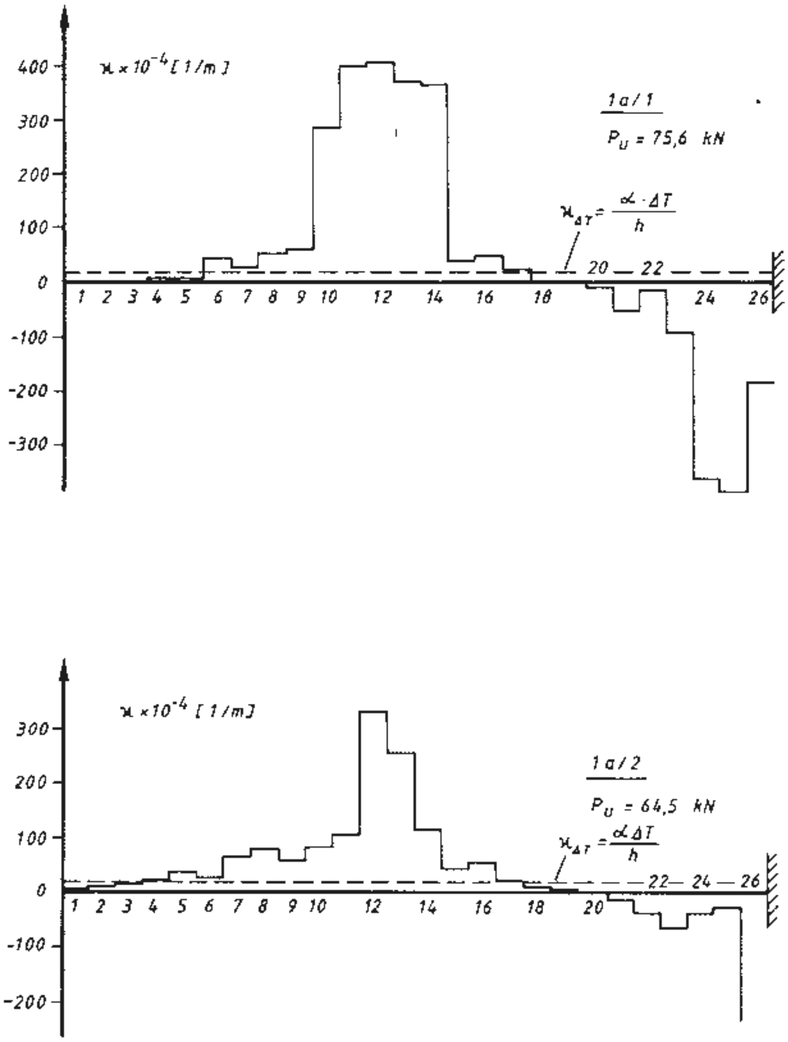


Fig. 6. Curvature distribution of the beams 1a/1 and 1a/2 at the ultimate state.

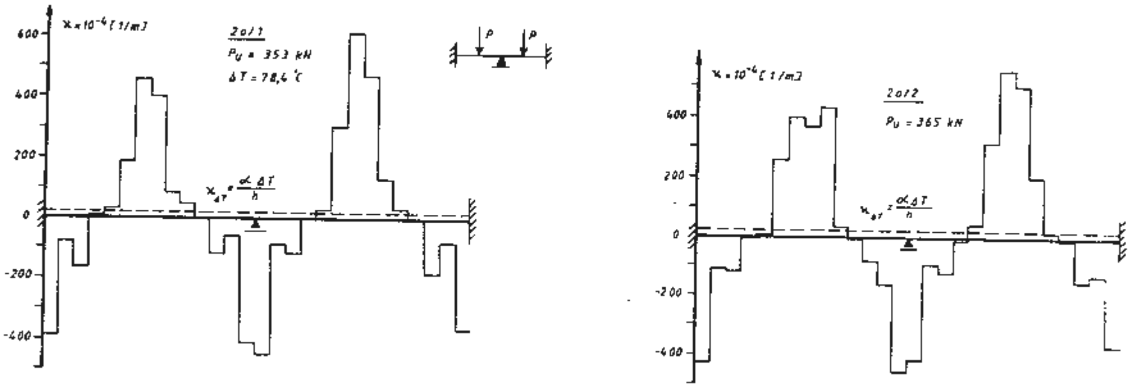


Fig. 7. Curvature distribution of the beam 2a/2 at ultimate state.

Table 5 shows the yield and ultimate loads of the beams obtained by the tests and the corresponding calculated values, which have been determined in accordance with the theories of elasticity and plasticity.

Table 5. Yield and ultimate loads (P_y and P_u) calculated on the basis of the values given in Table 3, and the corresponding test results (kN).

Beam No.	Calculated results						Test results	
	Elastic theory				Plastic theory			
	P and ΔT = 80 °C		only P		P			
	P_y	P_u	P_y	P_u	P_y	P_u	P_y	P_u
1a/1	-	-	54.8	81.4	61.8	91.9	60.0	81.0
1a/2	33.7	60.4	54.8	81.4	61.8	91.9	59.0	82.0
1a/3	53.4	85.5	54.8	81.4	61.8	91.9	60.0	81.0
1b	104.3	136.4	66.1	98.2	113.2	160.2	73.0	145.6
1c/1	-	-	147.7	206.1	166.7	232.6	138.0	193.1
1c/2	94.2	152.6	147.7	206.1	166.7	232.6	150.0	188.2
1c/3	145.9	216.3	147.7	206.1	166.7	232.2	150.0	201.0
1d	223.4	303.7	178.1	248.6	229.5	309.0	-	160.0
1e	57.0	64.4	78.0	85.4	86.4	87.0	94.5	

Table 6 gives the rotation capacities of the beams shown by the tests and determined by different means of calculation.

Table 6. Values and test results of calculated rotation capacities.

Beam No.	Theoretical rotation capacity $\cdot 10^{-3}$ (rad)					Test results
	Baker /7/	Corley /8/	CEB /5/	Plem /4/	Huovinen /6/	$\theta_p \cdot 10^{-3}$ rad
1a/1	14.7	42.0	52.1	42.6	27.2	47.9
1a/2	14.7	42.0	52.1	42.6	27.2	44.2
1a/3	14.7	42.0	52.1	42.6	27.2	35.2
1b	14.8	17.0	12.9	13.1	13.8	38.1
1c/1	12.3	19.3	17.3	14.2	11.4	41.8
1c/2	12.3	19.3	17.3	14.2	11.4	36.2
1c/3	12.3	19.3	17.3	14.2	11.4	30.3
1d	7.9	17.2	8.3	7.6	7.5	28.2
1e	15.1	33.6	41.7	23.5	18.7	-

The main reason for the rather large variation between different methods is in the generally large variation of the plastic deformations in reinforced concrete structures.

6. CONCLUSIONS

From the test results and calculations the following conclusions may be drawn:

1. The temperature difference corresponding to the opening of the first crack varied considerably (18...42 °C). This is mainly due to a great variance in the tensile strength of the concrete and the differences in the stiffness of the supporting.

2. The first cracks occurring in the beams under thermal load varied in width from 0.01 to 0.04 mm while the corresponding calculated values in accordance with the initiation of the 1st crack ranged from 0.01 to 0.09 mm.

3. According to the test results the crack widths, when the temperature difference is 80 °C, ranged from 0.02 to 0.12 mm. The crack widths, calculated in accordance with the cracking theory (CEB) requiring the stabilised cracking phase, would range from 0.03 to 0.12 mm, when the temperature difference is 80 °C. The corresponding crack widths calculated in accordance with the developed theory /2/ would range from 0.08...0.17 mm. These values are larger as the corresponding CEB-code values (see paragraph 1). According to calculations stabilisation had not yet taken place. The crack spacing in the test beams in the area of the largest moment was, however, clearly stabilised.

4. The effect of cracks on the stiffness of the beams is rather great. When the temperature difference was 80 °C, the stiffness of beams 1a and 1e was only 23... 33 % of the calculated uncracked stiffness of the corresponding beams. The corresponding stiffness proportion of beam 1b was 49 %, that of beam 1c 33...75 % and with beam 1d 42 %. The restraint moment at the fixed support was reduced by the same proportion.

5. The rotation capacity of all beams was sufficient for releasing the restraint moment caused by the temperature difference of 80 °C.

NOTATIONS

P	external load
P_y	value corresponding to the service state of external load
P_u	value corresponding to the ultimate state of external load
ΔT	temperature difference
f_c	compressive strength of concrete
f_y	yield strength of reinforcement
f_u	ultimate strength of reinforcement
s_r	crack spacing
y	coordinate
w	crack width

ρ	relative steel area of the lower side
ρ'	" " " of the upper side
l_b	bond transmission length
ε_s	relative strain in reinforcement
ε_c	relative strain in concrete
ε_{sm}	mean steel stress between two cracks
s_{rm}	stabilised crack spacing
w_m	mean crack width
EI_o	stiffness of the non-cracked cross section
EI_{cr}	" of the cracked " "

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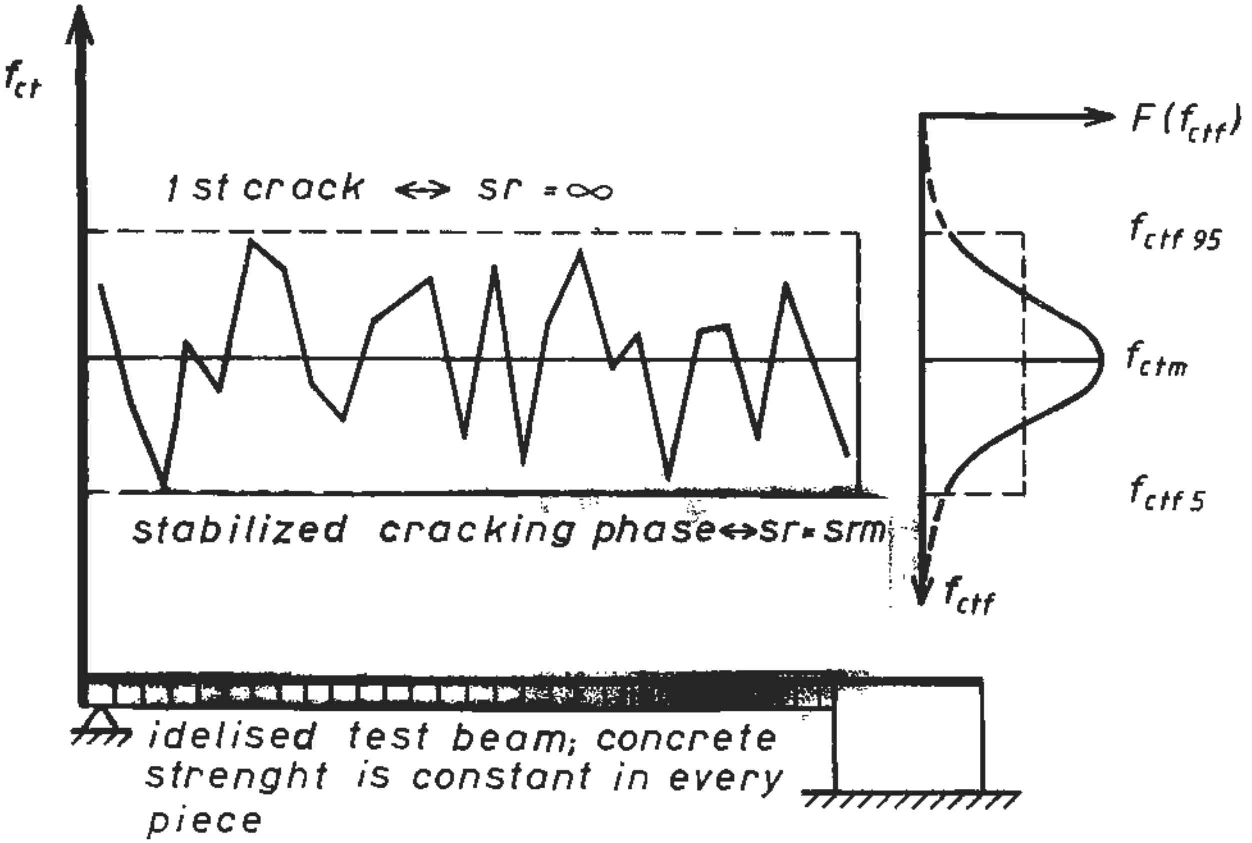
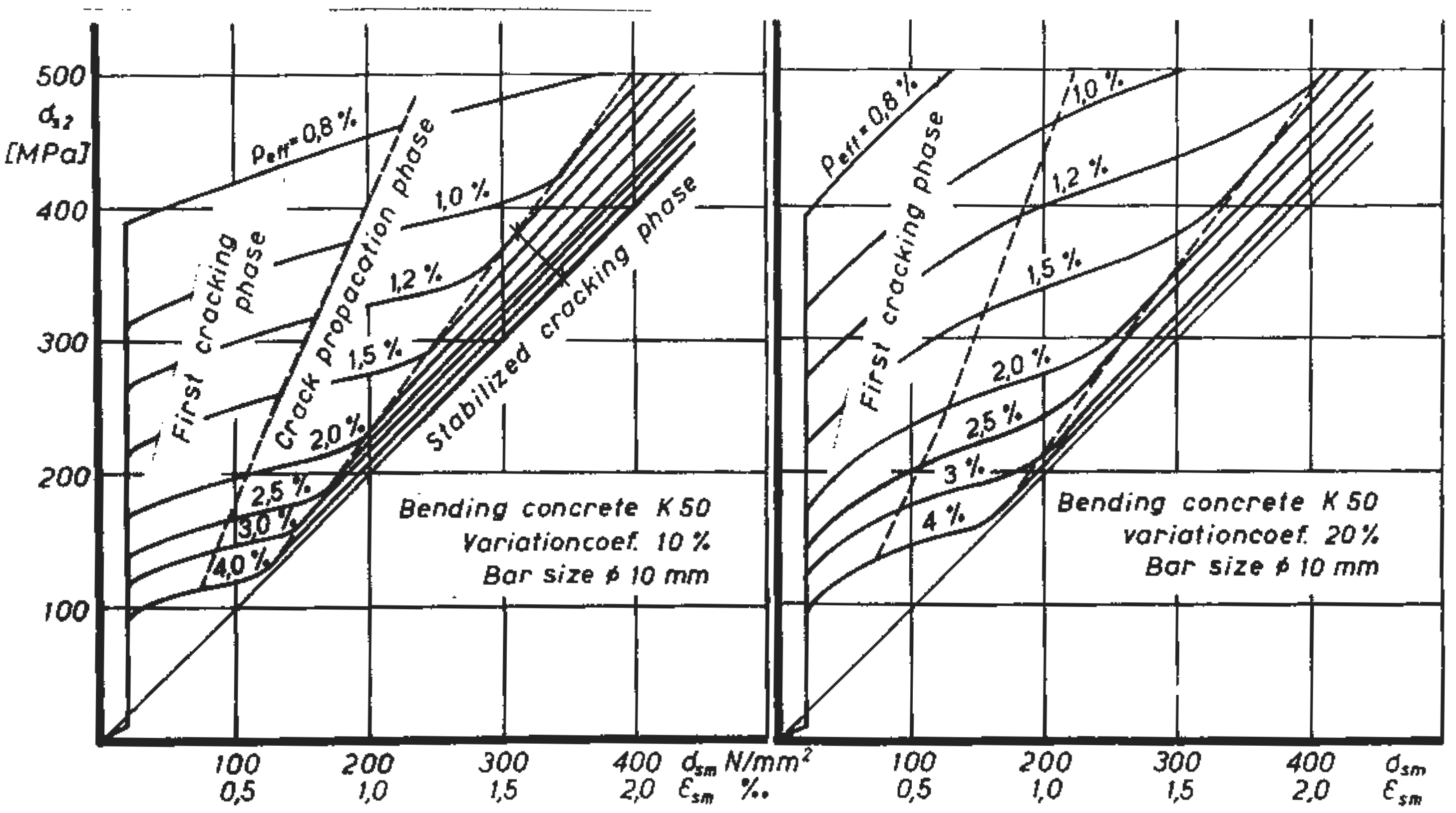

f_{ct}
1 st crack ↔ sr = ∞
$F(f_{ctf})$
$f_{ctf\,95}$
f_{ctm}
$f_{ctf\,5}$
stabilized cracking phase ↔ sr = srm
f_{ctf}
idelised test beam; concrete strenght is constant in every piece


500
400
300
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σs2
[MPa]
ρeff=0,8 %
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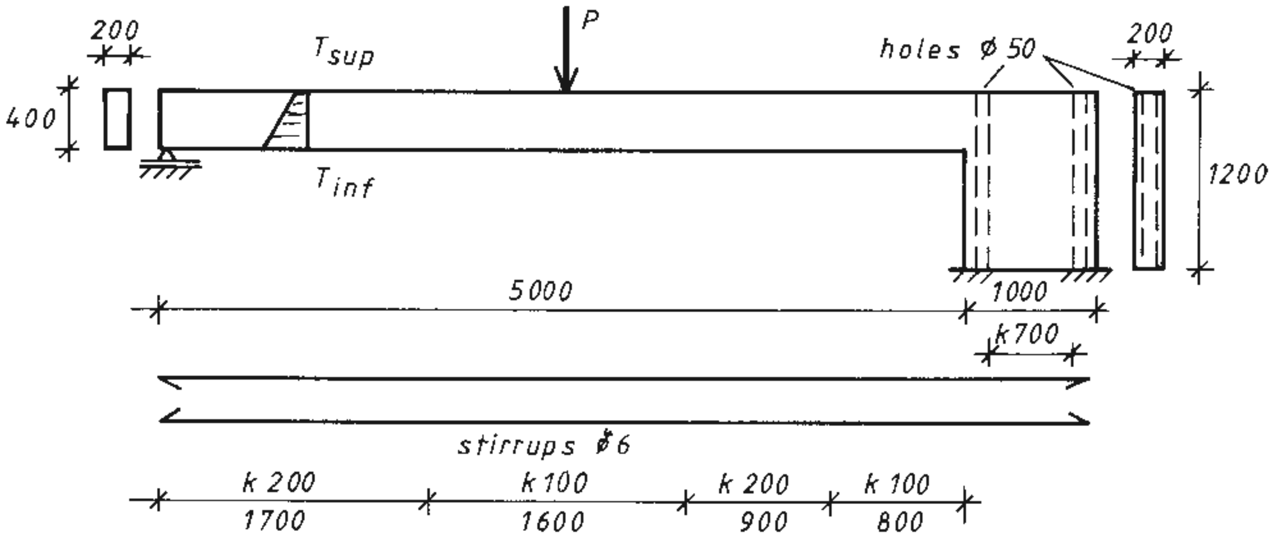
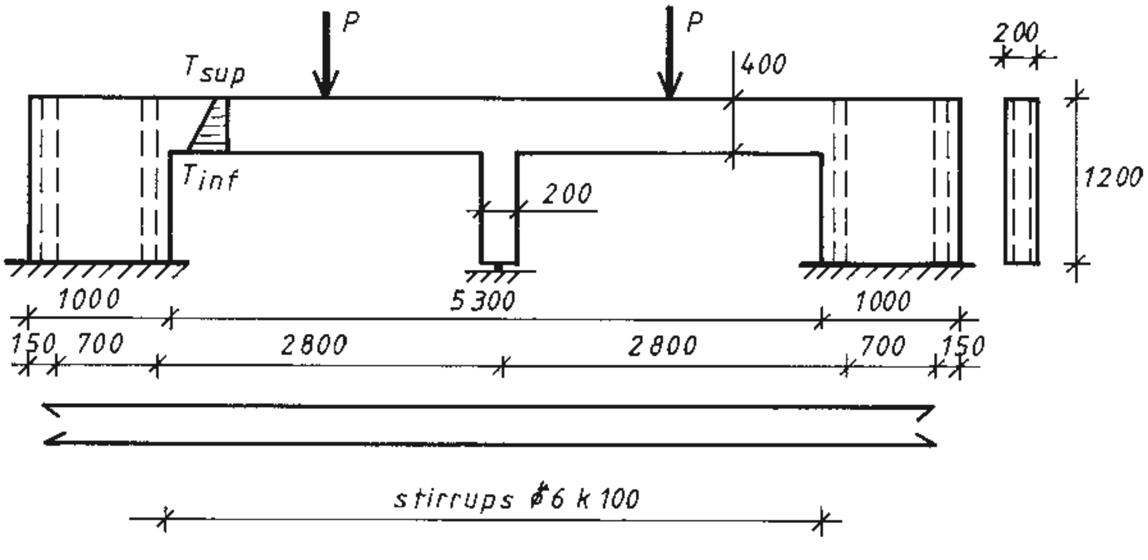
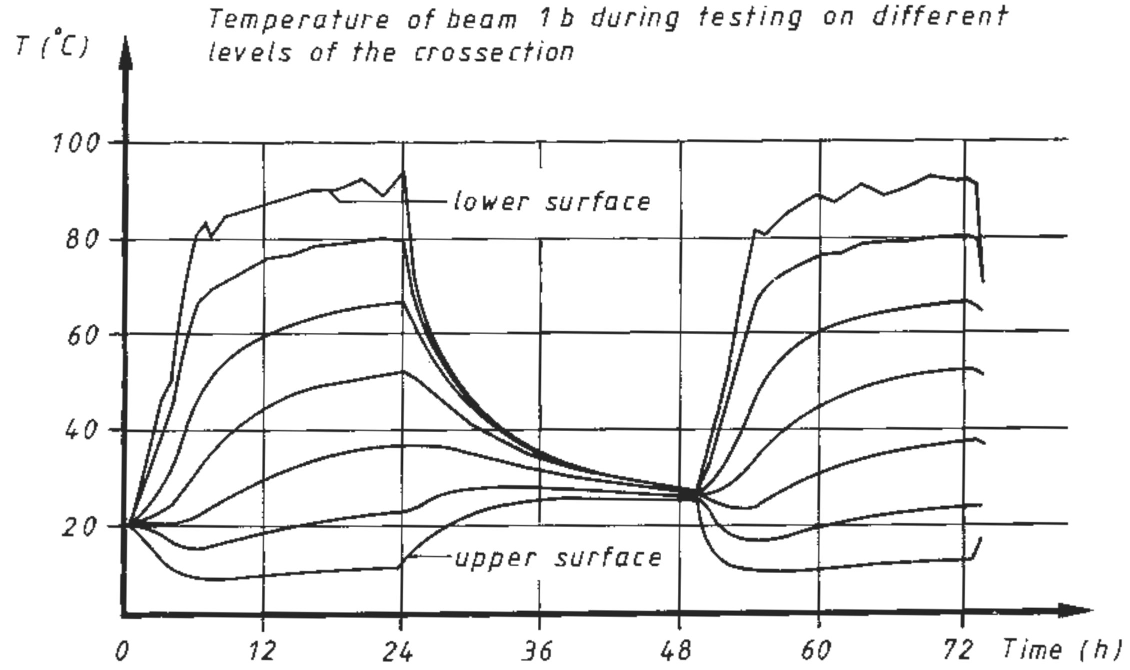
2,5 %
3,0 %
4,0 %
First cracking phase
Crack propagation phase
Stabilized cracking phase
Bending concrete K 50
Variationcoef. 10 %
Bar size ϕ 10 mm
100 200 300 400 σsm N/mm²
0,5 1,0 1,5 2,0 εsm ‰
ρeff=0,8 %
1,0 %
1,2 %
1,5 %
2,0 %
2,5 %
3 %
4 %
First cracking phase
Bending concrete K 50
variationcoef. 20 %
Bar size ϕ 10 mm
100 200 300 400 σsm
0,5 1,0 1,5 2,0 εsm

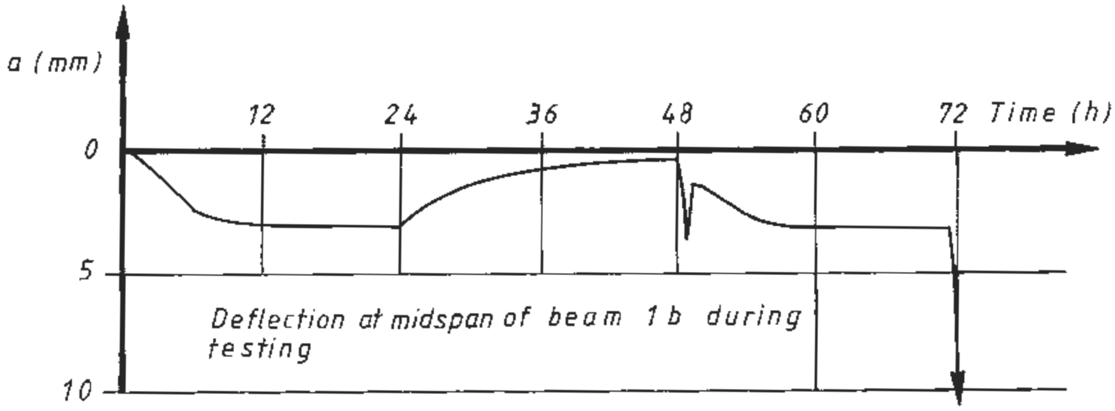
BEAMS TYPE 1

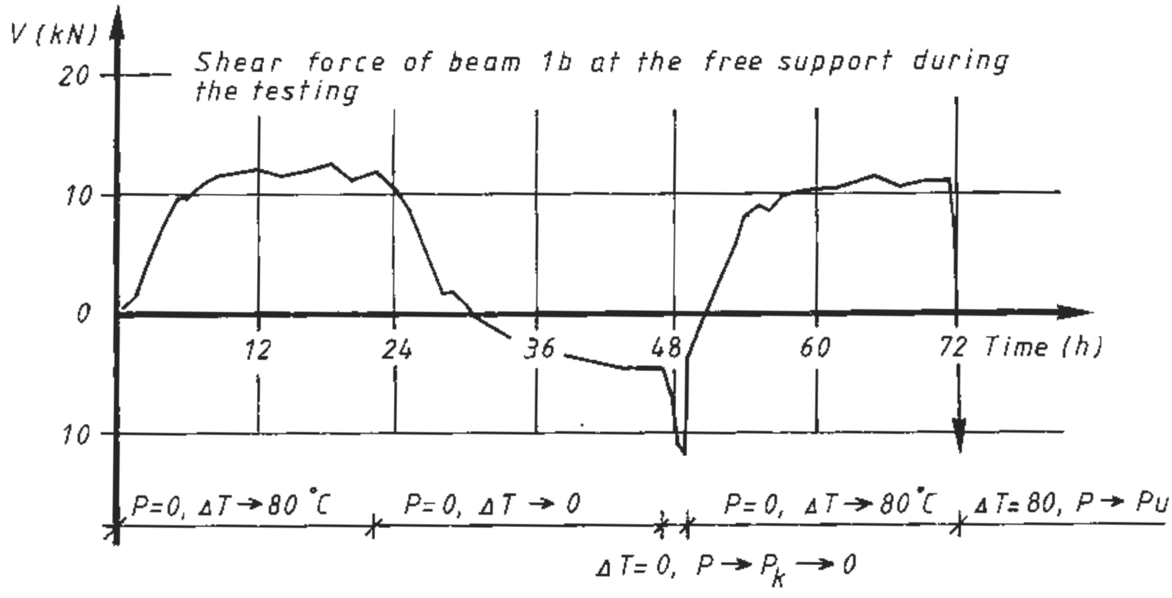


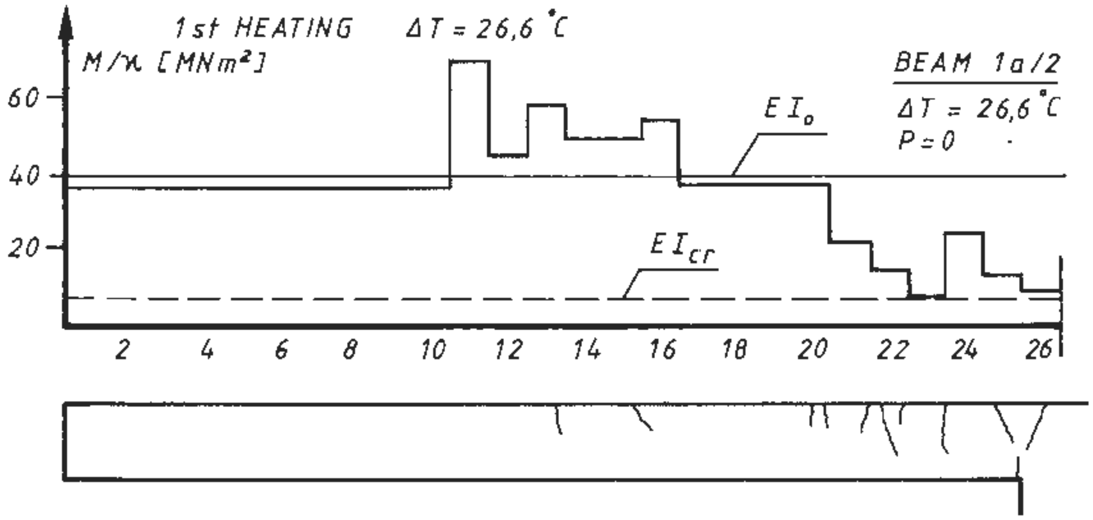
BEAMS TYPE 2




Temperature of beam 1 b during testing on different levels of the crossection


Deflection at midspan of beam 1 b during testing


Shear force of beam 1b at the free support during the testing


1st HEATING ΔT = 26,6 °C
M/κ [MNm²]
60
40
20
E I₀
E I_cr
BEAM 1a/2
ΔT = 26,6 °C
P = 0
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6
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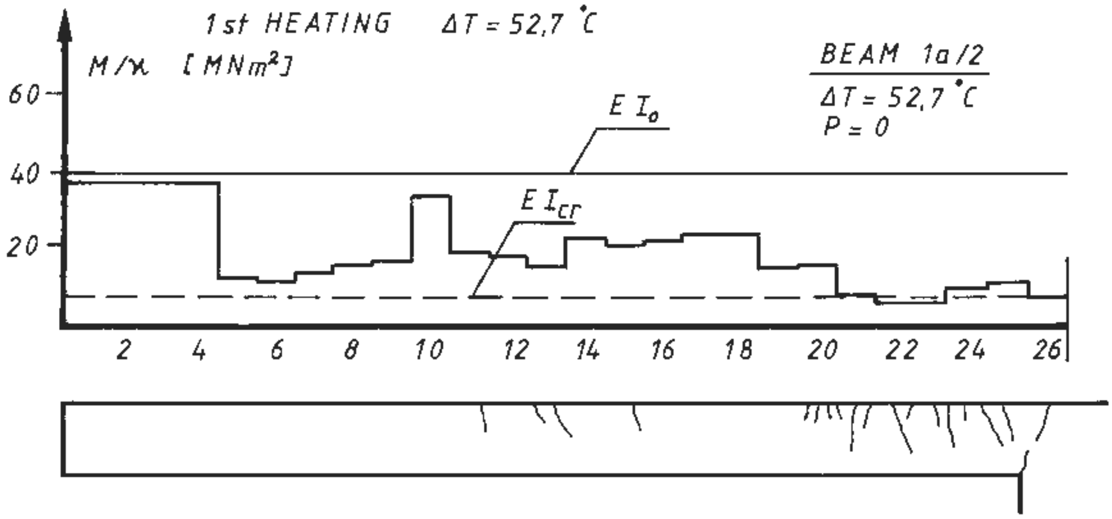
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M/κ [MNm²]
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40
20
E I₀
E I_cr
BEAM 1a/2
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P = 0
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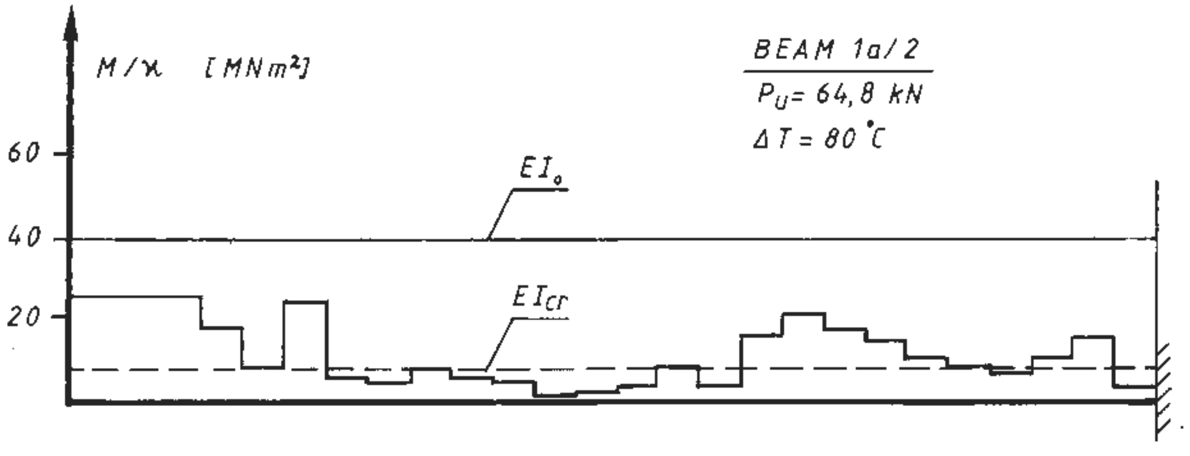
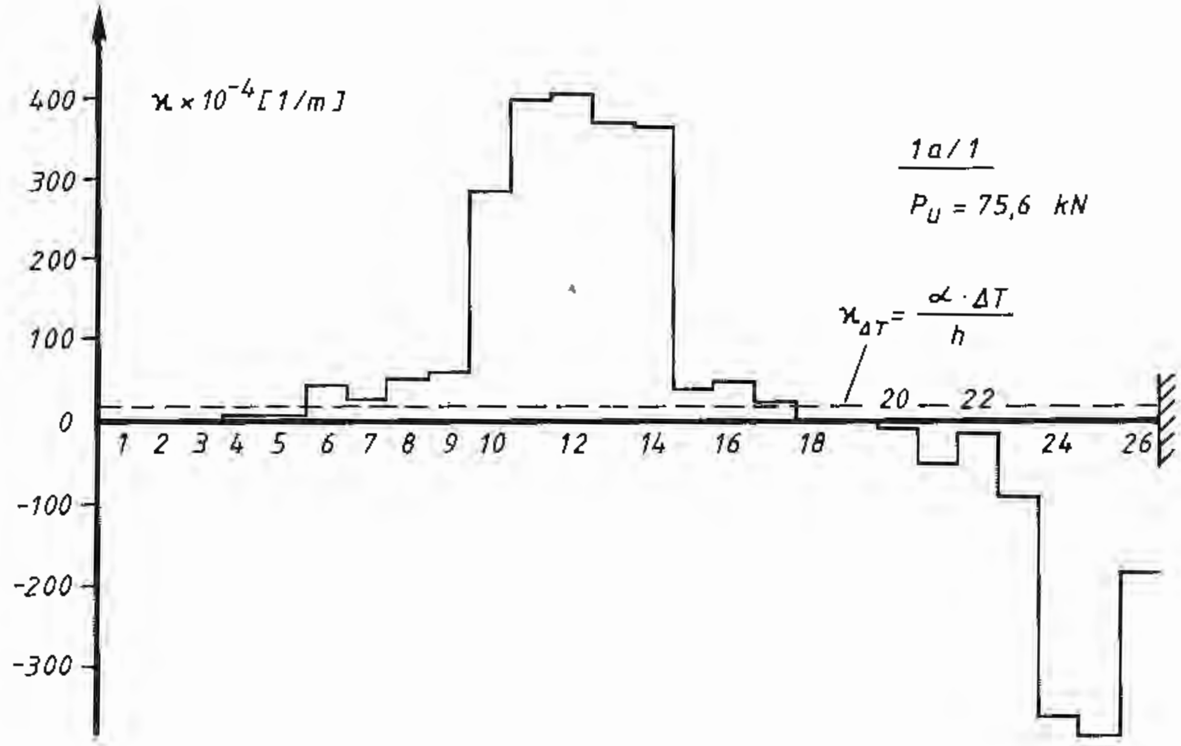

M/κ [MNm²]
BEAM 1a/2
P_U = 64,8 kN
ΔT = 80 °C
60
40
20
EI₀
EI_cr


ϰ × 10⁻⁴ [1/m]
400
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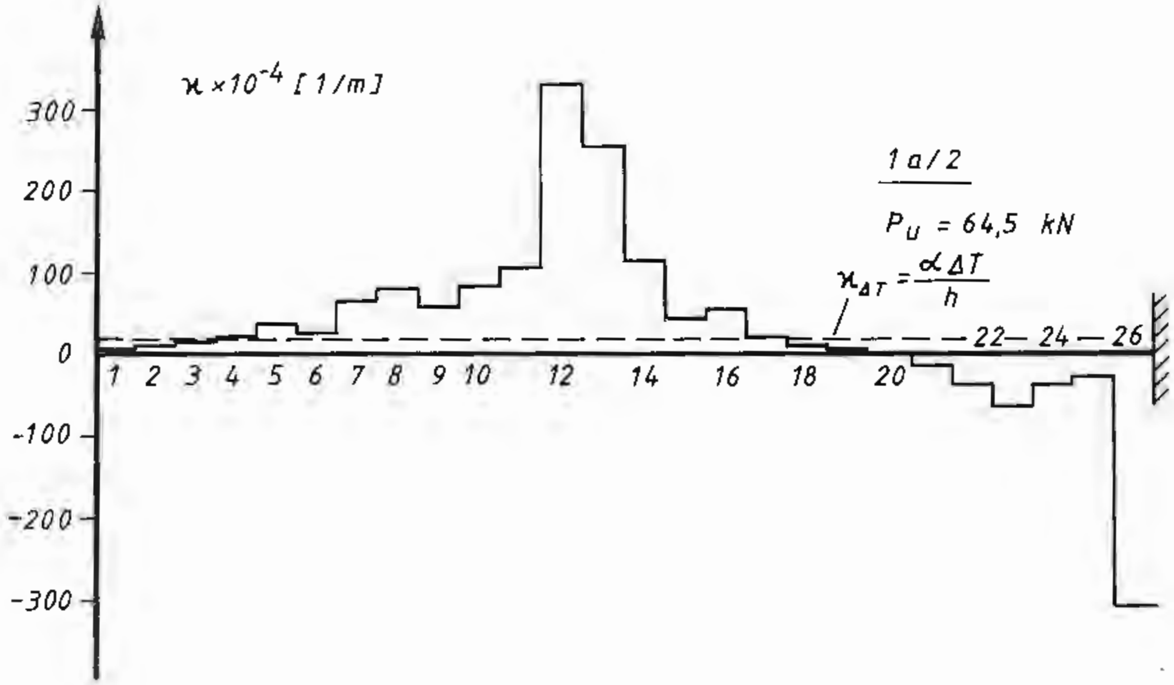
1a/1
P_U = 75,6 kN
ϰ_ΔT = α · ΔT / h


ϰ × 10⁻⁴ [1/m]
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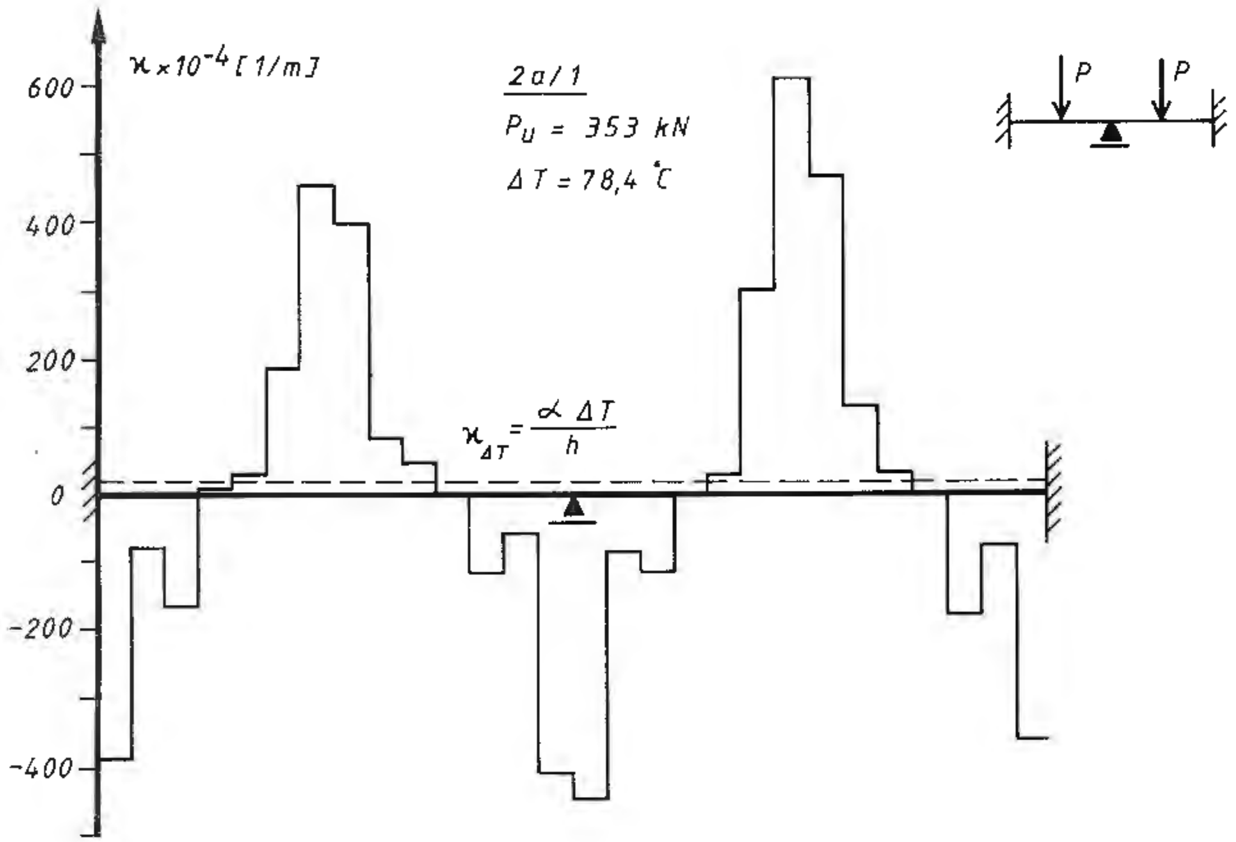
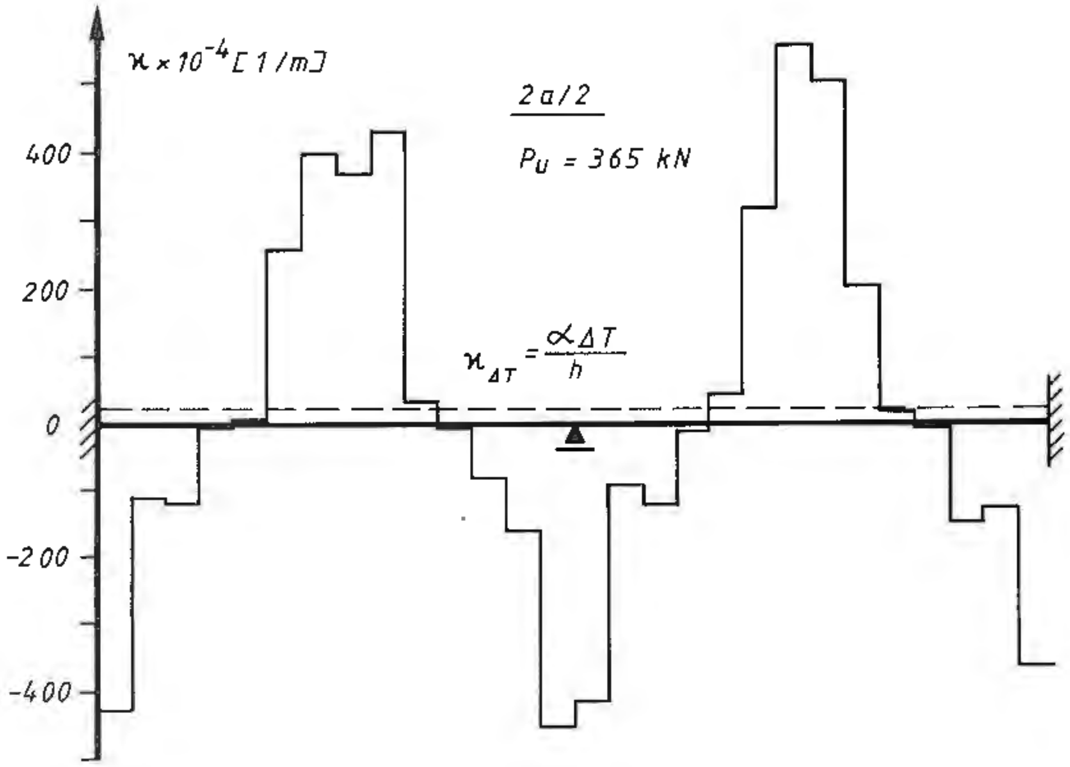
200
100
0
-100
-200
-300
1 2 3 4 5 6 7 8 9 10 12 14 16 18 20 22 24 26
1a/2
P_U = 64,5 kN
ϰ_ΔT = α ΔT / h


ϰ × 10⁻⁴ [1/m]
600
400
200
0
-200
-400
2a/1
Pᵤ = 353 kN
ΔT = 78,4 °C
ϰ_ΔT = α ΔT / h
P
P


ϰ × 10⁻⁴ [1/m]
400
200
0
-200
-400
2a/2
Pᵤ = 365 kN
ϰ_ΔT = α ΔT / h

Beam No.	Bar size mm	Calculated values					Measured values		
		ΔT, corresponds to the initiation of 1st crack 1) °C	ΔT, correponds to the initiation of 1st crack 2) °C	width of 1st crack 1) w_m mm	crack width when ΔT = 80 °C 2) w_m mm	crack width when ΔT = 80 °C 3) w_m mm	ΔT, corresponds to the initiation of 1st crack °C	width of 1st crack mm	crack width when ΔT = 80 °C mm
1a/1 4)	10	33.6	15.3	0.09	0.17	0.09	38	0.04	0.12
1a/2	10	33.6	15.3	0.09	0.17	0.09	23	0.02	0.12
1a/3 5)	10	33.6	15.3	0.09	0.17	0.12	39	0.01	0.09
1b	20	30.0	16.5	0.02	0.09	0.03	19	0.02	0.04
1c/1 4)	20	30.0	15.7	0.03	0.12	0.05	42	0.01	0.02
1c/2	20	30.0	15.7	0.03	0.12	0.12	25	0.02	0.06
1c/3 5)	20	30.0	15.7	0.03	0.12	0.06	41	0.01	0.07
1d	25	29.7	21.0	0.01	0.08	0.03	22	0.02	0.06
1e	10	34.9	15.3	0.09	0.17	0.10	18	0.02	0.08
variation		30... 34	15.3... 21.0	0.01... 0.09	0.08... 0.17	0.03... 0.12	18... 42	0.01... 0.04	0.02... 0.12