

AN INVESTIGATION OF REINFORCED CONCRETE BEAMS IN TWO SPANS  
SUBJECTED TO FATIGUE LOAD



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ABSTRACT

Static and fatigue tests have been carried out on 17 full scale two span beams. The investigation aimed to study the fatigue behaviour of reinforced concrete beams. Comparisons with the design specifications given in the Swedish code BBK 79 have been made. It has been found that BBK 79 seems to overestimate the shear capacity of beams subjected to cyclic loading.

Key words: Reinforced concrete beams, fatigue, limit state theory, shear failure.

1. INTRODUCTION

This paper deals with fatigue of concrete (in this case beams), and is a brief summary of the work reported in /1/. The main purpose of the investigation was to study the behaviour of a reinforced concrete beam subjected to fatigue load. In many previous investigations it is most common to find small test specimen and the specimen designed to fail in one type of failure, i.e. flexural failure, shear failure etc. In this investigation the test specimen has been made to almost full scale, which means that the dimensions are of the same magnitude as one can find in normal structures. The beams were designed according to the Swedish code BBK 79 /2/ (which was considered to incorporate the latest research findings). However, no safety coefficients were used. This means that no specific type of failure was expected to dominate.

With the test results on hand it was meant to confirm or further develop the design rules given in the code BBK 79.

It was also hoped that the test results would give a hint whether limit state theory is applicable in the design for fatigue or not.

## 2. BACKGROUND

Research on the subject of fatigue of concrete has been carried out since the turn of the century. The first fatigue test (on concrete) was made by de Joly in France in 1898. In the USA the first reports were published in 1903 and 1907 by Van Ornum. In the interwar period a great deal of investigations were made in the USA and Germany. Some of these were rather comprehensive regarding both the extent of the tests and the various variables that was studied.

Lately, most of the work carried out has been to explain the influence of different variables. Though, in many cases it has only been showed that a variable has an influence. The question how big that influence is, often remains unsolved.

Because of the great number of variables that affect the fatigue strength and the large spread that is characteristic of fatigue tests, one should perform large test series. To do that one need big resources, and therefore large fatigue tests have been carried out, mainly, in the USA.

The interest in fatigue of concrete and reinforced concrete has increased during the last decade or so. The reasons for this are

Higher material strength regarding static load and the use of refined calculation methods have caused a development towards slimmer structures. This means that the relative free share of the load increases, thereby causing that fatigue may occur.

The higher "static" material strength, first of all regarding the reinforcement, i.e. steel, means that the influence of other variables are greater than before. As an example, the stress concentration at lugs and weldjoints means that the fatigue strength of steel does not increase proportionally with the static strength. To make use of the higher material qualities one obviously has to study the effect of fatigue.

- In new types of structures, i.e. off-shore structures, windpower stations, watertower and high towers, which are subjected to wind and waveforces the effect of fatigue is obvious.
- In other types of constructions, such as machinebeds, it has been found lately that problems with, for example, deformations can be connected to fatigue.

In /3/, /4/ and /5/ a good survey of fatigue of concrete is given.

### 3. TEST SPECIMENS

Tests have been carried out on 17 reinforced concrete beams. The beams belonged to two test series. Series 1 consisted of one single span beam and four two span beams. Series 2 consisted of 12 two span beams. Nominal dimensions and reinforcing details are shown in Figure 1 to 5. The beams are of two types, called type A and B. Type A beams are designed for a distribution of the flexural moment according to the theory of elasticity for an uncracked homogenous material. Type B beams are designed to limit state theory in such a way that the middle support is reinforced for half the moment according to type A, while the field is reinforced so that the total bearing capacity equals type A.

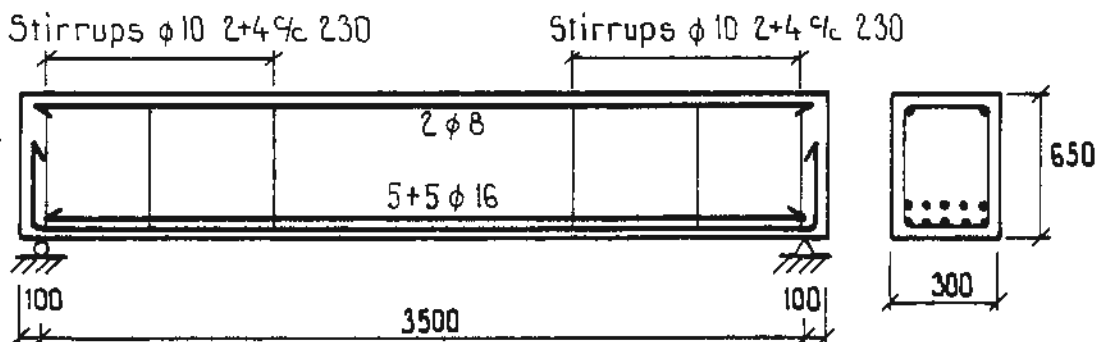


Figure 1. Beam 1

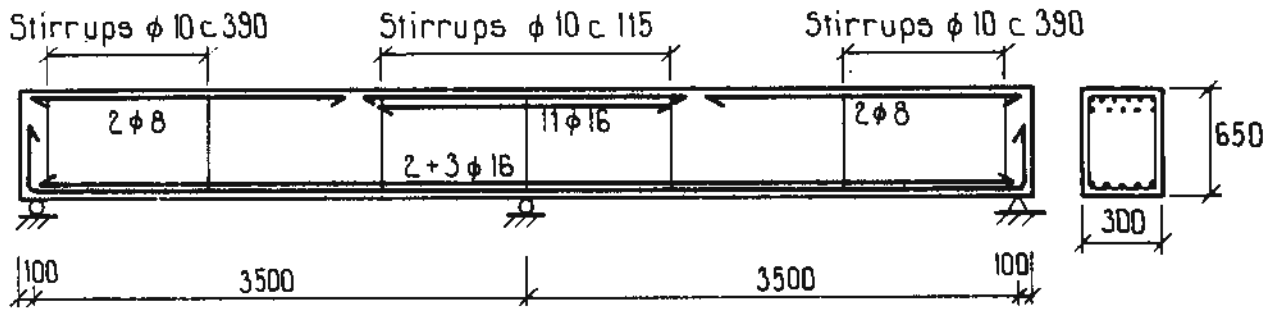


Figure 2. Beam 2 and 4. (type A)

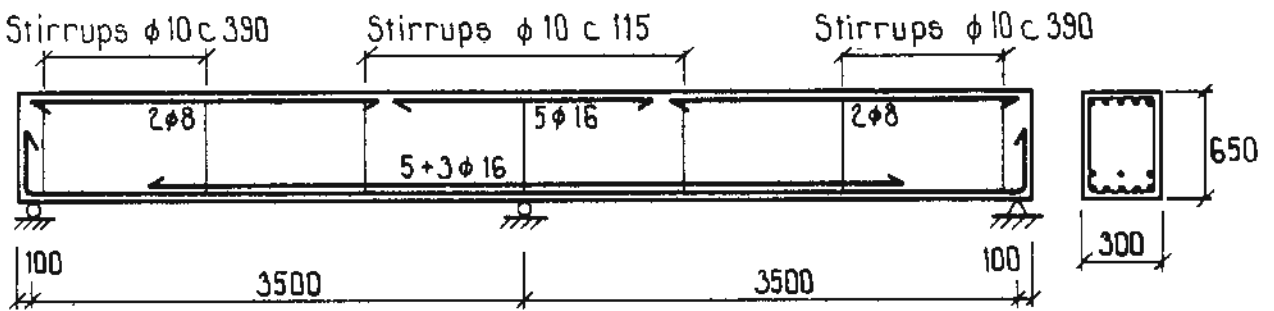


Figure 3. Beam 3 and 5 (type B)

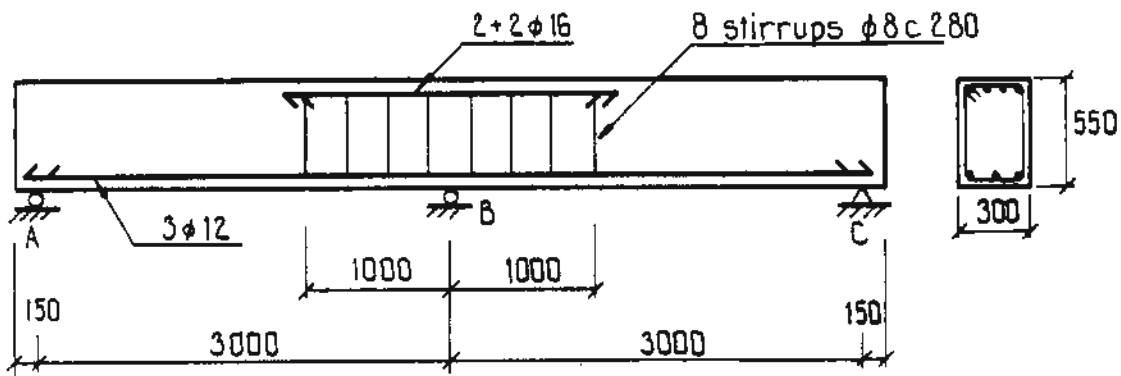


Figure 4. Beam A1 - A6 (type A)

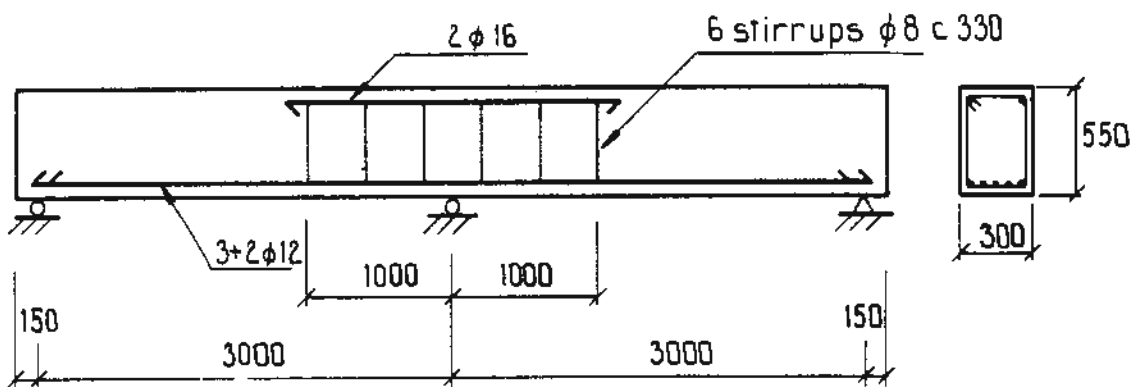


Figure 5. Beam B1 - B6 (type B)

The concrete mix was delivered by Sydsten in Hardeberga and the reinforcement from Domnarvet in Borlänge. For series 1 the concrete quality ordered was K 35 and the steel was Ks 600, except for the stirrups, which were Ks 400S. However, a much higher concrete quality was received from the factory. In series 2, the concrete quality ordered was K 25 and the reinforcement was Ks 400, except for the stirrups, which were Ks 400S. K stands for lugged bars and s 400 respectively s 600 stands for the yield limit. S means that the material may be welded.

The beams were cast at the institute by our staff. The concrete was delivered in one batch for series 1 and four batches for series 2. After having been cast, the beams were stored for four days in a wet climate, and there after in the laboratory (i.e. normal humidity and a temperature of 20°C). For every beam three standard cubes (150 x 150 x 150 mm<sup>3</sup>) were taken for compressive strength tests. The strengths listed in Table 1 are the average for each beam. Yield strengths and failure strengths for the bars are listed in Table 1. These tests were performed partly by Domnarvet and partly by our staff. The concrete compressive strength,  $f_{cc}$ , is calculated according to Degerman /6/ as

$$f_{cc} = 1.18 f_{cube}^{0.913} \quad (1)$$

where  $f_{cube}$  is the cube strength.

The stirrups for all beams in series 2 ( $\phi$  8) were bent cold around radii equal to 16 mm. For beams in series 1 ( $\phi$  10) a 24 mm radius was used.

#### 4. THE TEST SET-UP

The test set-up is shown in figure 6 and photo 1. The load was applied by a hydraulic loading machine, made by M.A.N. The loading frequency for the fatigue tests varied between 200 and 400 loadcycles per minute. The load cycled between a maximum and a minimum value with constant amplitude and frequency. The compressive strains were measured in the middle of the spans and over the middle support by strain gauges (150 mm long). The deflec-

tion of the beam was measured by potentiometers and the loads, i.e. applied load and reactions, were measured by load cells (made by Bofors).

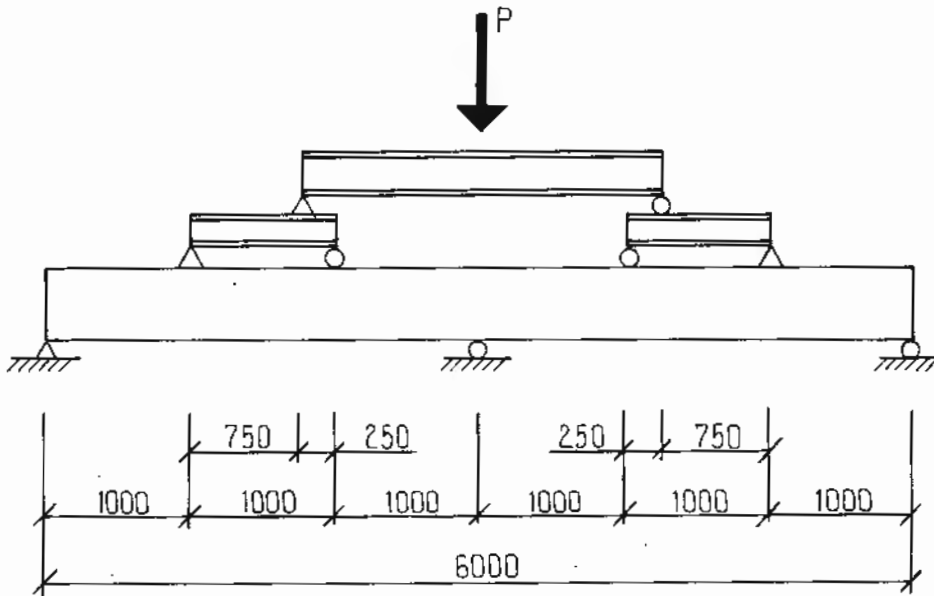


Figure 6. Test set-up for series 2

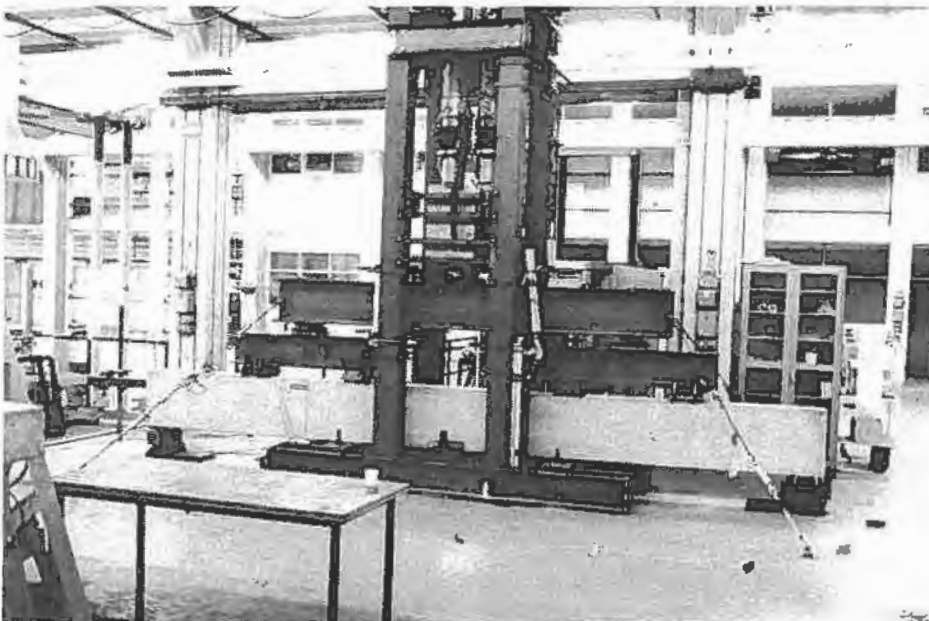


Photo 1. Test set-up

No strain gauges were placed on the bars. The reason for this is the following; To place a gauge on a bar, one has to refinish the surface of the bars and, perhaps, par away some of the lugs. This means that the fatigue strength is more or less affected.

Instead, it was chosen to calculate the steel stresses for the tensile reinforcement. They were calculated from the applied loads and reactions assuming elastic behaviour and a concrete section cracked in tension.

## 5. DESCRIPTION OF THE TESTS

Series 1 consisted of, as mentioned earlier, one single span beam and four two span beams. All the beams were subjected to cyclic loading. None of the beams failed due to fatigue, despite such high number of load cycles as, for the single span beam, 1.9 million. The stresses in both steel and concrete have been close to or below the calculated value of the fatigue limit. The fatigue limit is chosen as the fatigue strength at 2 million loadcycles. This means that the fatigue range limit of the bars are, according to Östlund /7/, 307 MPa.

For concrete, the fatigue limit is calculated from the expression (Tepfers /8/)

$$f_c^{fat} = f_c (1 - 0.0685 (1 - R) \log 2 \cdot 10^6) \quad (2)$$

where  $f_c$  is the static strength and  $R$  is the ratio  $P_{min}/P_{max}$ . Calculated stresses are presented in table 1. The ratio between the actual maximum shear force and the shear capacity at static load, according to Hedman and Losberg /10/, is also shown in this table. The loading history for the beams subjected to cyclic loading is indicated in table 2.

Of the 12 beams in series 2, eight were subjected to cyclic loading, while the remaining four were loaded statically to failure. All of the beams subjected to cyclic loading in this series failed. The number of loadcycles to failure varied between 2000 and 369 000.

Of the eight beams, which failed when subjected to cyclic loading, five different types of failure were recorded.

Table 1.

Beam	f <sub>cube</sub> MPa	f <sub>cc</sub> MPa	φ 8		φ 10		φ 12		φ 16		calculated stresses								V <sub>max</sub> V <sub>stat</sub>
			f <sub>st</sub> MPa	f <sub>stu</sub> MPa	f <sub>st</sub> MPa	f <sub>stu</sub> MPa	f <sub>st</sub> MPa	f <sub>stu</sub> MPa	f <sub>st</sub> MPa	f <sub>stu</sub> MPa	middle support				field				
											σ <sub>s</sub> <sup>max</sup> MPa	σ <sub>s</sub> <sup>min</sup> MPa	σ <sub>c</sub> <sup>max</sup> MPa	σ <sub>c</sub> <sup>min</sup> MPa	σ <sub>s</sub> <sup>max</sup> MPa	σ <sub>s</sub> <sup>min</sup> MPa	σ <sub>c</sub> <sup>max</sup> MPa	σ <sub>c</sub> <sup>min</sup> MPa	
1	59.0	48.8	-	-	475	600	-	-	590	875	-	-	-	-	253	39	20.4	3.1	0.19
2	59.0	48.8	-	-	475	600	-	-	590	875	273	27	23.3	2.3	169	18	10.4	1.0	0.4
3	59.0	48.8	-	-	475	600	-	-	590	875	385	39	23.0	2.3	150	15	10.7	1.1	0.4
4	59.0	48.8	-	-	475	600	-	-	590	875	273	27	23.3	2.3	169	18	10.4	1.0	0.4
5	59.0	48.8	-	-	475	600	-	-	590	875	385	39	23.0	2.3	150	15	10.7	1.1	0.4
A1	28.2	24.9	500	688	-	-	484	633	495	628	-	-	-	-	-	-	-	-	-
A2	30.3	26.6	500	688	-	-	484	633	495	628	485	82	23.1	3.8	367	37	10.4	0.9	0.73
A3	32.3	28.2	500	688	-	-	484	633	495	628	310	10	16.1	2.2	378	93	10.8	2.7	0.54
A4	32.3	28.2	500	688	-	-	484	633	495	628	446	70	21.1	3.3	406	97	12.0	2.8	0.70
A5	23.8	21.3	439	595	-	-	467	627	451	623	-	-	-	-	-	-	-	-	-
A6	24.0	21.5	439	595	-	-	467	627	451	623	-	-	-	-	-	-	-	-	-
B1	28.2	24.9	500	688	-	-	484	633	495	628	628	69	20.6	2.2	330	114	12.5	4.3	0.43
B2	30.3	26.6	500	688	-	-	484	633	495	628	628	185	21.7	5.8	247	27	9.4	1.0	0.74
B3	32.3	28.2	500	688	-	-	484	633	495	628	628	224	21.8	7.4	159	8	6.1	0.3	0.79
B4	32.3	28.2	500	688	-	-	484	633	495	628	628	154	22.7	5.3	189	49	7.2	1.9	0.82
B5	23.8	21.3	439	595	-	-	467	627	451	623	-	-	-	-	-	-	-	-	-
B6	24.0	21.5	439	595	-	-	467	627	451	623	-	-	-	-	-	-	-	-	-



Table 2.

Beam	type of test	Loading history		Number of cycles to failure	type of failure				Remarks	
		P <sub>max</sub>	P <sub>min</sub>		flexural failure	shear failure		stirrup failure		bond failure
		kN	kN			outer support	middle support			
1	fatigue	430	70	1 900 000*					*test broken off	
2	"	800	80	860 000*					* " " "	
3	"	800	80	1 470 000*					* " " "	
4	"	800	80	1 200 000*					* " " "	
5	"	800	80	930 000*					* " " "	
A1	"	714	233	2 000		x				
A2	"	570	70	195 000	x					
A3	"	470	70	369 000			x			
A4	"	560	110	4 900			x			
A5	static	762	-	-			x			
A6	"	760	-	-			x			
B1	fatigue	600	130	5 800				x		
B2	"	570	90	34 000			x			
B3	"	410	70	3 400			x			
B4	"	410	100	97 000			x		No active stirrup	
B5	static	726	-	-			x		" " "	
B6	"	709	-	-		x				

- 1) Shear failure in a non shear reinforced part of a beam were recorded in three cases. See photo 2.
- 2) Shear failure in a shear reinforced part of a beam, but with no failure of the stirrups, was recorded in one case. See photo 3.
- 3) In one beam a shear crack originated at the outer support and was followed by bond failure in two of the four longitudinal bars. As a direct cause of this the compressive zone collapsed. See photo 4.
- 4) Failure of the stirrups at the middle support were recorded in two cases. The fractures were in both cases of a brittle nature. See photo 5.
- 5) In one case a brittle fracture was recorded in one of the four bars over the middle support. See photo 6. After a further 25 000 load cycles all the flexural reinforcement in one span failed. See photo 7.



Photo 2. Shear failure in a non shear reinforced part of a beam.

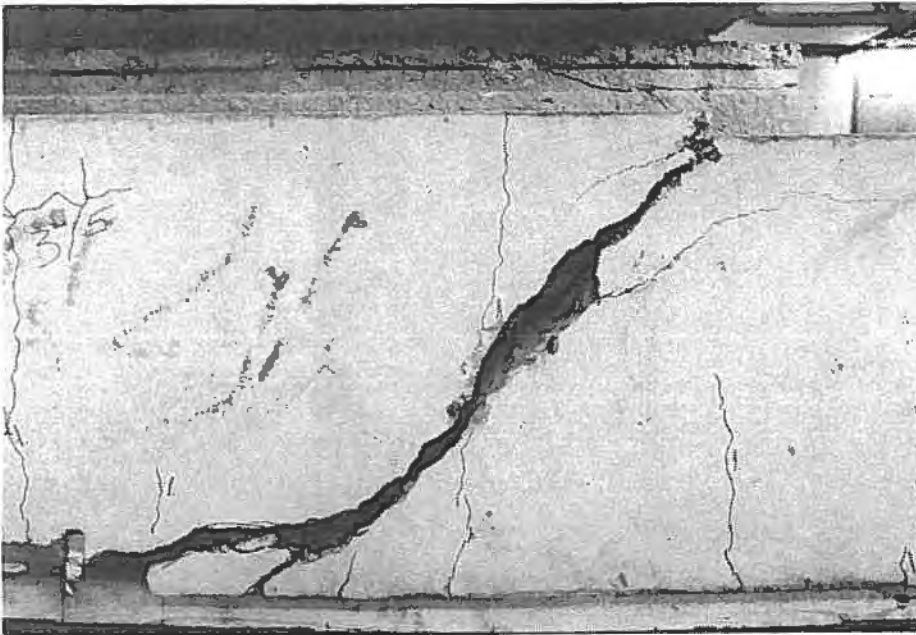


Photo 3. Shear failure in a shear reinforced part of a beam, but with no failure of the stirrups.

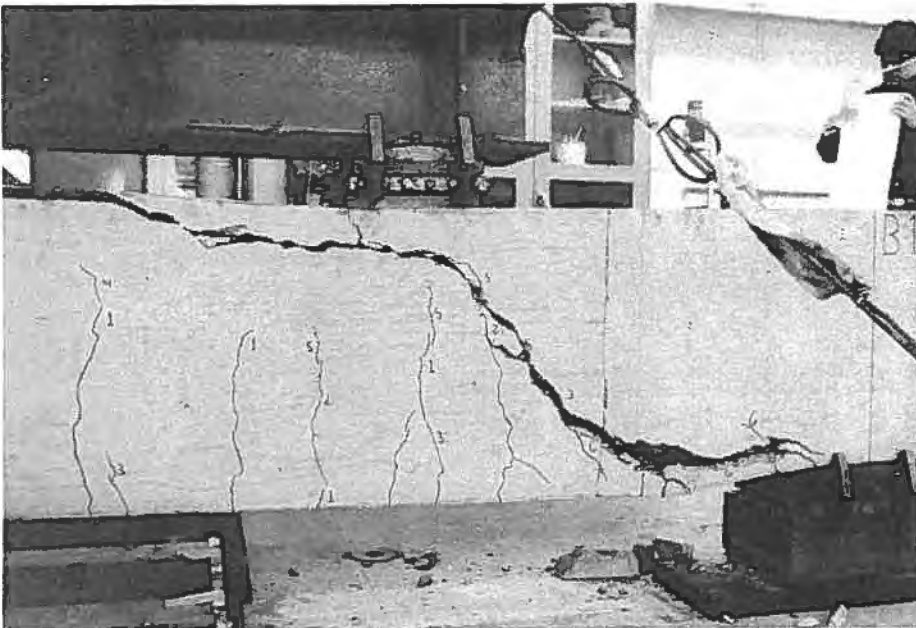


Photo 4. Bond failure.

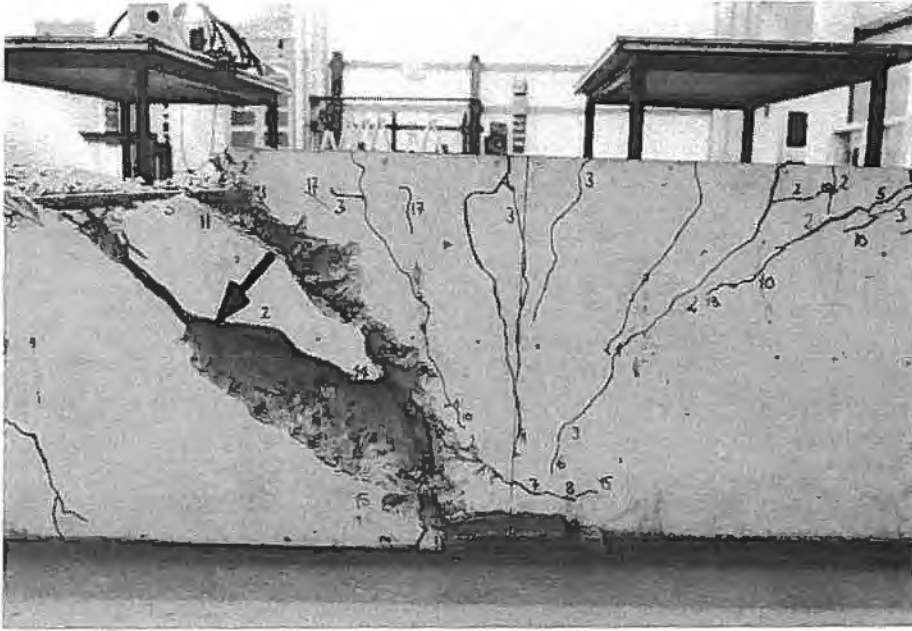


Photo 5. Failure of the stirrups. The arrow shows the position of the fracture.

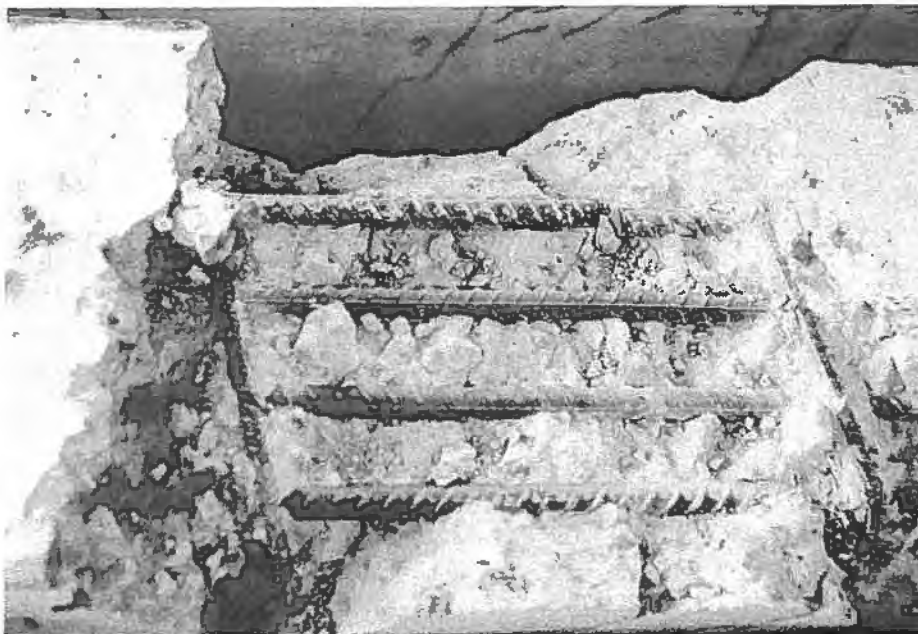


Photo 6. Brittle fracture in longitudinal bar.

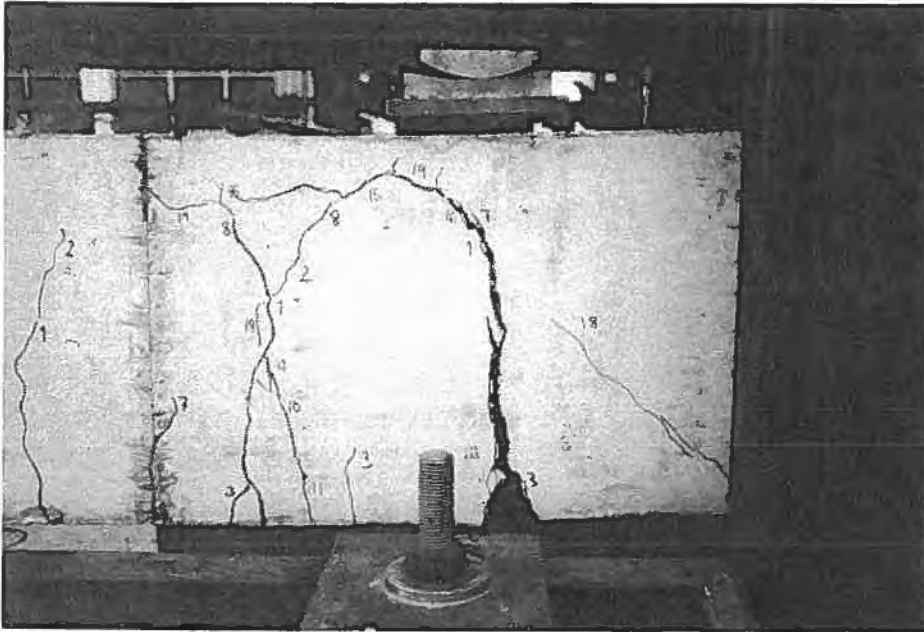


Photo 7. Flexural failure. Just prior to collapse.

Regarding the number of load cycles to failure, it can be noted that failure according to 1), 2) and 3) occurred at low or moderate number of loadcycles, while failure of the reinforcement occurred at moderate or high number of loadcycles.

The four beams in series 2, which were loaded statically to failure, all failed according to point 1), 2) or 4). However the flexural capacity was almost reached in all the beams, as the stress in the reinforcement had reached the yield limit.

Table 2 is a summary of the tests.

The crackpattern developed in a way similar to beams subjected to static load. The crack lengths and crack widths increased to some extent with the number of loadcycles. Beams of type A have a much larger crack pattern at the middle support than beams of type B. This is because the bars over the middle support have reached the yield limit. This means that the deformations over the support are concentrated to the yield zone, i.e. one crack.

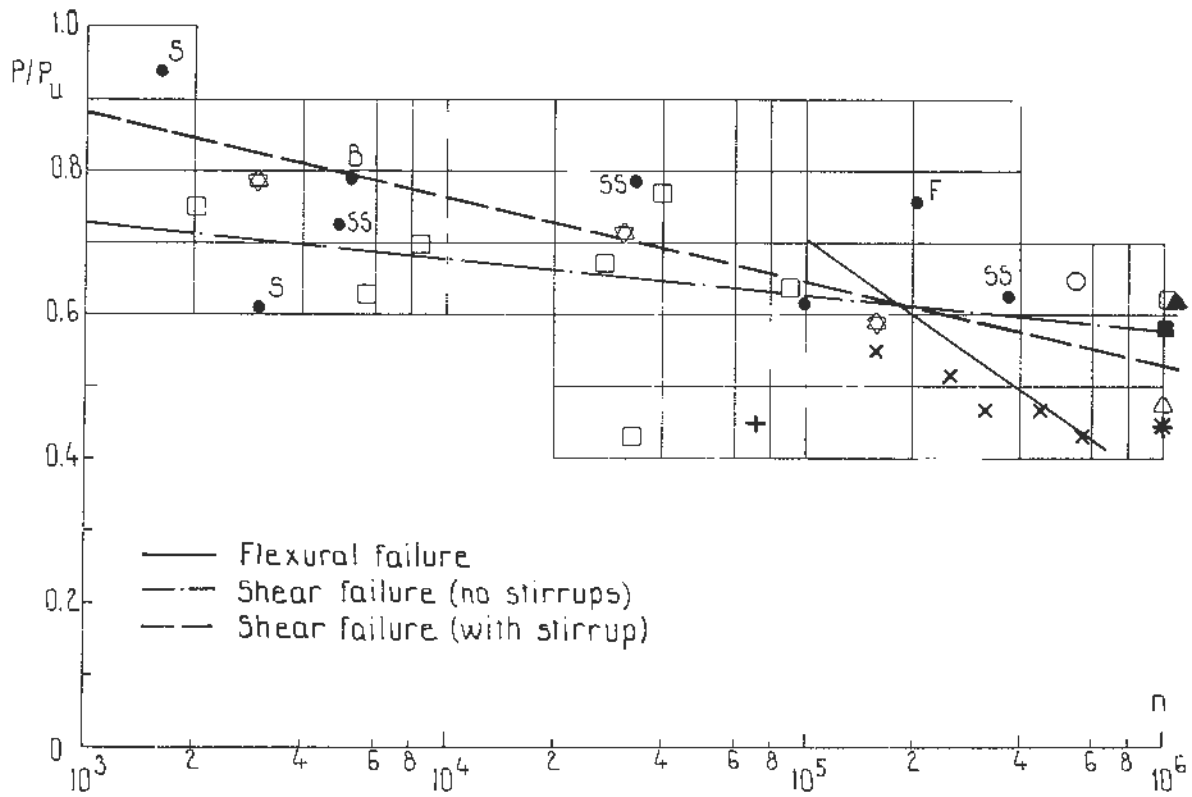
After the first load cycles, when the crack pattern had been stabilized, there was a certain difference between the largest and smallest moment at the middle supports as well as in the field. The ratio between these differences did not change as the test went on. This means that any interchange of moment between field and support has not taken place.

## 6. DISCUSSION

The results from this investigation and from some other investigations are presented in figure 7. The results are not quite comparable, as no regard has been paid to the lower load level. However,  $P_{\min}$  is less than 30 % of  $P_{\max}$  for all the presented results. Three lines of best fit are marked. One is for flexural failure, another for shear failure in a non shear reinforced part of a beam and the third for shear failure in a shear reinforced part of a beam. Figure 7 indicates that at high load levels (low number of load cycles to failure) it is fatigue of concrete which is the determining factor, while at low load levels (high number of load cycles) it is fatigue of the reinforcement which is the determining factor.

Some authors in the past, notably Hawkins /5/, have found that when a stirrup fails it does so where it is bent around the longitudinal bars. The fatigue strength for a bent bar is approximately half the strength of a straight one. Two of the beams in the present investigation failed due to stirrup fracture. However, in both cases the fracture occurred in the shear crack and not in the bend.

The explanation is probably that in the present investigation full scale test specimen have been used and stirrups made of lugged bars. In a bigger specimen there is more distance between where the stirrup bends around the longitudinal bars and consequently more available length to anchor the stirrup. A lugged bar needs much less anchorlength than a smooth one.



- Stelson & Cernica 1958      shear failure (no stirrup)
- Westerberg 1973              "              "
- △ Le Camus 1946                "              "
- Verna & Stelson 1963        "              "
- \* Le Camus 1946                shear failure (stirrup)
- ◉ Westerberg 1973              "              "
- × Westerberg 1973              flexural failure
- + Verna & Stelson 1963        bond failure
- Present investigation:
- S = shear failure (no stirrup)
- SS = shear failure (stirrup)
- F = flexural failure
- B = combined shear and bond failure

Figure 7. Comparison between the present investigation and some other.

It is a known fact that deformations increase with the number of load cycles. Figure 8 shows the deformations, here presented as  $\epsilon/\epsilon_{gu}$ , where  $\epsilon$  is the measured compressive strain and  $\epsilon_{gu}$  is the calculated strain corresponding to the fatigue limit. Figure 8 indicates that, for a value of  $\epsilon/\epsilon_{gu}$  of less than 0.5 the deformations do not increase with  $n$ . Assuming that the fatigue limit (i.e. the fatigue strength at two million load cycles)

is 60 % of the static strength, which is a widely used value ( $\sigma_{\min} = 0$ ), this implies that if the concrete stress is below 30 % of the static strength, the deformations will not increase with the number of load cycles.

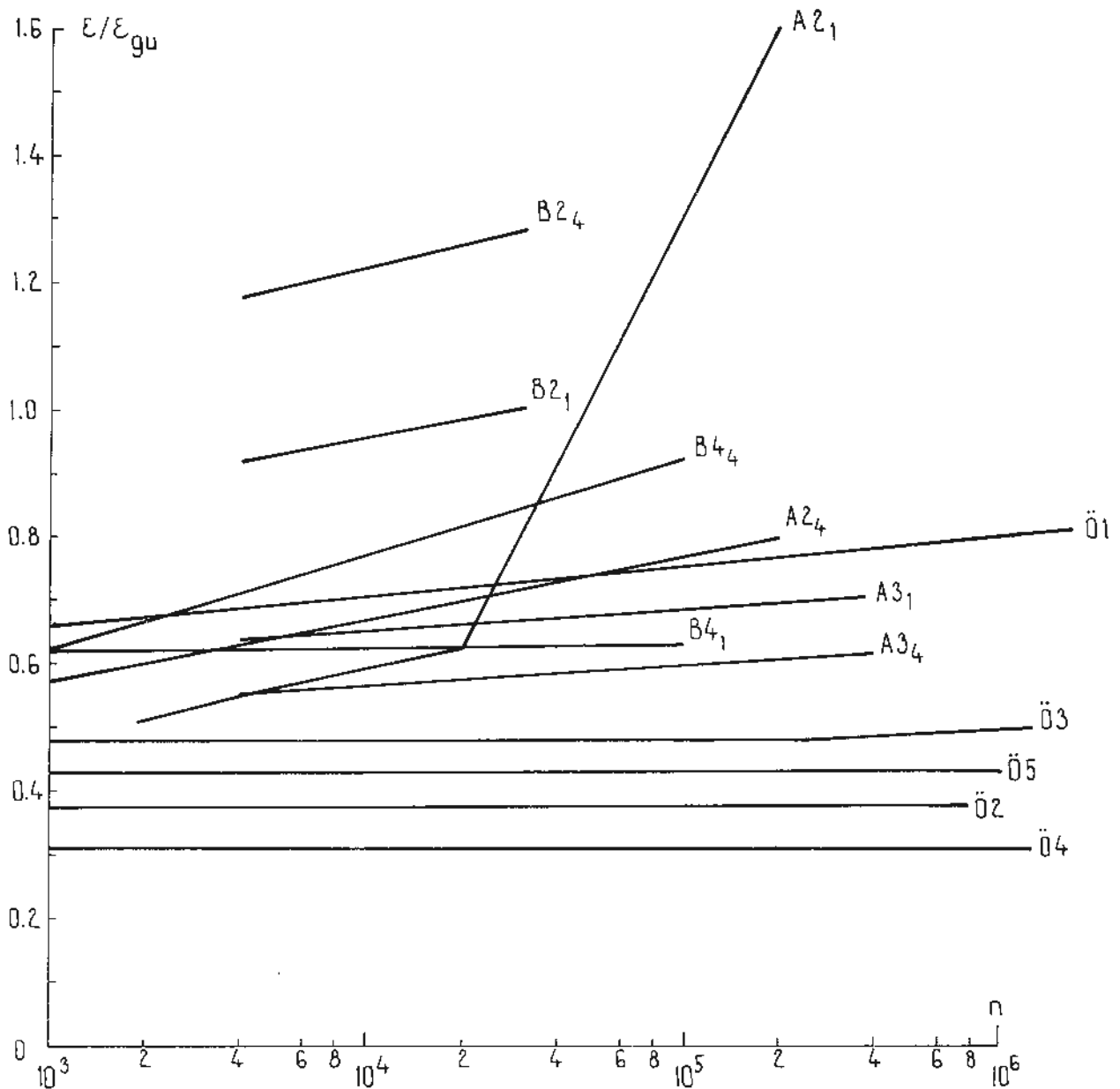


Figure 8. Deformations.

In the original report /1/, extensive calculations have been made to check out the design rules given in the Swedish code BBK 79. It has been found that the design rules given for designing for shear forces overestimates the shear capacity.



Figure 9 shows the design rule according to BBK 79 (dotted line) for a value of  $R = 0.19$  (where  $R$  is the ratio between minimum and maximum shear force) and no shear reinforcement. The straight line is the line of best fit for tested beams without shear reinforcement, taken from this investigation and from Westerberg /9/.  $V_{max}$  is the maximum shear force and  $V_{stat}$  is the calculated static shear capacity according to Hedman and Losberg. As can be seen, there is a considerable difference between the test results and BBK 79.

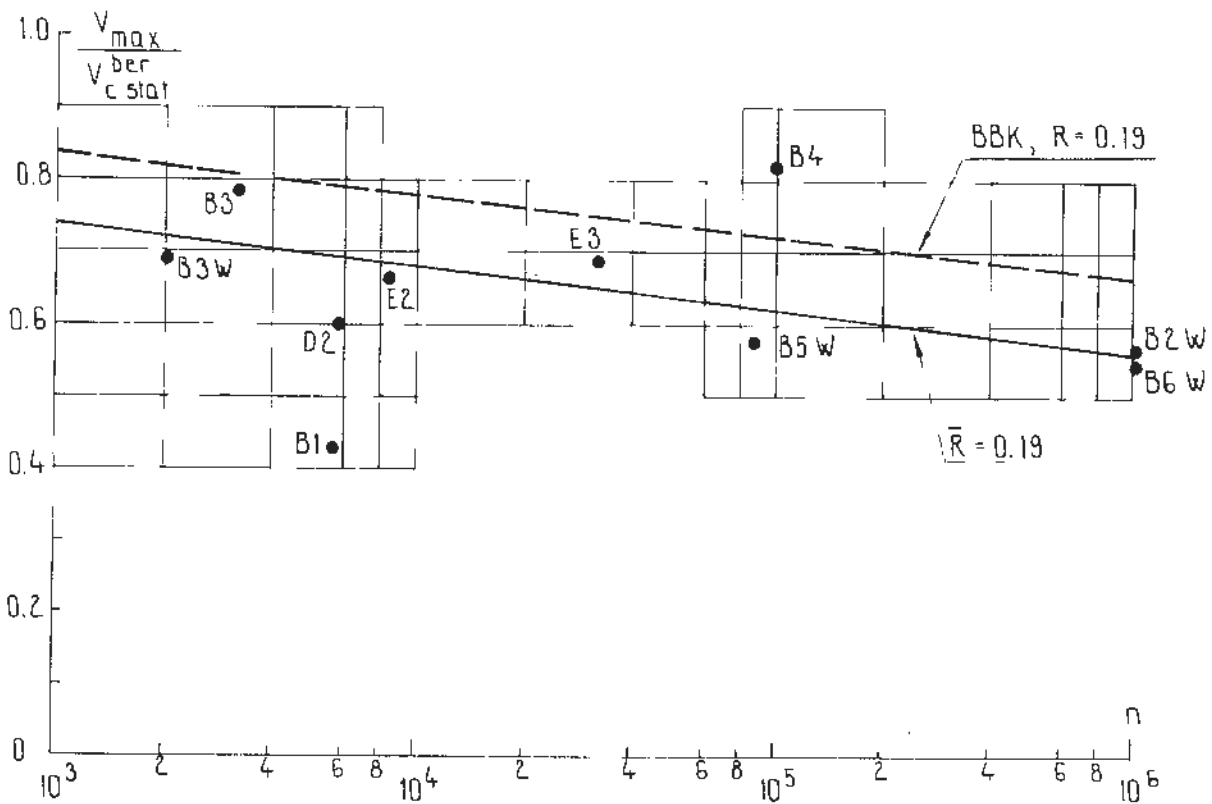


Figure 9. Comparison between the design specification in BBK 79 and test results.

According to BBK 79, the fatigue capacity of each component, i.e. concrete, longitudinal reinforcement and shear reinforcement, will be checked.

The calculated values for the beams with shear reinforcement, when using the straight line in figure 9 for the concrete shear capacity and the steel capacity according to BBK 79, stands in good agreement with the test results.

A total of six beams of type B have been subjected to fatigue load. When compared to the two beams of type B which have been loaded statically, it can be noted that no phenomena due to cyclic loading occurred. However, the deflections increase much more rapidly with  $n$  in type B than in type A. The spread is much larger for type B. The stress in the longitudinal bars over the middle support reaches the yield limit at an early stage of the test. This means that after the first load cycles the bars have been cold worked and consequently the material properties have been changed.

#### 7. CONCLUDING REMARKS

The fatigue failure is complex and there is always a large spread in the results of fatigue tests. This is why fatigue tests should be performed in large series with many specimens. However, from a practical and economical point of view, this is most often not possible. This is valid for the present investigation. It is desirable to carry out tests on more beams in the future, which will hopefully confirm the conclusions stated below.

- \* Shear failure is the most dangerous type of failure, as it can occur at quite low number of load cycles. Also, the rules given in BBK 79 seem to overestimate the shear capacity of a beam subjected to cyclic loading.
- \* Although a beam is reinforced with stirrups, a shear failure may occur without fracture of the stirrups.
- \* When a stirrup fails, it does so in a shear crack.
- \* If the concrete compressive stresses are below 30 % of the static strength, the deformations will not increase with the number of load cycles.
- \* The use of limit state theory in the design of a beam subjected to cyclic loading is not recommended.

\* It is necessary to develop a better knowledge of the behaviour of reinforced concrete subject to fatigue loads.

#### 8. ACKNOWLEDGEMENT

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