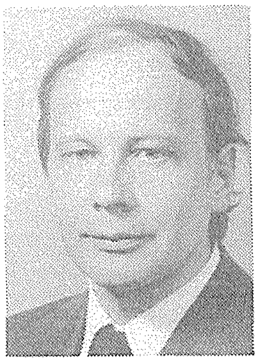

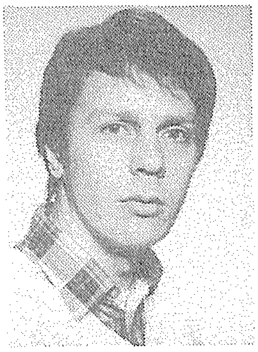

USE OF THE REAL FAILURE MECHANISM AND ROTATIONAL EQUILIBRIUM FOR CALCULATION MODEL OF THE SHEAR CAPACITY OF PRESTRESSED AND REINFORCED CONCRETE STRUCTURES

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Capacity of concrete structures is calculated using rotational equilibrium conditions in the cracked cross section of a structure. By this method it is possible to determine the capacity of the structure near the support as well as in the middle of a beam. Inclination of cracks is calculated by experimental formulae.

The results of the calculations in the case of prestressed hollow core slabs and prestressed I-beams have been compared to test results. The comparison shows a close agreement between calculations and test results.

Key words: shear, beams, slabs, modelling

SYMBOLS

A_c	the cross sectional area of concrete
A_s	the cross sectional area of tensile reinforcement
A_{sp}	the cross sectional area of the prestressing tension strands
A_{sv}	the cross sectional area of one stirrup
I_{eff}	the moment of inertia of the cross section with regard to the middle axis of the height of the cross section (the line at the height h/2 in figure 2)
I_w	the moment of inertia of the web = $\frac{b_w h^3}{12}$
M_c	moment capacity controlled by the compressed concrete zone above the inclined crack
$M_F (y + z \cot\alpha_y)$	the static moment of the external loads acting between the support and the point $y + z \cot\alpha_y$ with regard to the rotation centre
M_r	cracking moment capacity of the cross section in the direction of the crack
M_s	the rotation moment capacity of the reinforcement in the cracked state of cross section in the direction of the crack
M_{sv}	the moment of stirrups
Q	the shear force
T_s	the resultant force of the tension reinforcement
V	shear capacity
Wc	the section modulus of the concrete cross section
a_{sv}	the cross sectional area of the stirrup A_{sv} divided with the distance of two stirrups s
b	the breadth of the cross section
b_w	the breadth of the web
f_c	the compressive strength of concrete
f_{ct}	the flexural tension strength of concrete
f_y	strength of the steel
h	the height of cross section

k	the number of hollow cores
k_b	bond coefficient
r	the radius of the hollow core
s	the distance of stirrups in the direction of the beam axis
t	support bredth
u_s	the sum of the circumferences of the reinforcing bars or strands
x	the height of the compression zone of the cross section
y	distance from the support
z	the leverarm of the tension reinforcement
α	the direction angle between the crack and the beam axis
γ	the angle between the stirrup and the perpendicular line to the beam axis
η	variable depending on the amount of shear reinforcement
σ_p	the prestress of the strands
$\sigma_{p\infty}$	value of long time prestress of the strands
ϕ	nominal diameter of the strand

subscripts

o	value at the support
y	value at the distance y from the support
d	design value
u	ultimate value

1. INTRODUCTION

Many kinds of calculation models of the shear capacity of concrete beams and slabs have been presented and used in practice. The best known of these are the Mörsch truss analogy /4/ and its modifications and the addition principle of Regan /5/, which is presented also in the CEB (Comité Européen du Béton) model code and a modified version of it used in the ACI (American Concrete Institute) code. The fundamental principle of the rotation model has also been presented for example by Walther /6/, but this model has not been developed further into a suitable form for practical use.

The main problem in the development of the rotation model is the determination of the angle of the crack influenced by the shear force or combined shear force and bending moment.

2. PRINCIPLES

The most usual classical mode of failure of concrete beam or one-way slab is presented in figure 1. The failure occurs, when the concrete is cracked and the reinforcement is no longer able to keep the concrete blocks together, or the concrete fails at the compression side, when the structure becomes a mechanism and fails. The failure occurs usually by rotation of the concrete blocks between cracks. Near the supports the failure is called shear failure, further from the support, combined shear and bending moment failure and in the middle part of the span bending moment failure.

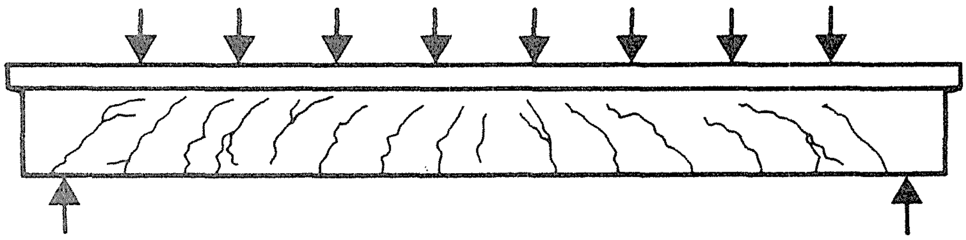

Figure 1. Classical mode of failure of reinforced or prestressed concrete beam or one-way slab.

The principle of this rotation calculation model is to examine the rotational equilibrium at each crack of the beam, approximating the direction of the crack separately at each point of the span through the empirical equation developed on the basis of test results. The direction of the crack is approximated as a function of the shear force and the bending moment at the point in question and of structural parameters, such as the reinforcement and the shape of the concrete cross section.

The rotation model leads to the uniform theory for pure shear, combined shear and bending and pure bending and can thus be used all over the span.

3. THE DIRECTION OF THE CRACK

In the extended tests of Leonhardt /2/ it has been observed that the direction of the crack depends on the shear reinforcement ratio, which is defined as 1 in the case of classical shear reinforcement design and on the ratio between the breadths of the concrete cross section at the flange and at the web. The crack direction does not depend on the amount of the longitudinal tension reinforcement of the beam.

On the basis of Leonhardt's test results the following equation for the crack direction α_o at the edge of the support is developed in our investigation:

$$\alpha_o = 20^o + \frac{l_{eff}}{l_w} 6^o + \eta \cdot 10^o \leq 45^o \qquad (1)$$

The variable η can be calculated using the equation:

$$\eta = \frac{a_{sv} \cdot z \cdot f_{yd}}{Q} \qquad (2)$$

The crack direction at a point distance y from the support edge is calculated using the equation

$$\alpha_y = \alpha_o + (1 - \frac{Q_y}{Q_o}) \cdot (90^o - \alpha_o) \qquad (3)$$

Using the equation (3) the crack direction angle varies between α_o and 90^o, when the shear force varies between Q_o and 0. The value $\alpha_y = \alpha_o$ presents pure shear and the value $\alpha_y = 90^o$ pure bending. We can see that the variation of the direction angle corresponds to the mode of failure of the beam presented in figure 1.

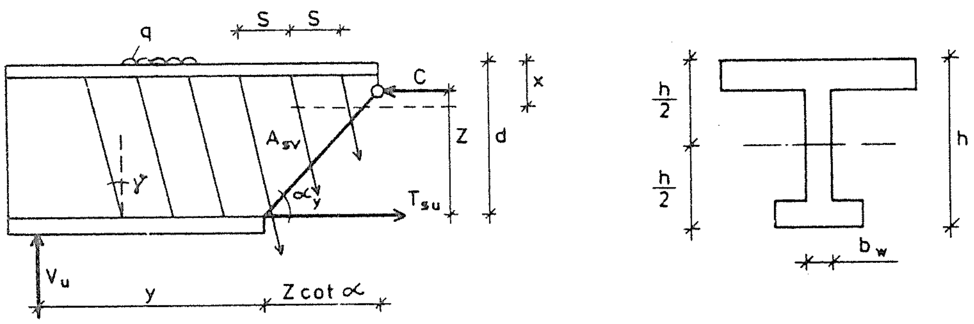

Figure 2. Forces and notations at the inclined cross section.

4. EQUILIBRIUM EQUATIONS

The loading capacity of the beam or one-way slab can be calculated at the cross sections in the cracking direction at each point of the span. The capacity depends on the cracking rotation capacity of the cross section and the ultimate rotation moment capacity of the reinforcement. The rotation capacities can be practically calculated using the intersection point of the crack direction and of the compression force resultant as rotation centre (point C in figure 2). The shear capacity is thus

$$V_u = \max \left\{ \frac{M_F \, (y + z \cot\alpha_y) + M_{rdy}}{y + z \cot\alpha_y}, \frac{M_F \, (y + z \cot\alpha_y) + M_{sdy}}{y + z \cot\alpha_y} \right\} \quad (4)$$

The shear crack usually begins at the tension side perpendicularly to the tension reinforcement. Therefore, the cracking capacity usually can be calculated in the direction perpendicular to the tension reinforcement.

The principle of equation (4) is presented schematically in figure 3. The cracking cpacity can be approximately calculated, for example, in the case of prestressed rectangular cross section using the equation

$$M_{rdy} = \frac{bh^2}{6} \cdot f_{ctd} + A_{sp} \cdot \sigma_{pdy} \cdot (d - \frac{h}{3}) \quad (5a)$$

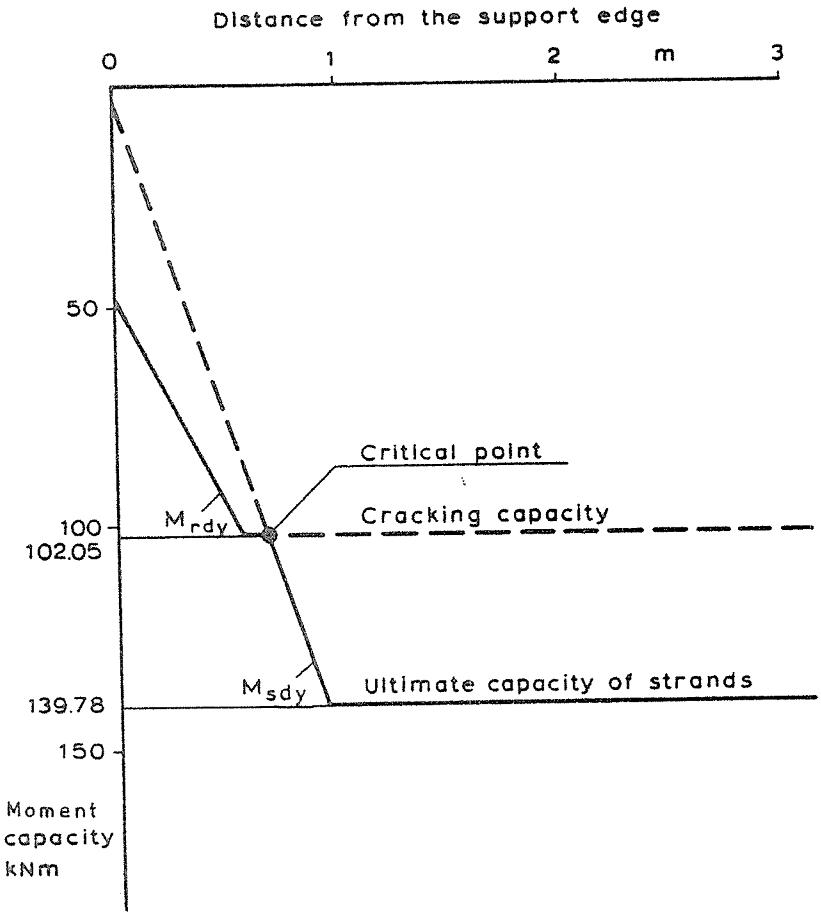

Figure 3. The principle of the determination of shear capacity.

In hollow core slab the cracking capacity can be calculated with the equation

$$M_{rdy} = W_c \cdot f_{ctd} + A_{sp} \, \sigma_{pdy} \cdot \left(\frac{1}{A_c} + \frac{d - \frac{h}{2}}{W_c}\right) \quad (5b)$$

The moment capacity of the cracked cross section is determined by the yield strength or tensile strength of reinforcement or by compressive capacity of concrete above the crack.

In the former case moment capacity could be calculated by equations

$$M_{sdy} = z \cdot T_{sd} + M_{svd} \leq M_{cdy} \tag{6}$$

$$T_{sd} = k_b \cdot u_s \cdot (y + f) \cdot f_{ctd} \leq A_s \cdot f_{yd} \tag{7}$$

$$M_{svd} = \frac{A_{sv}}{s} \cdot f_{yd} \cdot (d - x) \cdot (\tan \gamma + \cot \alpha_y)^2 \cdot (z - \frac{d - x}{2}) \cdot \cos \gamma \tag{8}$$

In equation, (8) it is assumed that all stirrups are yielding in the crack in the failure state.

In the latter case, when the compressive capacity of concrete is determinative, moment capacity could be calculated by equation (9)

$$M_{cdy} = z \cdot x \cdot b \cdot f_{cdy} \tag{9}$$

The determination of the height of the compressed zone of the cross section (x) is quite complicated, because the structure near the support does not behave in accordance with the bending theory.

For example, Ohja /7/ has suggested a method for the determination of deformation conditions near the support. The method by Ohja is being so improved at VTT that it can be applied to the rotation method.

Equation (6) is very suitable for the cases of prestressed structures and reinforced structures with heavily reinforced web. Equation (9) is applicable to reinforced structures without web reinforcement.

5. SHEAR CAPACITY CALCULATION

The method is rather complicated for manual calculation. Therefore, a computer program has been developed, which makes the calculations according to equations 1...9 presented above. The calculations are made at equally distributed cross sections from the support to the middle of the span. Near the support, in usual loading conditions the capacity is controlled mainly by shear, at greater distance by combined shear and bending moment and in the middle of the span by pure bending moment. All these cases can be calculated with the same equations 1...9.

According to the equation (4) the ultimate shear capacity is defined either by cracking moment capacity of the concrete section M_{ry} or by the moment capacity of the reinforcement M_{sy} as presented in figure 3.

Cracking moment capacity M_{ry} is not much higher near the support than the moment capacity of reinforcement M_{sy}, but it is necessary to take it into account in order to simulate the real behaviour of structures. When the capacity of the structure is controlled by the cracking moment, the type of failure is brittle. That is why an additional coefficient of security for the tensile strength of concrete is used.

The usual case in design is that we have to calculate the amount of shear reinforcement, usually stirrups. It can be made by modifying the equations 6...9 presented above. So we find in the case controlled by tension reinforcement the equation for the amount of stirrups needed in the form

$$\frac{A_{sv}}{s} = \frac{M_{ud} - T_{sud} \cdot z}{(\tan\gamma + \cot\alpha_y)^2 \cdot (d - x) \cdot (z - \frac{d - x}{2}) f_{yd} \cdot \cos\gamma} \qquad (10)$$

6. EXAMPLES OF APPLICATIONS

6.1 Hollow core slabs

The method has been applied to the calculation of shear capacity curves of prestressed hollow core slabs. An example of these curves is presented in figure 4.

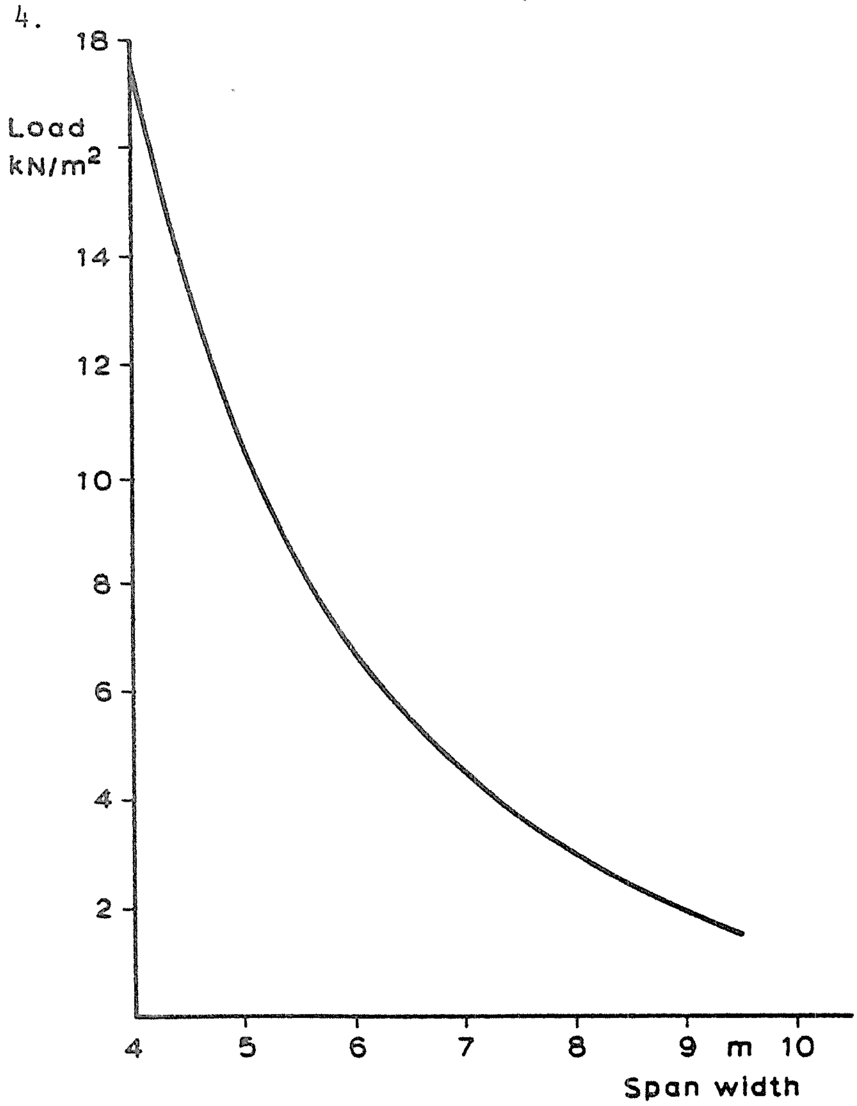

Figure 4. Example of a shear capacity curve for prestressed hollow core slabs

For the examination of the accuracy of the method the calculated capacities were compared with the shear test results of hollow core slabs in 34 cases. In these cases the mean difference between the calculated characteristic shear capacities and the test results was -12 % and the difference varied between -29 % to +15 %. The comparison of the test results and calculated values is presented graphically in figure 5. As the calculated values are characteristic values we can see that they correspond very well to the usual accuracy need.

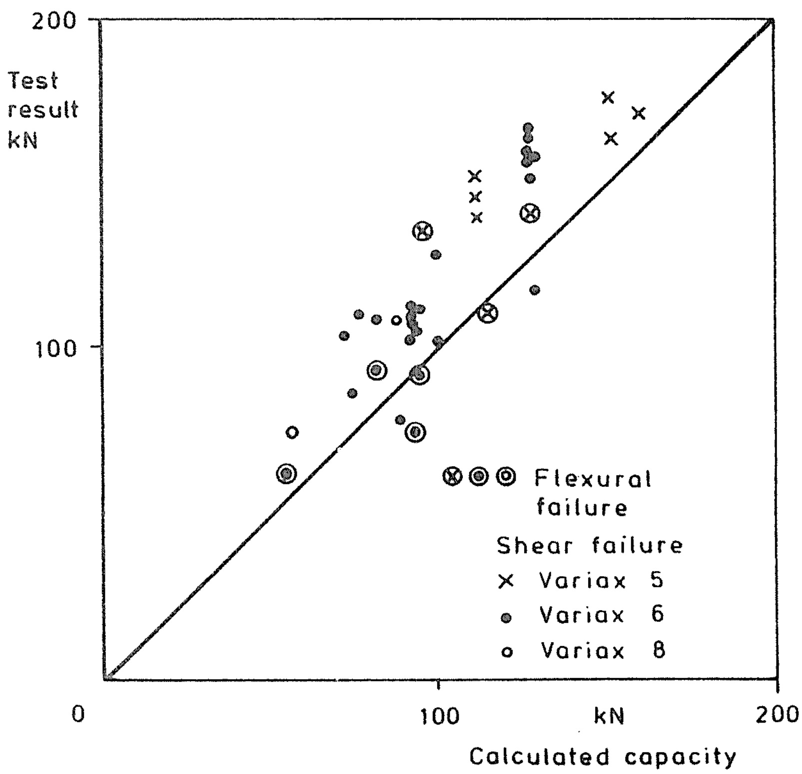

Figure 5. Comparison of the calculated shear capacities of prestressed hollow core slabs with the test results.

6.2 Prestressed I-beams

The results of the beam tests made by prof. Thürlimann /1/ were compared with calculated capacities in 12 cases. In these cases the mean difference between the calculated characteristic shear capacities and the test results was -3.6 % and varied between -13 % to +6 %. In these cases the accuracy was very good. The same kind of comparison was made with the results of the beam tests made by Leonhardt, Koch and Rostasy /3/. In these cases the mean difference between the calculated characteristic shear capacities and the test results was +5.0 % and varied between -3.5 % and +17.1 %. The comparisons are presented graphically in figure 6.

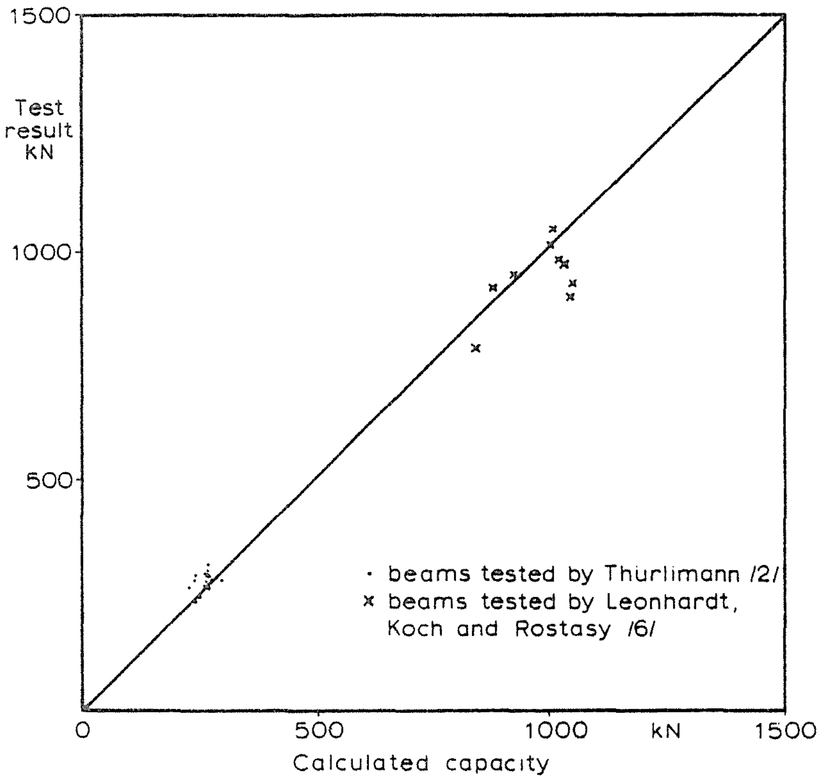

Figure 6. Comparison of the calculated shear capacities of prestressed beams with the test results.

7. PROGRAM FOR FURTHER RESEARCH

The main needs for further research in the development of this method are the following:

1° Application of the method to reinforced concrete structures with and without shear reinforcement.

2° Application of the method for continuous prestressed and reinforced structures. Here the cracking behaviour at the continuous supports must be studied.

3° The secondary modes of failure and the principles of their calculation methods. The secondary modes of failure are very much dependent on the structure and they can usually be prevented with suitable structural shape and reinforcement. The most common secondary modes of failure are web-crushing failure (Fig. 7a), longitudinal cracking of the structure avobe the level of reinforcement (Fig 7b) and the longitudinal cracking on the level of reinforcement (Fig. 7c). Here the main principle of the rotation model can also be applied in the calculations.

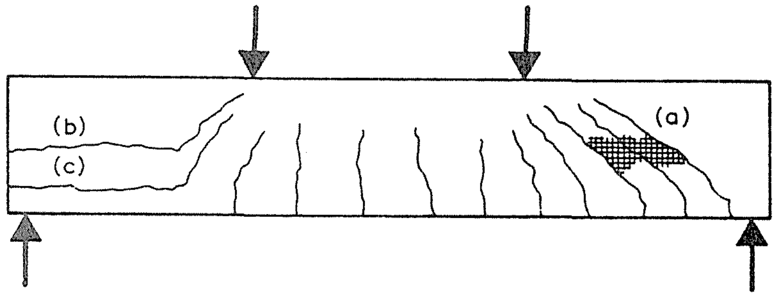

Figure 7. Secondary modes of failure
a) web-crushing failure
b) longitudinal cracking above the level of reinforcement
c) longitudinal cracking in the level of reinforcement.

LITERATURE

1. Caflisch, R. & Thürlimann, B., Schubversuche an teilweise vorgespannten Betonbalken. Zürich 1970. Eidgenössische Technische Hochschule Zürich, Bericht 6504-2. 177 p.

2. Leonhardt, F., Schub bei Stahlbeton und Spannbeton - Grundlagen der neueren Schubbemessung. Beton- und Stahlbetonbau 72 (1977) 11, p. 270-276 and 72 (1977) 12, p. 295-302.

3. Leonhardt, F., Koch, R. & Rostásy F., Schubversuche an Spannbetonträgern. Berlin 1973. Deutscher Ausschuss für Stahlbeton, Heft 227. 179 p.

4. Mörsch, E., Der Eisenbetonbau, seine Theorie und Anwendung. Stuttgart, Verlag K. Wittwer, 1923. 490 p.

5. Regan, P., Shear in reinforced concrete. London 1971. CIRIA (Construction Industry Research and Information Association), Report.

6. Walther, R., Über die Berechnung der Schubtragfähigkeit von Stahl- und Spannbetonbalken. Schubtheorie. Beton- und Stahlbetonbau 57 (1962) 11, p. 261-271.

7. Ohja, S., K., The shear strength of rectangular reinforced and prestressed concrete beams. Magazine of Concrete Research 19 (1967) 60, p. 173-184.

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The shear stress at the web must also be limited to the value of the design shear stress of plain concrete in such cases when the web is without shear reinforcement as in the case, for example, of the hollow core slabs.

The usual case in design is that we have to calculate the amount of shear reinforcement, usually stirrups. It can be made by modifying the equations 6...9 presented above. So we find in the case controlled by tension reinforcement the equation for the amount of stirrups needed in the form

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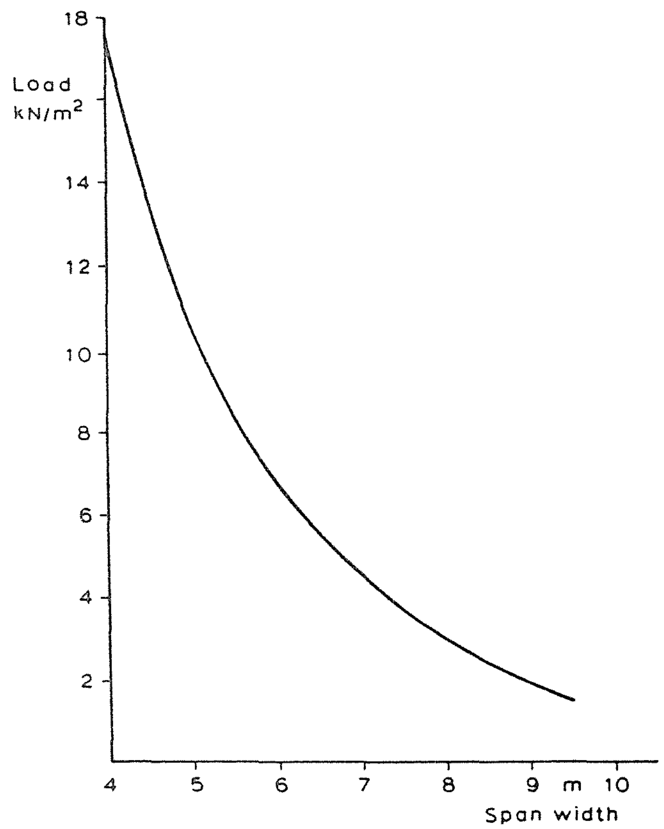

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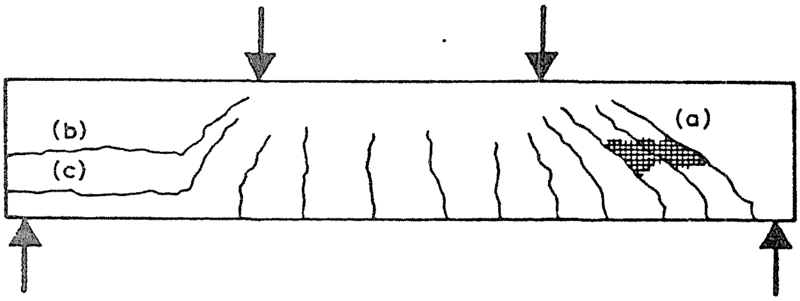

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7. Ohja, S., K., The shear strength of rectangular reinforced and prestressed concrete beams. Magazine of Concrete Research 19 (1967) 60, p. 173-184.

8. Sarja, A. & Nykyri, P., Rotational model for the determination of the shear capacity of reinforced and prestressed concrete structures. Espoo 1982. Technical Research Centre of Finland. Research Reports 76. 19 p.